FINAL TECHNICAL REPORT
MATHEMATICS OF
GEODETIC SECOR DATA PROCESSING

FTR/71-2

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Placed by

U. S. Army Engineer Geodesy, Intelligence
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Fort Belvoir, Virginia

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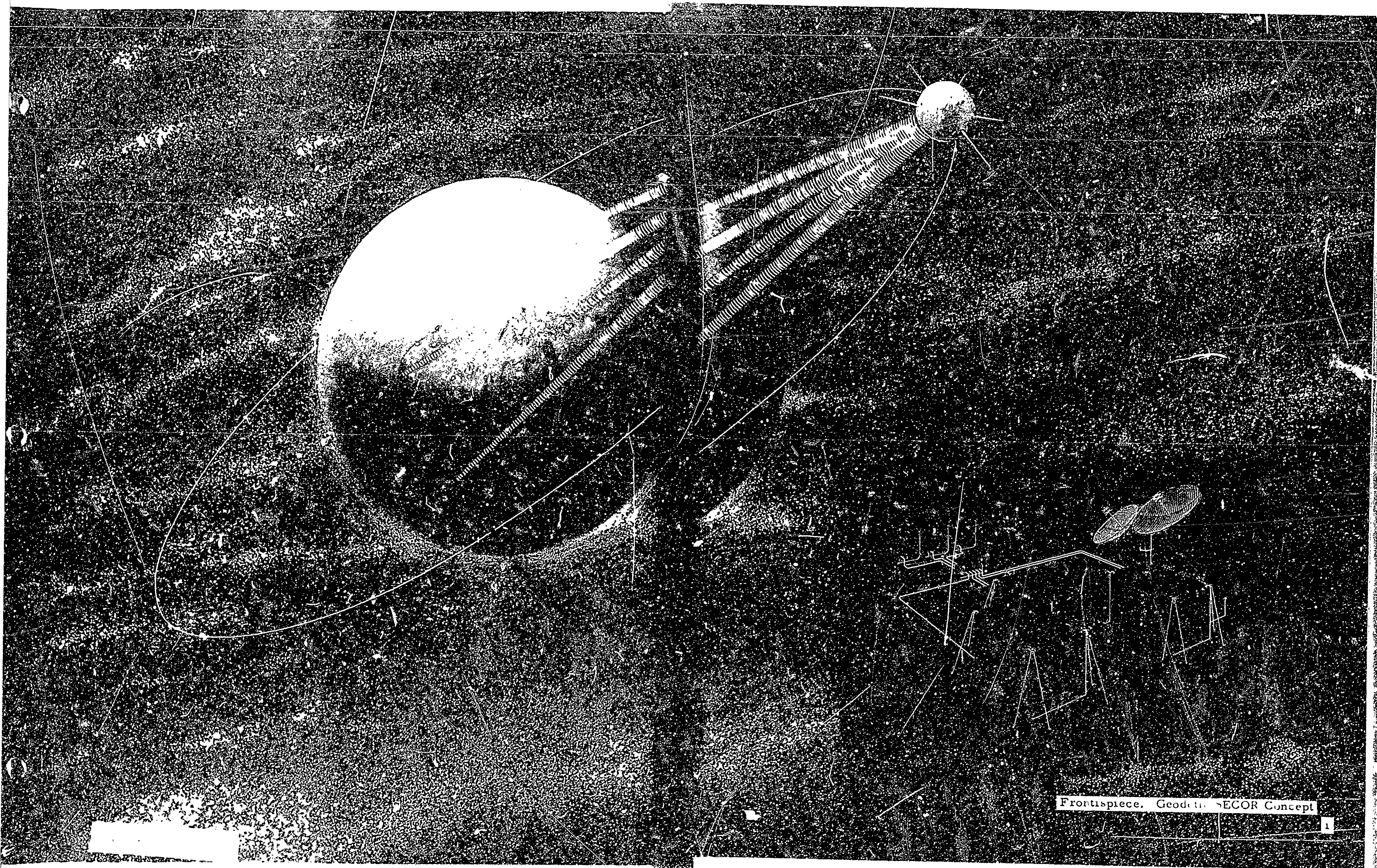

Frontispiece. Geodetic SECOR Concept

FOREWORD

Cubic Corporation developed the mathematics and methods for processing Geodetic SECOR USA-2 satellite tracking data obtained during the equipment test-service test (ET/ST). The ET/ST commenced with the USA-2 satellite launch in January 1964, and continued through May 1964. This report contains the mathematics, and a general discussion of the methods employed and results obtained in processing Geodetic SECOR USA-2 satellite tracking data. The report is prepared in compliance with the requirements of Department of the Army Contract DA-49-018-ENG-2390, Modification 24, Addition II to Exhibit A, paragraph 1d.

Cubic Corporation was the prime contractor, responsible for the implementation of all contract provisions. All work was administered under the supervision of the U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency (GIMRADA), Fort Belvoir, Virginia.

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SECTION I
SUMMARY

1.1 Introduction. This report contains a discussion of the data processing techniques employed by Cubic Corporation in the reduction and analysis of Geodetic SECOR tracking data. The data processed was taken during the period from 15 January 1964 to 24 April 1964 using the transponder aboard the USA-2 satellite. Several ground station configurations and tracking modes were used during this period, and many of the possible types of solution were performed with the data.

In the main text, the data processing techniques themselves are discussed, together with a summary of some of the results obtained. Recommendations for further processing techniques, and for modifications to the existing techniques are included. Details concerning the design or operation of the Geodetic SECOR system are not part of this document. Refer to Cubic engineering reports for the design characteristics of SECOR equipment.

To permit familiarization of the reader with the over-all processing techniques without becoming overburdened with mathematical detail, the mathematical discussions and operational procedures have been incorporated as appendices. Supplementing the report are two copies of the program listings given in Appendix T and one copy of the computer programs on punched cards. The programs, which include many general purpose subroutines developed in conjunction with this and other projects, have been extensively tested and refined to provide both accuracy and speed. Appendix A gives the constants, units, rotations, and translations used in the text.

For additional information concerning the results of the Geodetic SECOR data processing and analysis, refer to the following Cubic Corporation reports:

Geodetic SECOR Simulation Study, Satellite USA-2, Cubic Document ES/71-2, June 1964

Geodetic SECOR Data Processing Summary, USA-2 Satellite Orbits 463-1448, Cubic Document SR/71-1

Geodetic SECOR Range Accuracy Study

Geodetic SECOR Maximum Ranging Capability Study

1.2 Purpose of the Processing. Data processing of the Geodetic SECOR data (satellite USA-2) by Cubic Corporation was designed (1) to provide a rapid check of the system operation, (2) to provide an indication of the quality of the range data, and (3) to evaluate the data processing techniques employed with data from the various modes of system operation.

1.3 Modes of Operation. The Geodetic SECOR system operates in either the simultaneous mode or the orbital mode. For either mode, the data obtained can be used in one or more types of solution as described in the following paragraphs.

1.3.1 Simultaneous Mode. In the simultaneous mode, all four trackers take simultaneous range data over the same portion of one or more satellite passes. Thus, only the portions of the satellite orbit which are line-of-sight with all four trackers can be used in simultaneous mode operation.

1.3.1.1 3-3 CORDEX Solution with Simultaneous Mode Data. In the simultaneous mode 3-3 CORDEX (COoRDinates X, the unknown station) solution, the range measurements made over two or three satellite passes by three known sites and the CORDEX site are used to determine the position of the CORDEX site. (See figure 1-1.)

1.3.1.2 3-2 CORDEX Solution. The 3-2 CORDEX solution is similar to the 3-3 CORDEX solution, except that the height of the CORDEX station is assumed to be known and is constrained in the solution. In this solution, the CORDEX site may be located using only one orbital pass as shown in figure 1-2. This solution is advantageous where good geometry may not be obtained for a 3-3 CORDEX solution, or where the height of the CORDEX site has been well-established by other means.

1.3.1.3 Line Crossing Computation. The line crossing computation is used to determine the distance along a reference spheroid (i.e., the geodesic) between the CORDEX site and one or more of the known sites. (See figure 1-3.) These line length measurements may be used in a network adjustment program to provide a check on the other CORDEX solutions, or even to furnish a solution for the CORDEX site relative to some assumed spheroid. In the line crossing solution, the three known sites are used to establish the satellite's height while the simultaneous ranges taken from the ends of the baseline provide the primary control of the line length.

1.3.2 Orbital Mode. The orbital mode differs from the simultaneous mode in that the CORDEX site measures ranges to the satellite either before or after the satellite is line-of-sight with the three known sites. The positions of the satellite corresponding to the CORDEX site range measurements are determined by orbital prediction using orbital elements determined from the simultaneous range data taken by the three known stations.

By using the orbital mode, CORDEX sites much farther from the established survey grid can be located since the requirement for simultaneous line-of-sight conditions is eliminated. The results obtained using the orbital data indicate the presence of one or more sources of error which may include base site survey, range calibration, ionospheric refraction correction bias,

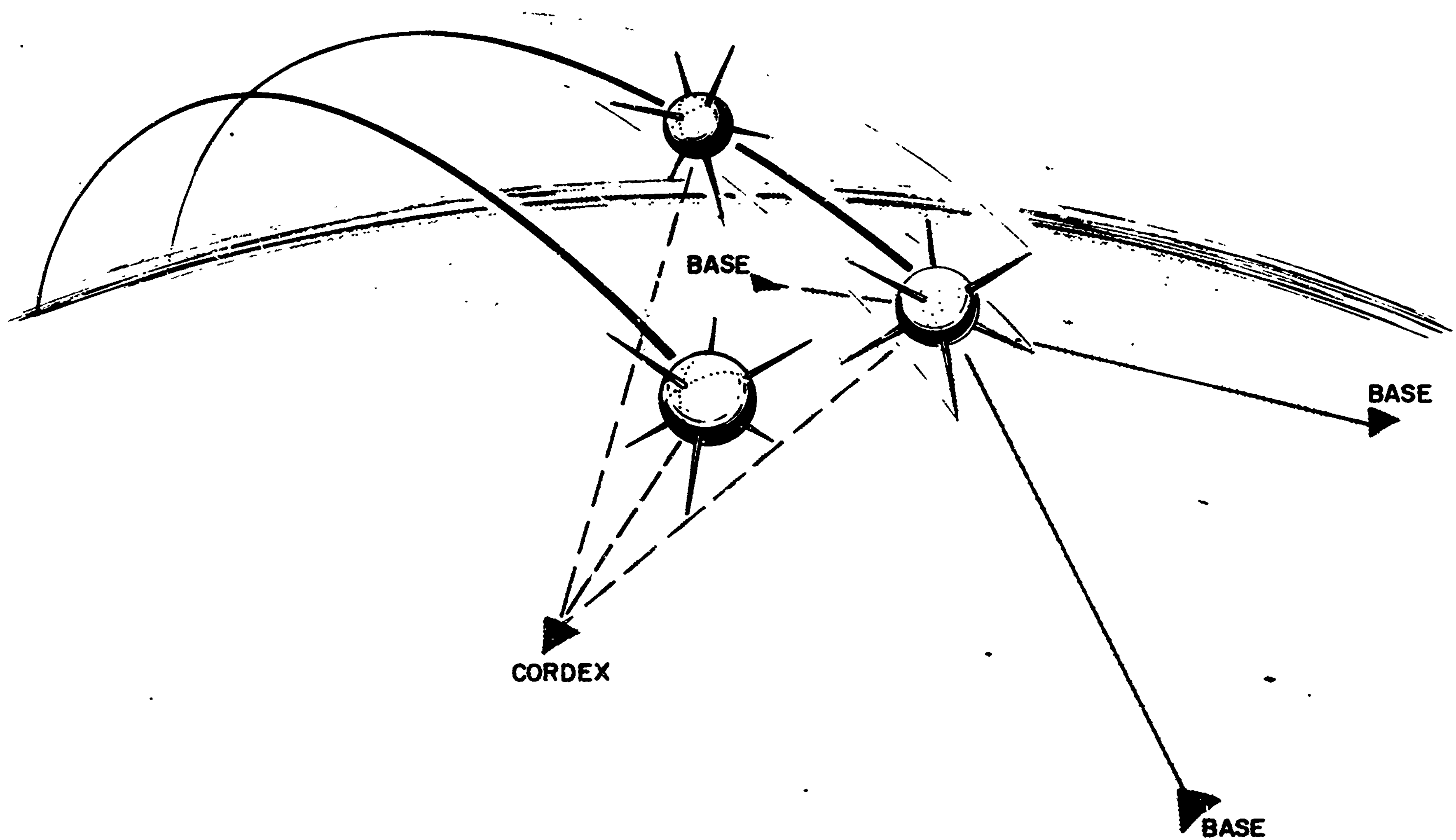

Figure 1-1. 3-3 CORDEX Solution

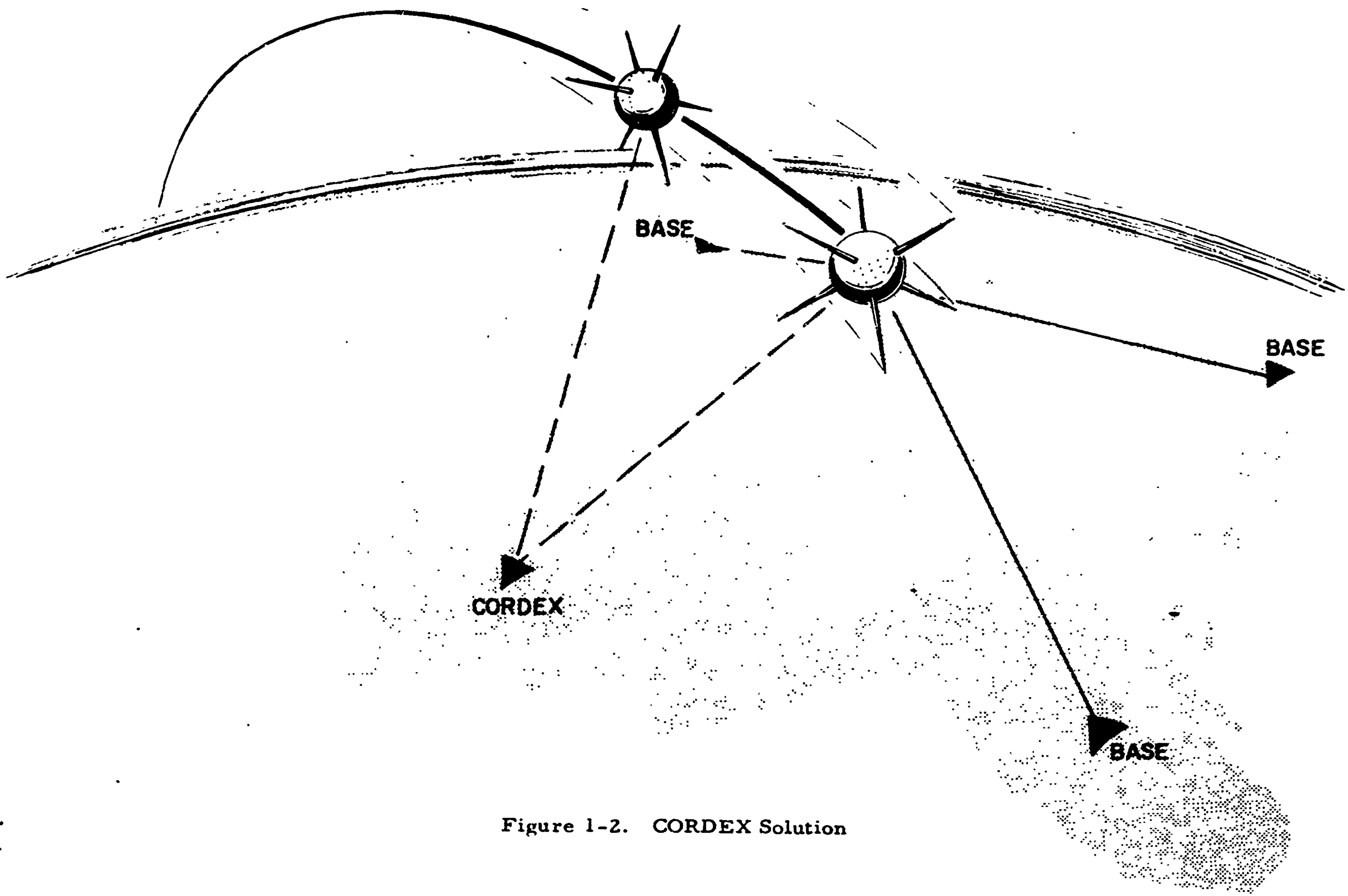

Figure 1-2. CORDEX Solution

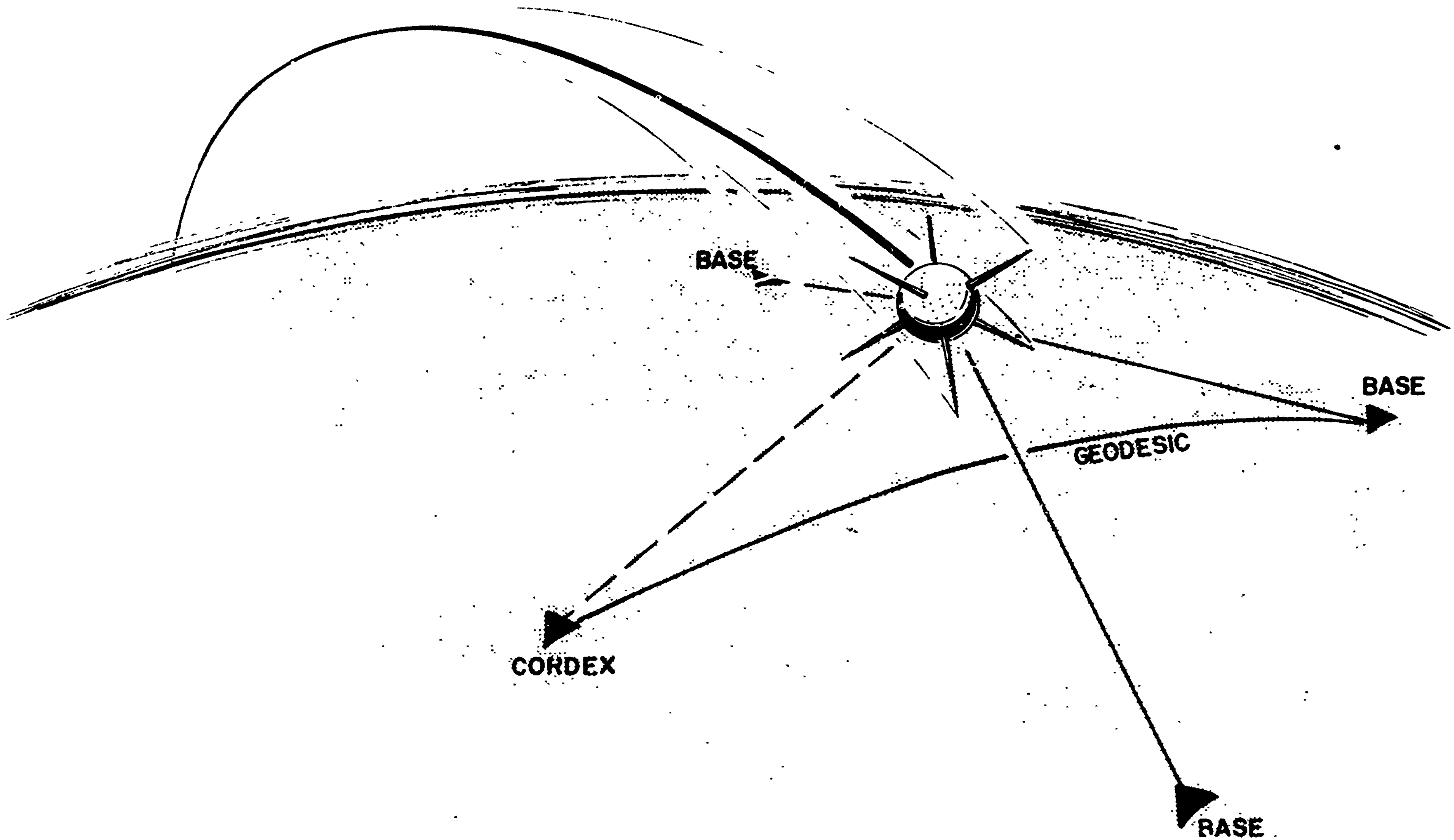

Figure 1-3. Line Crossing Mode

and orbital prediction techniques. It is recommended that further investigation be undertaken to identify the sources of bias, and to eliminate them through extended solutions which solve for the residual biases. Also, orbital prediction techniques utilizing longer fitting spans (or even multiple orbit fitting) should be attempted to improve the long range prediction accuracy.

Only the 3-3 CORDEX solution as described in the following paragraph was performed with the orbital mode data.

1.3.2.1 3-3 CORDEX Solution with Orbital Mode Data. This solution is similar to that performed with the simultaneous mode data, except that satellite positions are determined from orbital prediction instead of from direct measurement.

1.4 Data Processing Facilities. Data processing was accomplished at the computer center of the University of California at San Diego (UCSD) on a Control Data 1604 computer system. The CDC 1604 computer has a core memory of 32,000 48-bit words, and an average cycle time of 4 μsec. The peripheral equipment includes a 160A satellite computer, twenty magnetic tape units, a card reader and punch, and a 1000-line-per-minute printer. In addition, off-line key punches, a card lister, duplicator, and interpreter are available for those using the computer.

The data processing programs were written in FORTRAN 63 and in CODAP assembly language. Operational descriptions of these programs are included as Appendix T, and sample listings constitute Appendix S of this report.

SECTION II
COMPUTATIONAL PROCEDURE

2.1 Introduction. Most of the data processing programs were written prior to the launching of satellite USA-2. Since the data in this report were obtained in the first truly operational test of the system, the various processing steps were set up to run on separate computer passes to allow inspection of the intermediate results before further processing steps were attempted. The results obtained during each computer pass were listed and recorded on magnetic tape for use in the subsequent processing steps. Figure 2-1 is the over-all data processing flow diagram. Each box in the diagram indicates a computer pass, and the arrows (unless otherwise indicated) designate the magnetic tape reels used. The tape reels, except for the original raw tapes, were identified with a letter, orbit number, and station number for processing purposes. For example, R 132.1 is the raw tape from orbit 132, station 1. The letter designations of the various tapes are in parentheses next to the arrows.

In the following paragraphs, the processing steps accomplished during each computer pass (as shown in figure 2-1) are discussed.

2.2 Copy Raw Tapes. The magnetic tapes recorded at each tracking site were forwarded to Cubic Corporation through the GIMRADA representative. The tapes were copied and the originals were returned to the GIMRADA representative for shipment to Army Map Service (AMS). Because the end-of-record gap on the original raw tapes was not sufficiently long for use by the CDC 1604 computer, copying the tapes could not be accomplished directly on the computer. The original tapes had an end-of-record gap of only 5/8-inch, whereas the 1604 computer system tape units stop at the end of each physical record and require a 3/4-inch end-of-record gap. Therefore, the tapes were copied on the CDC 160A satellite computer with a CDC 163-2 tape unit which reads continuously and requires a shorter inter-record gap. The tapes output by this program (R tapes) were compatible with the 1604 tape units, and were used for the subsequent processing steps.

Some delay in processing time was experienced as a result of this procedure because the 160A computer is not generally available for use except as an integral part of the 1604 computer system. The time which would be saved in processing would probably justify an investigation into a tape format change to make the magnetic tapes compatible with standard tape units.

2.3 Raw Data Listing. Each raw tape was listed as a preliminary check of data quality and as a means of locating regions of usable data. The listing (program EXAM1) involved unpacking the raw tape data format (subroutine FORMAT), and converting it into a more convenient format; resolving the ranges (subroutine RESOLVE); and listing this information. Copies of the raw data listing were forwarded to the GIMRADA representative.

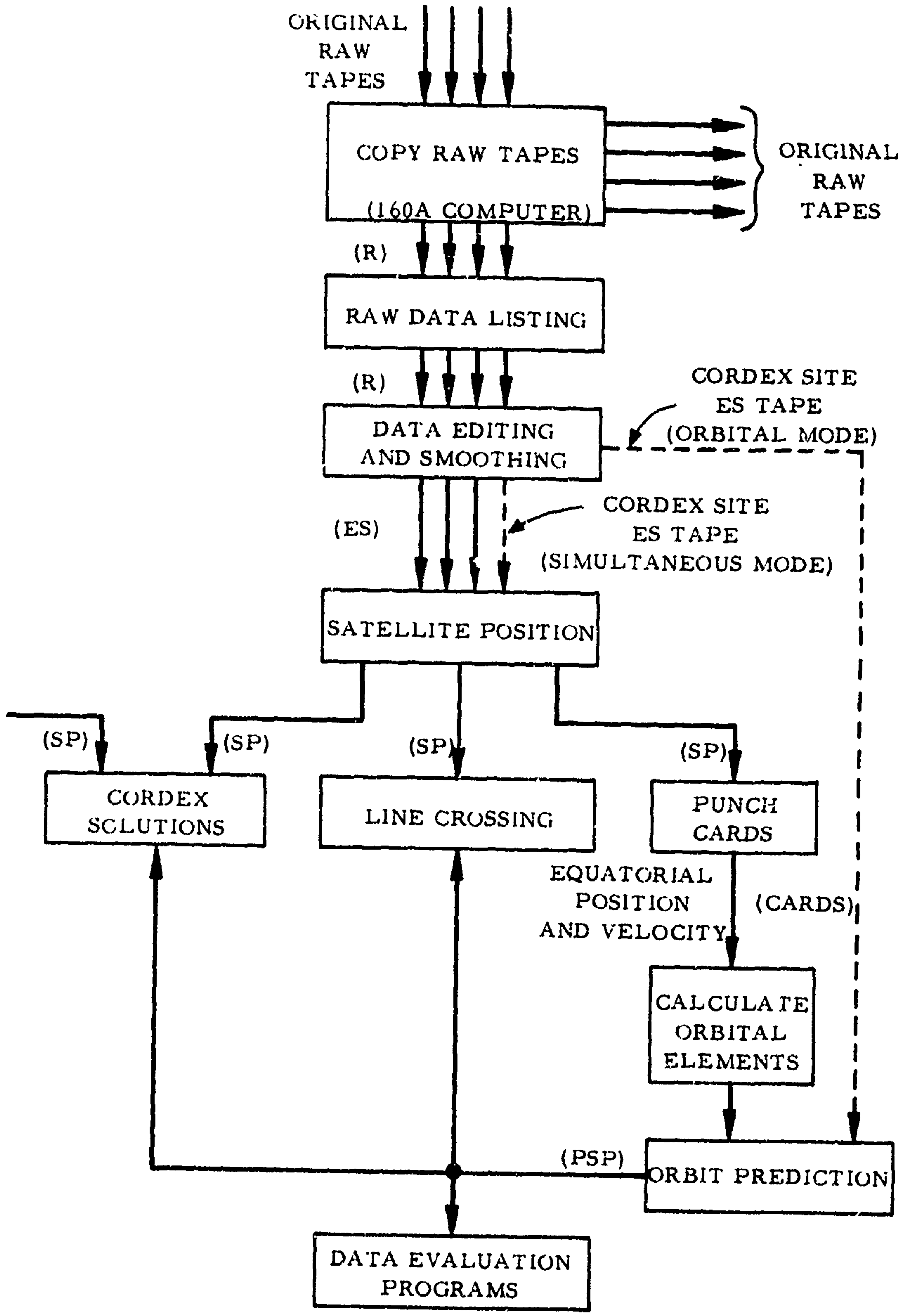

Figure 2-1. Data Processing Flow Diagram

The method used to resolve the ranges is discussed in Appendix B, and the sample raw data listings (as previously noted) appear in Appendix S.

The raw data listings were made available within 24 hours of the receipt of the original raw tapes by Cubic Corporation. This turn-around time was continued throughout most of the data processing, and provided valuable assistance both to the GIMRADA representatives and to the Cubic field engineers.

2.4 Data Editing and Smoothing. During the data editing and smoothing pass (program PASS2), the raw tape from each tracking site was read, and the following operations performed:

(1) calibration constants were applied to the raw range and to the measured ionospheric correction (IC),

(2) the raw ranges (plus calibration) were edited for ambiguities and spurious bad samples,

(3) the edited ranges were smoothed using a least squares moving span filter,

(4) the range rate and range acceleration were determined as a by-product of the range-smoothing process,

(5) the smoothing residuals (differences between the smoothed and edited ranges) were calculated and used to determine a standard deviation (rms error) for each block of 132 samples.

The information from step (5) was used in the subsequent data evaluation. The editing and smoothing data were generally available within two days of the receipt of the original raw tapes.

2.4.1 Calibration Constants. The calibration constants to be applied to the data were calculated for each orbit in the field and forwarded to Cubic Corporation through the GIMRADA representative. These calibration constants included one for the very fine (VF) channel and one for the very fine ionospheric correction (VFIC) channel. These constants were input on cards during the data editing and smoothing pass (program PASS2).

The final calibration constants represented an estimate of the phase shift within the station-satellite loop other than that phase shift attributable to the range. In practice, these phase shifts were measured in four steps:

(1) Each station measured the range to a test transponder (TT) over a known distance (cable phase shifts included) and noted the offset ($\varphi_{STA} + \varphi_{TT}$).

(2) The test transponder of the station was compared with the transponder calibration unit (TCU) and the difference was noted ($\varphi_{TT} + \varphi_{TCU}$).

(3) The transponder calibration unit was compared with the satellite transponder ($\varphi_{TCU} + \varphi_{SAT}$).

(4) The phase delay from the satellite antenna to the satellite transponder was measured ($\varphi_{SAT\ ANT}$).

The final calibration constant was found from:

$$\varphi_{CALIB} = (\varphi_{STA} + \varphi_{TT}) - (\varphi_{TT} + \varphi_{TCU}) + (\varphi_{TCU} + \varphi_{SAT}) + \varphi_{SAT\ ANT}$$

2.4.2 Data Editing. The data editing process removed ambiguous and spurious bad samples from the raw range data. The editing was accomplished by comparing each first difference with a first difference predicted from previously edited ranges. Where the agreement was within the noise tolerance (25 meters), the corresponding range was passed unchanged. If the agreement was within the noise tolerance of an integral number of 256-meter ambiguities, the total ambiguity was removed from the output range. Where neither of these conditions existed, the sample was considered to be a spurious bad sample, and it was replaced by an extrapolated range value. Since extrapolation depends on the use of 'good' samples, only five successive bad samples were allowed before a search for a new starting span was initiated.

The process of editing based on first differences requires that the editing process begin in a region of nonambiguous data. In order to find such a region, the average second difference of a span of five samples (the starting span) was computed and compared with a predetermined maximum value (10 meters/(0.1 sec)^2). If the maximum value were not exceeded, editing commenced; if the maximum value were exceeded, the next five samples were examined. A detailed description of this data editing technique, together with a flow diagram of the process, is included as Appendix C.

The edited Geodetic SECOR range data exhibited two types of ambiguity, sporadic and consistent. The sporadic ambiguous samples occurring in 3 to 5 per cent of the ranges posed no problem, and they were completely eliminated from the data. The consistent ambiguities caused a constant offset of the data for an extended interval. These ambiguities sometimes resulted in an offset of the entire span of edited range data if the editing procedure started in an ambiguous span of data. These ambiguities were generally in the extended range (524,288 meters) and resulted in an offset which was readily recognized from an approximate knowledge of the orbit, or by examining the permuted satellite positions. When such an offset occurred, it was removed during the satellite position calculation by applying the offset as a calibration constant.

An example of the edited range data is shown in figure 2-2. In this data sample, the editing process was started in a region of consistently ambiguous data, resulting in an offset of +524,288 meters in the edited ranges. The presence of the offset is obvious from the predicted range data supplied by the NASA Goddard Space Flight Center, and it was removed as a calibration offset in subsequent processing steps. The figure also illustrates one spurious ambiguity in the fifth sample; this was removed during the editing process.

2.4.3 Data Smoothing. The edited range data were smoothed to reduce the random noise content of the data. As a byproduct of the smoothing process, the range rate and the range acceleration were also determined. The smoothing filter used was a twenty-five, second degree, midpoint filter. This filter effectively fits a second degree polynomial to a span of twenty-five ranges, and from this polynomial a smoothed thirteenth range is calculated. By successively shifting the range data and repeating the process, a series of smoothed ranges were determined. The range rate and the range acceleration were then determined using the time derivative of the polynomial. This type of filter is discussed in more detail in Appendix D where plots indicate the theoretical noise reduction and frequency response.

The differences between the edited and smoothed ranges (the smoothing residuals) were calculated and output as an indication of the data quality. A plot of a typical set of residuals is shown in figure 2-3. The horizontal dashed line in this figure indicates the probable error of a single observation based on these smoothing residuals (0.26 meter).

A more comprehensive understanding of the noise removed by the smoothing process may be gained by examining the frequency distribution of the smoothing residuals. A typical spectral density plot of the residuals is shown in figure 2-4. This plot is normalized in such a way that the area under the curve is equal to the sample variance of the residuals. The tapering off of the curve at low frequencies must be attributed to a combination of the spectral characteristics of the noise and the filter response.

2.5 Satellite Position. The satellite position program (program PASS3) time-synched either three or four ES tapes (edited and smoothed data tapes), and produced an output tape consisting of the station data plus the computed satellite position. During this computer pass the ranges were corrected for constant offsets, tropospheric refraction, ionospheric refraction, and transit time. These corrected ranges along with the range rates were then used to compute the final position of the satellite. In addition, when simultaneous mode data were used, an internal range consistency check was computed.

2.5.1 Calculation of Satellite Position and Velocity. The calculation of satellite position and velocity was performed twice. The first solution was performed with the smoothed ranges from the ES tapes plus the correction for constant offsets. This initial solution was used to calculate

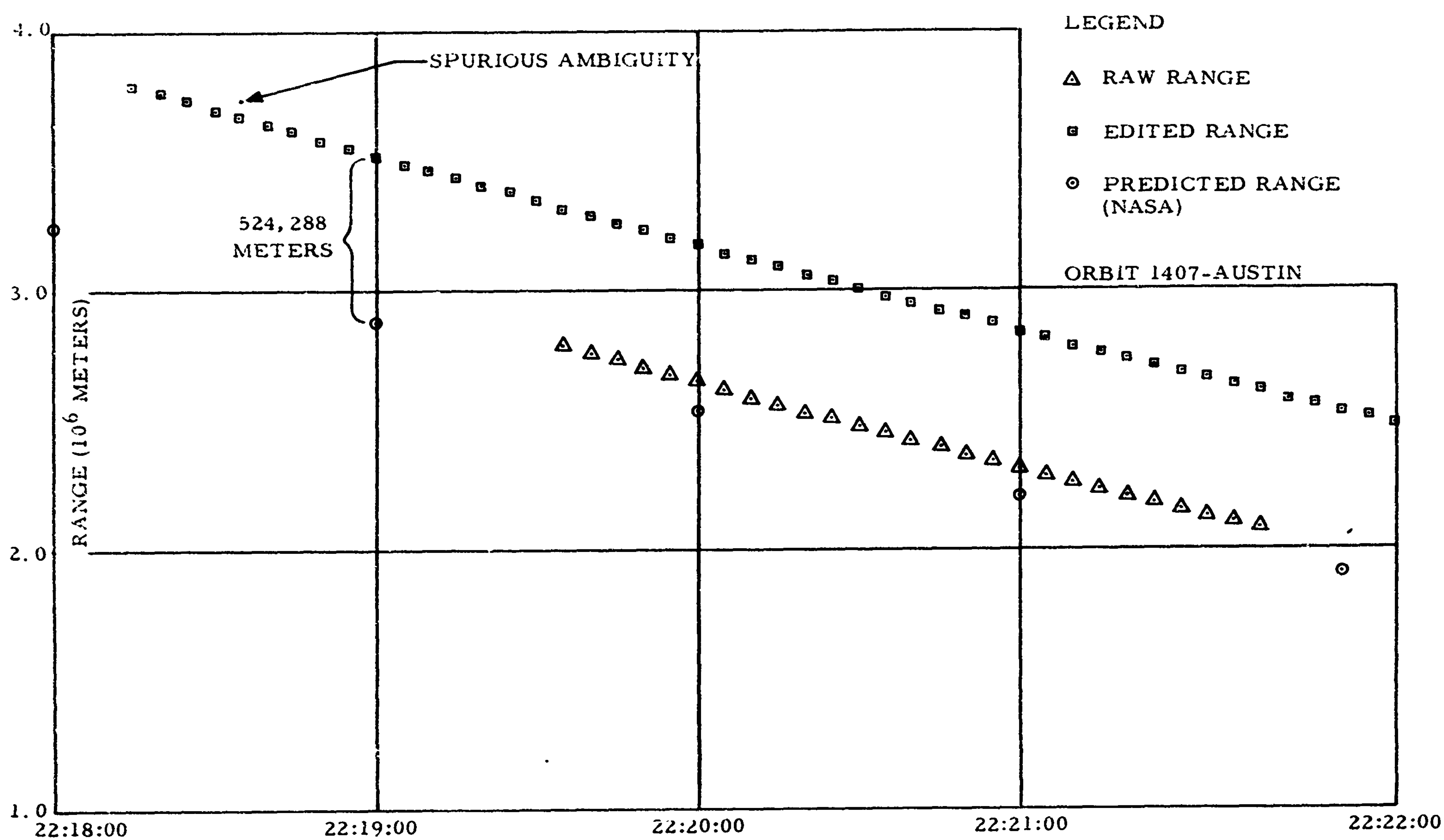

Figure 2-2. Geodetic SECOR Edited Data

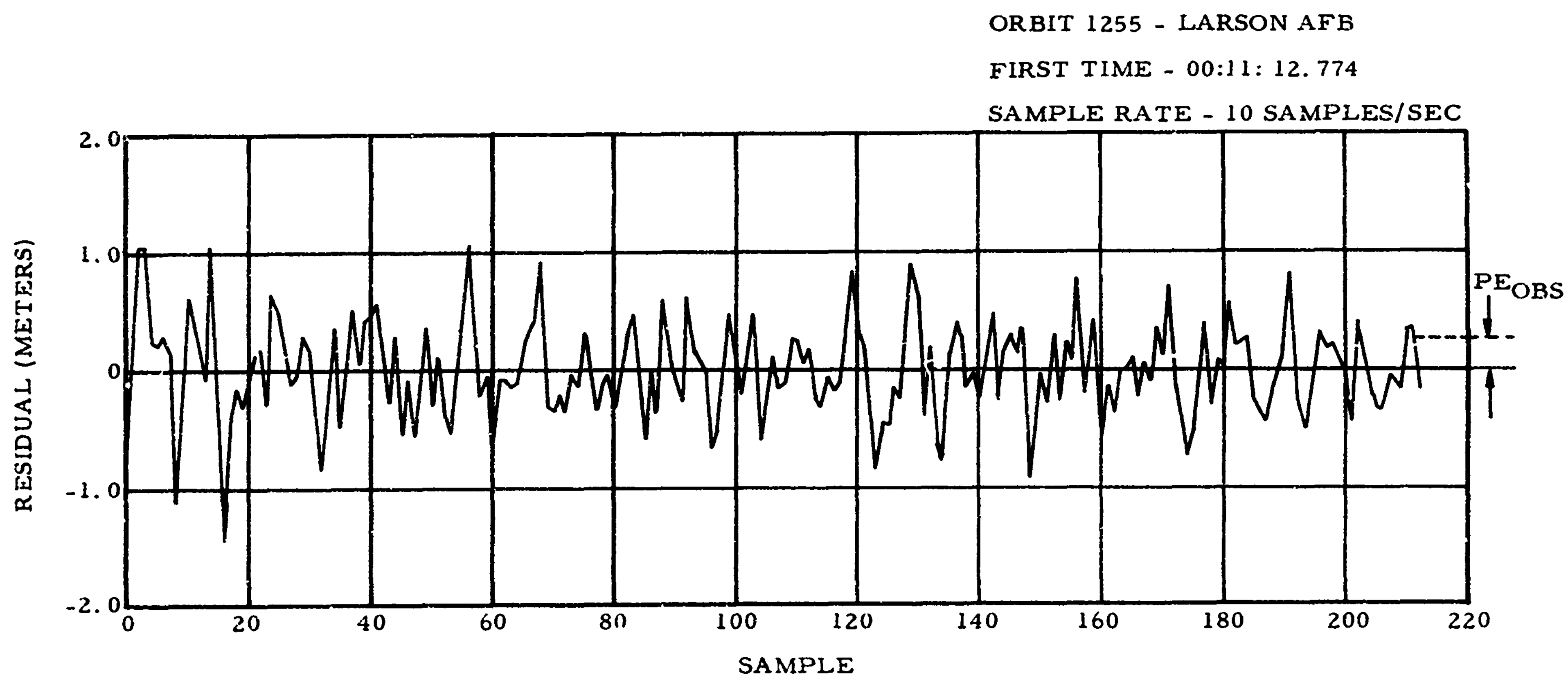

Figure 2-3. Range Smoothing Residuals

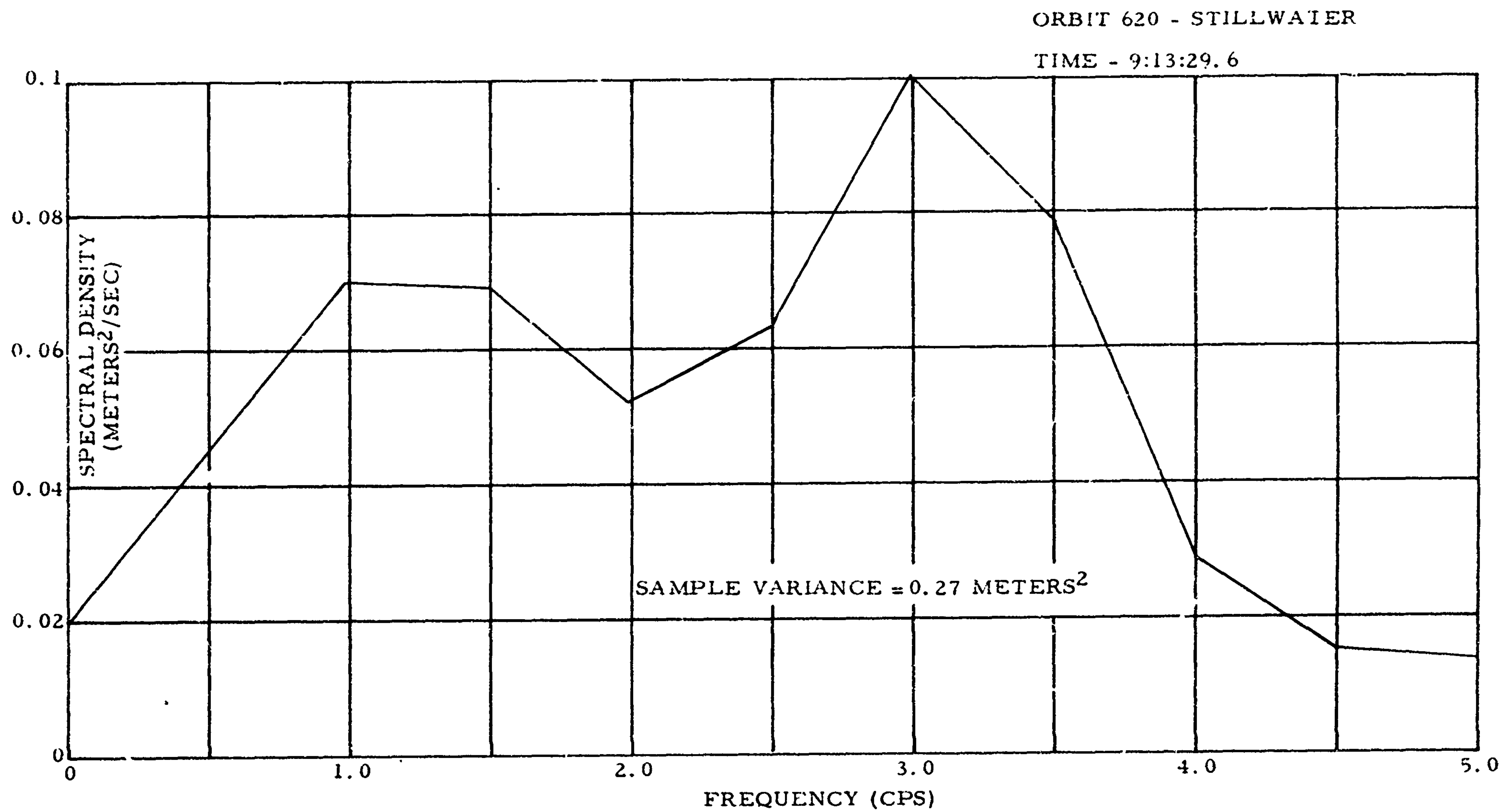

Figure 2-4. Spectral Density VS Frequency

the range, elevation angle, and the range rate of the satellite as observed at each tracking site. These parameters were then used (as described in the next paragraph) to compute the various range corrections. From the corrected ranges, the final satellite position was computed.

The mathematics for the calculation of satellite position using the simultaneous ranges measured at the three known stations is derived in Appendix E. This solution may be geometrically interpreted as the intersection of the three spheres, with the radii defined by the three ranges and centered at the three tracking sites.

The satellite velocity was determined using the simultaneous ranges and range rates from the three known sites and the set of linear equations derived in Appendix F.

2.5.2 Range Corrections.

2.5.2.1 Correction for Constant Range Offset. Since the data editing, at times, produced ranges which were offset by a constant ambiguity, provision was made to apply a calibration constant to each range during the satellite position calculation. The set of range corrections (if any) was input on cards by the PASS3 program and applied to the input ranges before any computations were made.

2.5.2.2 Tropospheric Refraction Correction. The tropospheric refraction range correction was made using the analytic model (subroutine REF) discussed in Appendix G. The correction was computed using the range and elevation angle at each site determined from the initial satellite position computation as the model parameters.

2.5.2.3 Ionospheric Refraction Correction. The ionospheric correction was made using the analytic model for the ionospheric correction (subroutine IONCR) described in Appendix H except for one orbit (1365) where the measured IC was directly applied to the data. (Refer to Appendix I.) The use of the analytic model was adopted because of the relatively high noise content of the measured IC compared to the measured very fine channel, and the consistent loss of IC lock at the Austin site.

In addition to the range and elevation angle, the analytic model for the ionospheric correction used the maximum electron density of the F2 layer and a slope constant, K2. These parameters were determined by a least squares adjustment to the measured IC values from all sites (program IONITR). Figure 2-5 shows the distribution of the maximum electron density plotted as a function of local time. These results indicate a probable residual error of about 20 per cent of the total ionospheric correction, which normally represents about five meters range error.

2.5.2.4 Transit Time Correction. The transit time correction (Appendix J) was applied to the ranges (program PASS3) to

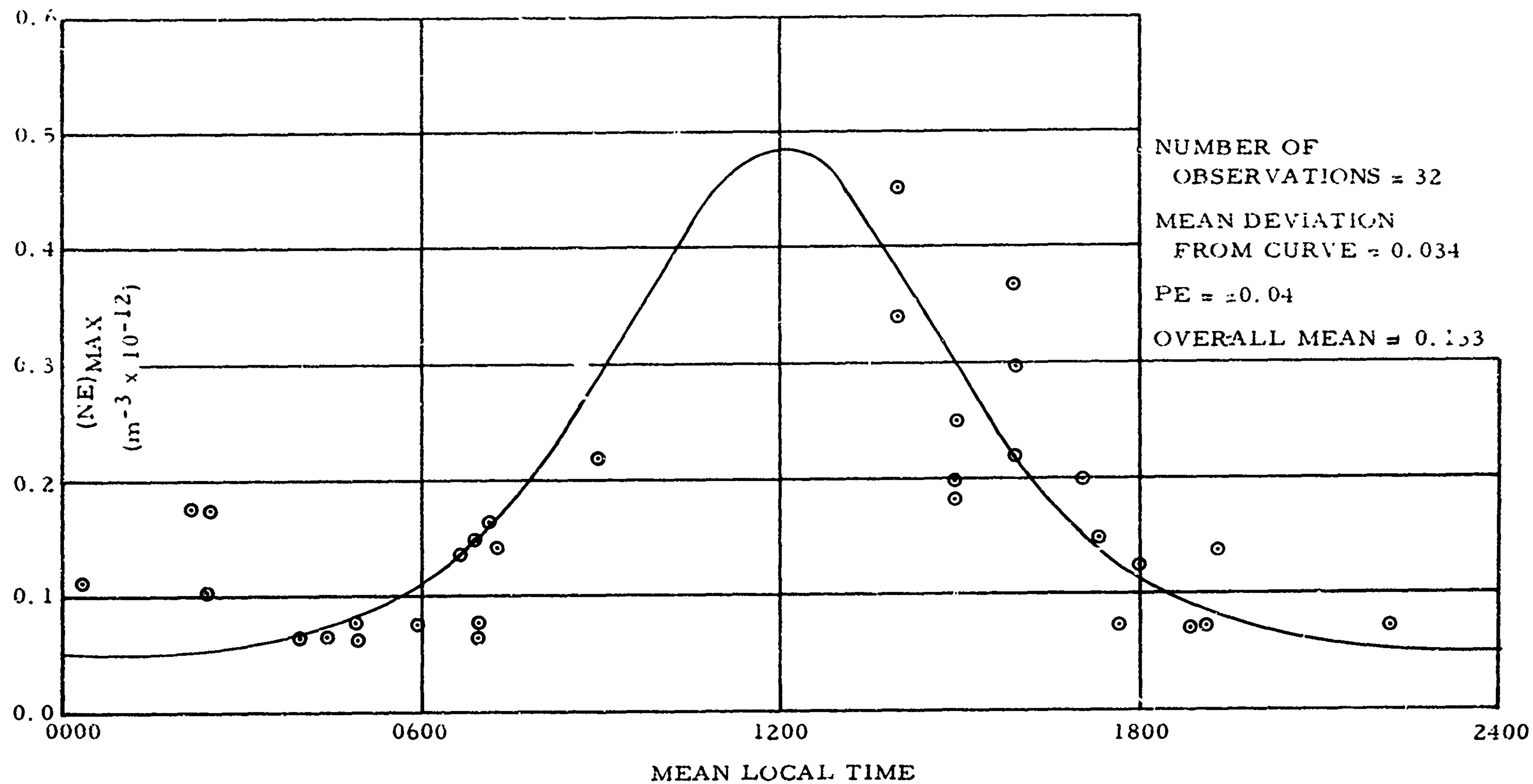

Figure 2-5. Maximum Electron Density of F Layer Input to Ionospheric Correction Model VS Local Time

establish a consistent time base for the observations. This correction was applied to all range data used for satellite position determination, although it is only required when orbital prediction is to be employed.

2.5.3 Internal Consistency. When simultaneous mode data was used in the satellite position program, an internal consistency check was made by performing four different satellite position calculations with the four sets of three ranges. Comparing these four solutions with the average solution provided a measure of the consistency of the range measurements. The results of this internal comparison showed agreement in satellite position within a few meters in regions of good geometry.

2.6 Determination of Orbital Elements. The various solutions using orbital mode data required the determination of the position of the satellite corresponding to the time at which the range measurements were taken at the CORDEX site. This determination of satellite position depended on orbital prediction techniques based upon simultaneous range measurements made by the three known stations. From range data taken by the known stations, a set of orbital elements were derived for the orbital prediction program.

2.6.1 Punch Cards. For convenience, the equatorial satellite coordinates of position and velocity determined from measured data were punched onto cards (program SPUNCH) from the satellite position (SP) tape. The cards were then used in the orbital fit program discussed in the following paragraph.

2.6.2 Orbital Elements by Least Squares Trajectory Fit. The orbital prediction techniques employ the particular set of orbital elements known as injection vectors. These are the equatorial position and velocity of the satellite at a certain time (injection time). The choice of injection time usually corresponded to the first time for which satellite position was calculated. Thus, the measured satellite position and velocity at the injection time yielded a first estimate for the injection vectors. The orbit fitting technique is discussed in Appendix K, and a general discussion of the least squares adjustment techniques is included (for information) in Appendix L.

Orbit fitting (program PCMPTJ) consisted of adjusting the components of the injection vectors so that the differences between the measured and predicted equatorial position and velocity were minimum (in the least squares sense). The method of obtaining the predicted coordinates is discussed in a later paragraph. The results of a typical orbital fit are illustrated in figure 2-6 where the equatorial position residuals (differences between measured and predicted values) are plotted for the samples used in the fitting span.

Similar orbital fitting techniques were attempted, fitting only to the satellite position, and directly to the difference between predicted and measured ranges. (See figure 2-7.) However, all of these techniques produced similar results.

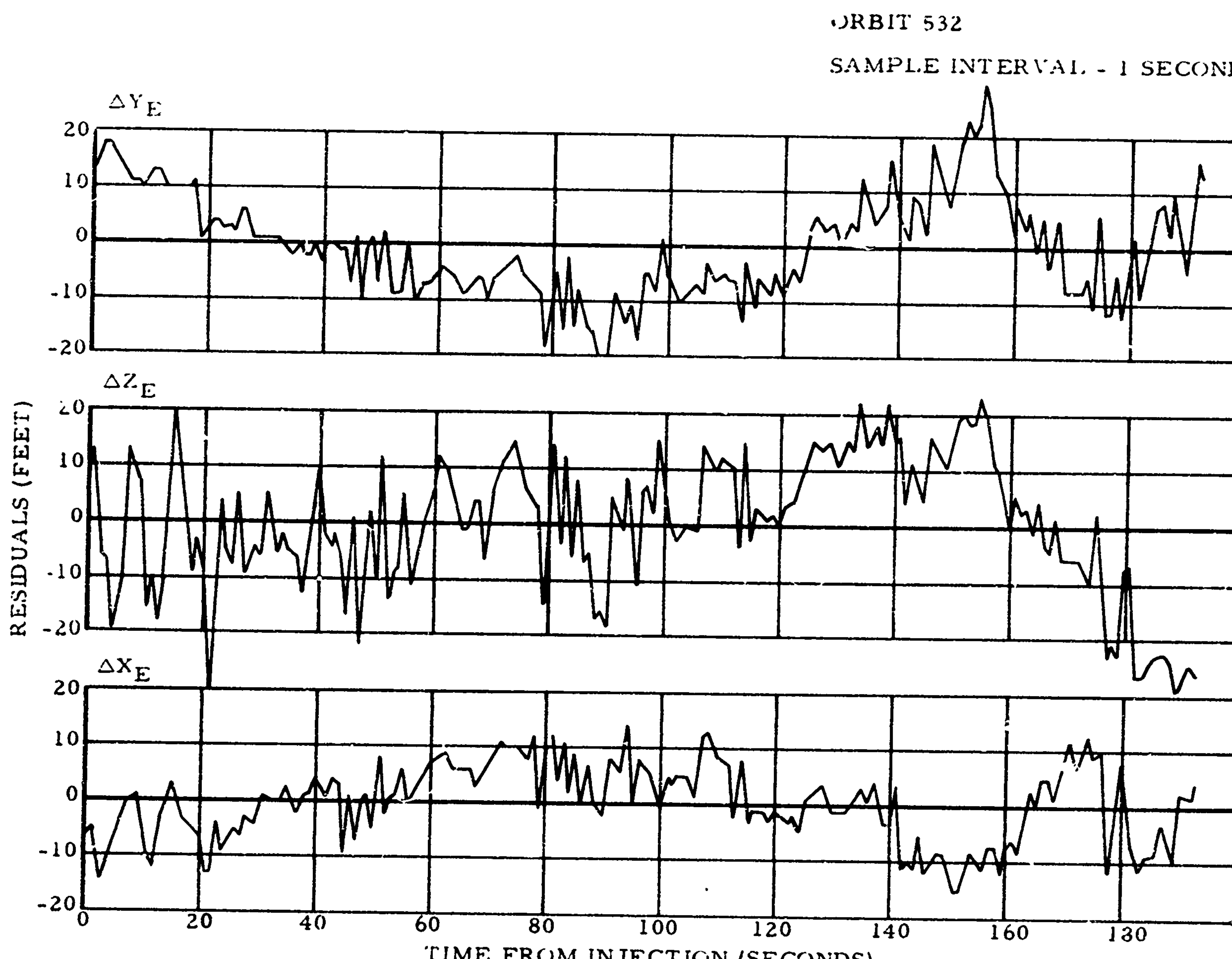

Figure 2-5. Orbital Fitting Residuals Fit to Position and Velocity

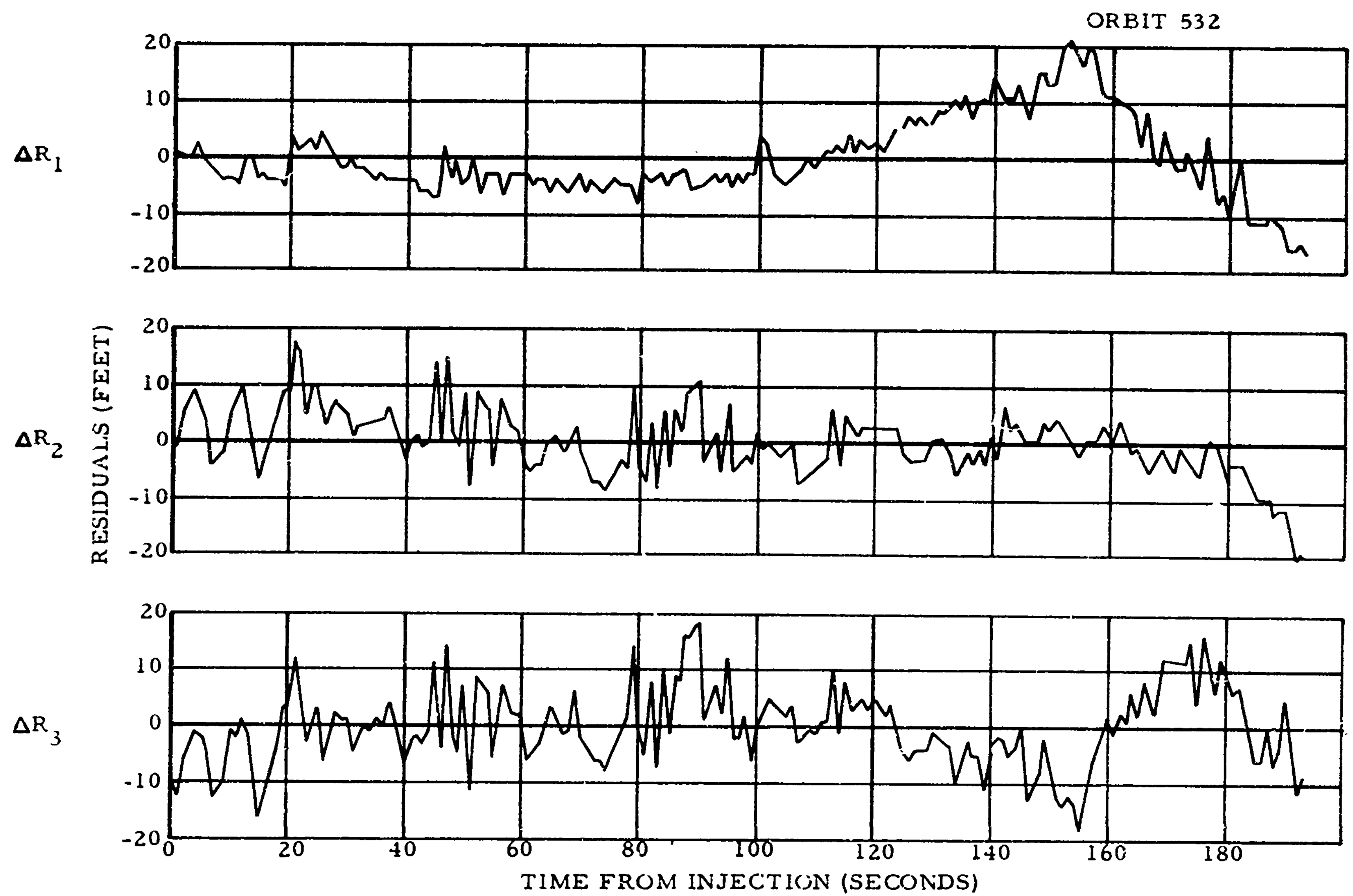

Figure 2-7. Orbit Fitting Residuals, Range Fit

2.7 Satellite Position by Orbital Prediction. During the orbital prediction pass (program GSORB) the injection vectors determined from the trajectory fitting were used to predict the satellite position for each data sample read from the CORDEX site ES tape. These data were then packed on a predicted satellite position (PSP) output tape which was simiiar in format to the SP tapes, and was compatible with the various CORDEX solution programs.

The actual prediction of satellite position was performed by numerically integrating the total force field, or by numerically integrating only the perturbation accelerations and adjusting a two-body reference orbit. (Refer to Appendix M.) Both these techniques are described in detail in Appendix N, and the two-body prediction techniques are described in Appendix O. The perturbations mentioned refer to forces other than the two-body central force field, and include the second through the ninth zonal harmonics of the earth's gravity, atmospheric drag, and lift. The last two perturbations are discussed in Appendix P, although, as expected, the effect was found to be negligible for the prediction intervals and vehicle altitudes involved.

The primary perturbation arises from the higher order terms of the gravitational force field. The gravitational perturbation was calculated using the first nine zonal harmonics as described in Appendix Q (subroutine GRAVITY).

The satellite position and velocity predicted were used to form a predicted range and range rate which could then be compared with the measured values. Figure 2-8 shows the comparisons over about three minutes of track for orbit 504.

2.8 CORDEX Solutions. The solution for the position of the CORDEX site is the primary function of the Geodetic SECOR system. In processing the data two types of solution (3-3 and 3-2 CORDEX solutions) were performed. Although each of these solutions may be performed with either simultaneous mode or orbital mode data, only the 3-3 CORDEX solution was actually attempted with orbital mode data.

The initial computation is the same for either solution. That is, the position of the satellite must be found at points along the orbit at which ranges to the CORDEX site are available. For the simultaneous mode data, all this information was available on the SP tapes. In the orbital mode, the satellite positions were found by orbital prediction, and were recorded along with the corresponding CORDEX site ranges on the PSP tapes.

In processing both the CORDEX solutions, discrete solutions were computed by choosing three (or two) spans of data and performing the solution with successive triads (or pairs) of data points. The resulting solutions were compared with the mean solution, and with the surveyed position of the CORDEX site in order to estimate the quality and consistency of the solutions.

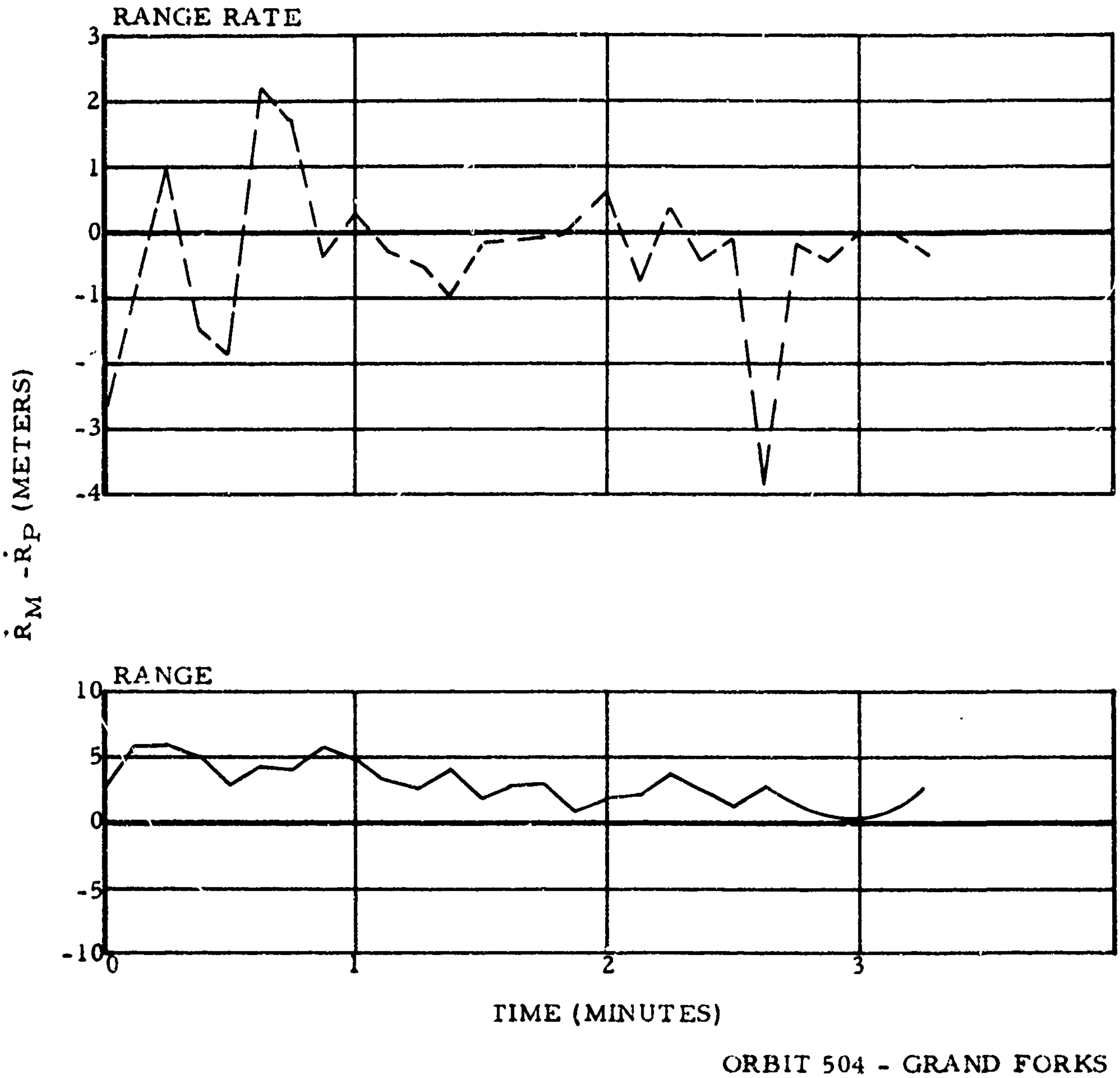

Figure 2-8. Measured VS Predicted Ranges and Range Rates

2.8.1 3-3 CORDEX Solution. In the 3-3 CORDEX solution (program PASS4) the position of the CORDEX site was determined by trilaterating to the CORDEX site from three satellite positions using the corresponding ranges to the CORDEX site. The mathematics of this solution is identical with that used to solve for the satellite position except that the three satellite positions form the reference sites and the CORDEX site position is solved for. (Refer to Appendix F.)

Figure 1-1 shows the geometrical arrangement for the simultaneous mode 3-3 CORDEX solution where two orbital passes are used. An example of the results of the 3-3 CORDEX solution is illustrated in figure 2-9. In this figure, the difference between the CORDEX site latitude and longitude determined from range measurements and the survey values are plotted on the left. Each point of this plot represents a solution determined using a different triad of satellite locations. The approximate geometry for the solutions is illustrated at the right. The geometry for this set of solutions was good; hence, the distribution of solutions is quite symmetrical. Less favorable geometry tends to distribute the random errors within an elliptical region.

2.8.2 3-2 CORDEX Solution. The 3-2 CORDEX solution (program PASS432) is similar to the 3-3 CORDEX solution except that the height of the CORDEX site is assumed to be known; thus, the height replaces one range measurement. The trilateration to the CORDEX site requires two satellite positions plus the height of the CORDEX site. The mathematics of the two-range-and-altitude solution is derived in Appendix F, and the geometrical configuration using one satellite pass is shown in figure 1-2.

2.9 Line Crossing Solution. The line crossing mode illustrated in figure 1-3 consists of determining an estimate of the geodesic (shortest distance along the spheroid) between two of the tracking sites. In operation, four sites tracked the satellite simultaneously as it crossed the baseline. From the range data taken by three of the sites, the satellite's distance from the center of the earth was determined. One of these three sites and the fourth site formed the ends of the baseline. The mathematical details of the line crossing technique are contained in Appendix R.

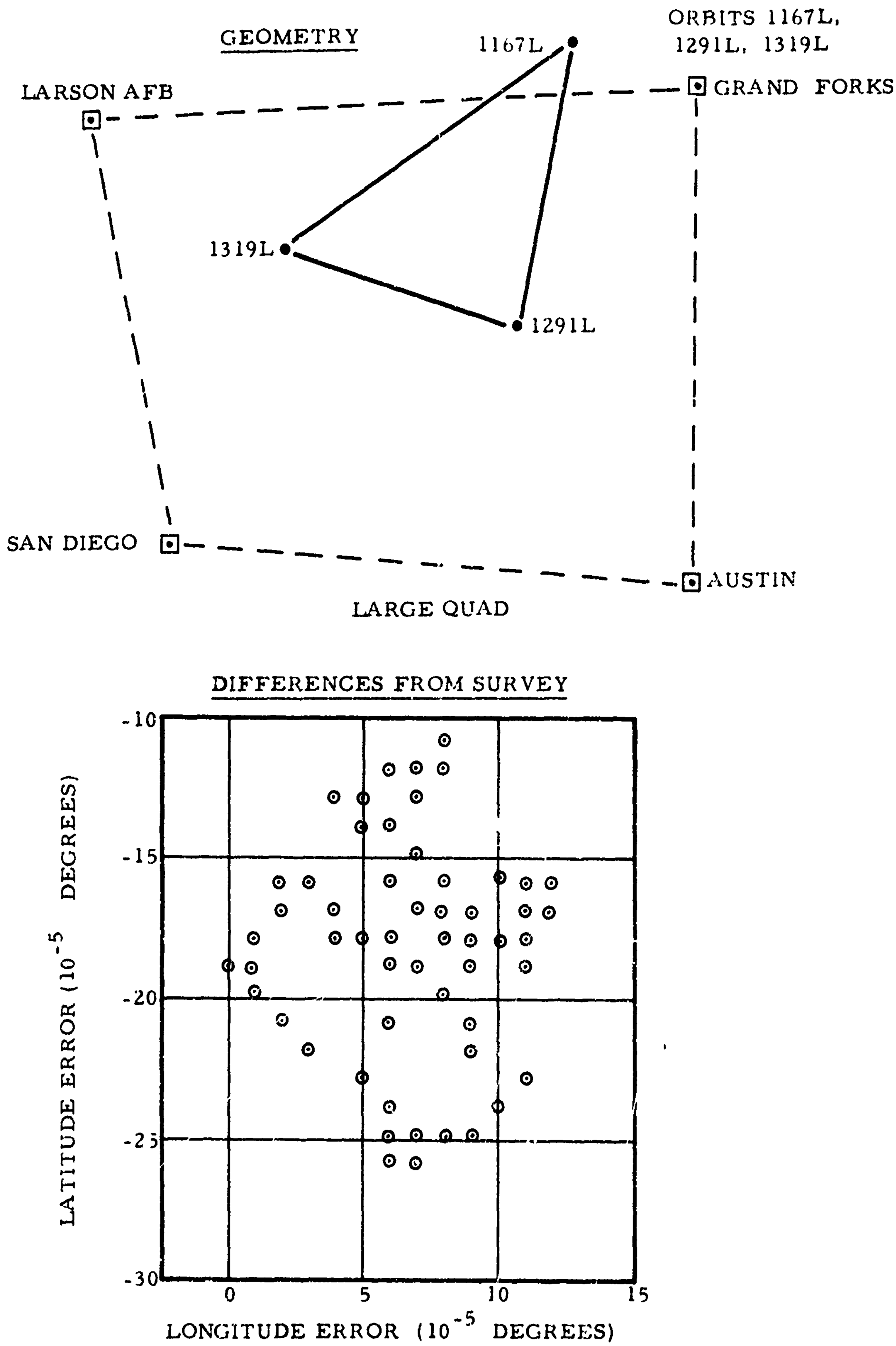

Figure 2-9. 3-3 CORDEX Solution Error (Larson AFB)

SECTION III
SUMMARY OF RESULTS

3.1 Introduction. The method of computing the unknown site solutions for the Geodetic SECOR USA-2 satellite data processing did not weigh solutions or discriminate between them on the basis of optimum geometries. Because geometry bears a dramatic relation to accuracy, theoretical error propagations were processed which corresponded to the actual geometries used in the data reduction. A comparison of the theoretical accuracies and the observed results, therefore, provides a means of normalizing solutions. In many solutions, disagreement can be anticipated if poor results are predicted in the error propagation.

It is important to remember that the theoretical error propagation is based on a statistical model, implying a large sample space. Actual solutions, on the other hand, are discrete cases or represent an average over a relatively small number of discrete cases. Observed solutions should approach the theoretical predictions in characteristics and in magnitude over many solutions, provided the theoretical error model is valid. The theoretical error propagation does not take into account that the standard (in this experiment, the assumed position of the unknown site) might be incorrect. The difference between the computed and surveyed site coordinates, therefore, includes some constant error because of the uncertainty in the assumed standard.

The assumed error models used in the theoretical error analysis are listed in table 3-1.

TABLE 3-1
ASSUMED ERROR MODELS

Model Number	σ System (feet)	σ Tropo (per cent)	σ Scale (ppm)	σ Survey (ppm)	σ Iono (per cent)	σ Site Height (feet)
CORDEX 1	9	5	1	4	5	0
CORDEX 2	15	5	1	4	5	0
CORDEX 3	15	5	1	10	5	0
Line Crossing	9	5	1	15	5	15

3.2 Small Quad 3-3 CORDEX Solutions. The base stations used in the small quad were Stillwater, Oklahoma; Las Cruces, New Mexico; Austin, Texas; and Fort Carson, Colorado (the unknown site). Table 3-2 includes the major portion of the CORDEX solutions processed on the early USA-2

TABLE 3-2

GEODETIC SECOR USA-2 SATELLITE 3-3 CORDEX SOLUTIONS

FORT CARSON SITE SOLUTIONS (3-3 SIMULTANEOUS MODE) SMALL QUAD

Solut. No.	Orbits Used for Solution	No. of Indiv. Compu.	SECOR-SURVEY LATI m	SECOR-SURVEY LONG m	SECOR-SURVEY HEIGHT m	SECOR-SURVEY RSS m	STANDARD DEVIATIONS LATI m	STANDARD DEVIATIONS LONG m	STANDARD DEVIATIONS HEIGHT m	STANDARD DEVIATIONS RSS m	COMPARISONS	COMPARISONS TOTAL RSS m	COMPARISONS THEORETICAL (1) RSS m	COMPARISONS THEORETICAL (2) RSS m
1A	132,173,173	46	1.1	-2.6	3.2	4.3	6.7	15.1	4.2	15.3		15.9	22.7	30.6
5A	242.1,173.2,270.2	86	3.3	9.7	14.6	17.8	2.2	3.5	1.8	4.5		18.4	13.5	18.2
6A	270.1,132,173.2	74	-8.9	8.8	6.3	14.0	4.4	1.8	2.6	5.4		15.0	11.5	15.1
7A	132,270.1,173.1	74	-7.8	11.4	6.0	15.1	3.3	1.8	2.6	4.6		15.8	11.7	15.5
8A	132,173.1,187.2	74	4.1	-11.4	0.3	12.1	4.4	5.3	1.5	7.0		14.0	14.6	19.5
9A	234,173.2,270.2	104	3.3	9.7	14.8	18.0	4.4	3.5	2.6	6.2		19.0	21.1	28.3
10A	187.1,173.2,270.2	104	12.3	8.8	18.4	23.8	1.1	3.5	1.3	3.9		24.1	15.1	20.0
11A	132,270.1,242.2	56	-7.8	14.0	5.0	16.8	3.3	1.8	2.1	4.3		17.3	11.7	15.5
13A	132,201,242.2	56	-5.6	11.4	4.4	13.4	2.2	1.8	2.4	3.7		13.9	11.7	15.7
14A	201,132,173.2	74	-6.7	7.0	5.7	11.2	3.3	3.5	2.8	5.6		12.5	11.6	15.3
15A	132,201,173.1	74	-6.7	8.8	5.6	12.4	3.3	1.8	2.9	4.7		13.3	12.0	15.7
16A	187.3,220,201	102	1.1	24.5	0.7	24.5	5.6	4.4	3.8	8.1		25.8	17.8	23.9
17	132,187.2,173.2	74	4.4	-4.4	1.3	6.4	4.4	4.4	1.7	6.5		9.1	12.6	16.8
18	220,132,187.1	74	4.4	21.0	3.5	21.7	2.2	4.4	2.9	5.7		22.4	12.8	16.7
19	187.1,463.2,173.1	93	-1.1	-9.7	-3.9	10.5	1.1	3.5	1.4	3.9		11.2	17.8	23.7
	MEAN		-.70	7.1	5.7	14.8	3.5	3.9	2.4	6.0		16.5	14.5	19.4
	STANDARD DEVIATIONS		6.0	10.1	4.8									

satellite orbits. SECOR - SURVEY differences are the result of taking an average of a sequence of actual Geodetic SECOR solutions and subtracting the U. S. Coast and Geodetic Survey geodetic site coordinates from this average solution. The standard deviations shown per solution are computed from the residuals, where the residuals are the differences between each SECOR - SURVEY offset, and the average of the offsets for one particular solution. RSS refers to root sum square, and indicates the composite bias and noise error. The RSS is computed by squaring the mean offsets and the standard deviations, adding, and taking the square root.

Hence,

$$RSS = (\Sigma\Delta^2 + \Sigma\sigma^2)^{1/2}$$

where

$$\sigma^2 = \frac{\sum_{i=1}^{n} (\Delta - \bar{\Delta})_i^2}{n - 1} = \text{sample variance}$$

Δ = SECOR - SURVEY = residual

$\bar{\Delta}$ = average residual

n = number of individual solutions

$$SEP = \sqrt{\frac{\sigma_\varphi^2 + \sigma_\lambda^2 + \sigma_h^2 + \Delta_\varphi^2 + \Delta_\lambda^2 + \Delta_h^2}{3}}$$

with Δ_φ, Δ_λ, Δ_h, σ_φ, σ_λ, σ_h expressed in meters.

Note that in table 3-2 the over-all observed results fall between the theoretical error propagations given by error models 1 and 2. The controlling terms in these models were the system and survey errors. In both cases, a base site survey with an accuracy of 4 ppm was assumed. A ranging accuracy of approximately 3 and 5 meters was used in models 1 and 2, respectively. This indicates (nonconclusively) that over-all accuracy (ranging, correcting, processing, etc.) was approximately ±4 meters, and site survey was approximately 4 ppm for this quad.

Several solutions which were processed for the small quad are not included in this summary. Deleted solutions were adjudged nonrepresentative either because of geometry, or because of peculiarities in the data. Refer to the tabulations and listings for detailed information concerning the solutions.

3.3 Large Quad 3-3 CORDEX Solutions. In the large quad 3-3 simultaneous mode CORDEX solutions, the base stations were located at San Diego, Austin, Grand Forks, and in some cases, Fort Carson. Larson

Air Force Base in the state of Washington was the unknown station. Results tabulated are in the same format as those presented and explained for the small quad operation.

From table 3-3 it is apparent that solutions are not quite as accurate for the large quad operation. The error propagation, however, predicts reduced accuracy for the geometries used. It had been anticipated and proven by the error propagations that the large quad would give the opportunity for improved geometries and, consequently, improve solutions over those encountered in the small quad. In the actual operation, intervals of simultaneous track and the selection of orbits limited the geometries that could be used to obtain comparison data.

Considering the large quad results with respect to the theoretical error propagation, improved results were obtained over those experienced on the small quad. As a test criterion, if the total RSS observed is divided by the theoretical RSS means (using error model one), then from tables 3-2 and 3-3,

$$\text{Small Quad Ratio} = \frac{\text{Observed}}{\text{Theoretical}} = \frac{16.5}{14.5} = 1.14$$

$$\text{Large Quad Ratio} = \frac{\text{Observed}}{\text{Theoretical}} = \frac{34.0}{75.7} = 0.45$$

3.4 Orbital Mode 3-3 CORDEX Solutions. Table 3-4 includes the results of the orbital mode 3-3 CORDEX solutions. Sites in the small quad were used to determine the satellite position and velocity to which injection vectors were computed by an iterative least squares technique. Satellite positions were then predicted forward to times synchronous with ranging observation times at the Grand Forks station. With the predicted satellite positions and the measured ranges, the position of Grand Forks was computed. Error propagations of the Grand Forks CORDEX solution in the orbital mode were not processed. Solution results, therefore, have to be qualified subjectively.

It appears that the reduced accuracy in the orbital mode has three primary sources:

(1) relatively small orbit fitting spans,

(2) system and base site survey bias errors,

(3) internal timing.

Small fitting spans allow any error in the data to upset the vector fitting and give less accuracy in the injection vector determination. The forward prediction will then deteriorate rapidly. Timing error arises from the use of independent local time sources. A time offset will mean that the predicted

TABLE -3

GEODETIC SECOR USA-2 SATELLITE 7-3 CORDEX SOLUTIONS (SIMULTANEOUS MODE)

LARSON AFB SITE SOLUTIONS (3-3 SIMULTANEOUS MODE) LARGE QUAD

Solut. No.	Orbits Used for Solution	No. of Indiv. Compu.	SECOR-SURVEY LATI m	SECOR-SURVEY LONG m	SECOR-SURVEY HEIGHT m	SECOR-SURVEY RSS m	STANDARD DEVIATIONS LATI m	STANDARD DEVIATIONS LONG m	STANDARD DEVIATIONS HEIGHT m	STANDARD DEVIATIONS RSS m	COMPARISONS	COMPARISONS TOTAL RSS m	COMPARISONS THEORETICAL (1) RSS m	COMPARISONS THEORETICAL (2) RSS m
1	1305,1319,1167	50	-15.6	-19.1	-36.4	44.0	2.2	3.8	5.0	6.7		44.5	91.1	101.5
2	1305,1333,1167	55	-11.1	14.5	14.9	23.6	2.2	2.3	2.8	4.2		24.0	61.0	68.3
3	1305,1305L,1319L	86	-15.6	9.1	0.5	18.1	3.3	2.3	3.3	5.2		18.8	83.6	94.5
4	1305,1319L,1291L	81	-14.5	3.8	-2.2	15.2	3.3	3.0	2.4	5.1		16.0	64.8	73.5
5	1167L,1291L,1319L	81	-20.0	5.3	-5.7	21.5	4.4	2.3	4.0	6.4		22.4	64.3	75.1
6	1291L,1167, 1319	50	-5.6	-9.1	-16.4	19.6	3.3	2.3	6.1	7.3		20.9	95.0	108.1
7	1269,1269M,1291L	81	3.3	14.5	40.0	42.7	4.4	3.0	3.8	6.5		43.2	84.7	96.4
8	1269M,1269,1305L	91	-11.1	38.1	45.2	60.1	2.2	3.8	4.8	6.5		60.5	84.0	93.8
9	1269,1291L,1167L	81	-22.2	22.9	28.1	42.5	4.4	1.5	3.9	6.1		42.9	59.0	67.0
10	1269,1291L,1305L	81	-31.1	25.9	23.7	46.9	3.3	2.3	3.4	5.3		47.2	69.0	79.5
	MEAN		-14.4	10.6	9.2	33.4	3.3	2.7	4.0	6.0		34.0	75.7	85.6
	STANDARD DEVIATION		8.9	15.7	24.3									

TABLE 3-[illegible]

[illegible] 3-3 [illegible] MODE)

GRAND FORKS SITE SOLUTIONS — (3-3 ORBITAL MODE) — SMALL QUAD

Solut. No.	Orbits Used For Solution	No. of Indiv. Compt.	SECOR-SURVEY LATI m	LONG m	HEIGHT m	RSS m	STANDARD DEVIATIONS LATI m	LONG m	HEIGHT m	RSS m	TOTAL RSS m
1-0	463,463,477	54	-73.4	-9.7	-3.8	74.1	3.3	2.2	1.7	4.3	74.2
2-0	477,477,532	54	-86.7	36.6	-60.4	98.9	3.3	5.2	1.0	6.2	99.1
3-0	377,477,463	54	-116.8	-11.2	-15.0	118.3	3.3	5.2	1.1	6.3	118.5
4-0	532,532,477	54	-61.2	-29.1	-18.3	70.2	4.4	2.2	0.5	4.9	70.4
5-0	463.1,463.2,477.2	142	-27.8	-111.2	-13.8	115.5	14.5	12.7	7.3	20.6	117.3
6-0	620.1,463.1,463.2	130	37.8	20.1	9.2	43.8	2.2	16.4	2.0	16.7	46.9
7-0	725,463.1,463.2	185	12.2	-20.9	-0.6	24.2	4.4	14.9	3.7	16.0	29.0
8-0	725,477.1,477.2	139	-25.6	8.2	-38.9	47.3	8.9	4.5	2.0	10.2	48.4
9-0	532.2,477.1,477.2	139	-52.3	-36.6	-24.4	68.3	21.1	18.7	5.2	28.7	74.1
10-0	532.1,532.2,477.2	132	-12.2	-89.5	-21.0	92.7	12.2	29.8	5.5	32.4	98.2
11-0	532.1,532.2,477.1	132	21.1	-21.6	-10.9	32.1	5.6	14.2	2.1	15.4	35.6
12-0	620.2,727,463.2	167	4.4	73.9	3.8	74.1	5.6	6.7	3.8	9.5	74.7
13-0	504,463.1,463.2	195	15.6	-12.7	0.3	20.1	8.9	6.0	5.2	11.9	23.4
14-0	504,532.1,532.2	132	17.8	-29.1	-12.0	36.2	4.4	3.7	2.3	6.2	36.7
15-0	504,620.2,463.2	167	-62.3	64.2	-17.1	91.1	20.0	26.9	17.9	38.0	98.7
16-0	504,620.2,532.2	167	-82.3	79.8	-30.5	118.6	51.2	50.7	38.8	81.8	144.1
17-0	620.2,727,477.2	142	-61.2	152.9	-67.4	178.0	2.2	34.3	22.2	40.9	182.6
MEAN			-32.5	7.2	-17.1	76.7	10.3	15.0	7.1	20.6	80.7
STANDARD DEVIATION			44.1	61.4	17.5						

satellite positions and the measured ranges are not synchronous. Time synchronization is not encountered while the Geodetic SECOR equipment is used in the simultaneous mode. System and survey bias will give a slight misorientation of the injection vectors and therefore will affect the forward predictions.

The experience and extensive testing undertaken in the data reduction and computer program development has indicated that the orbit fitting and prediction techniques are extremely accurate. Over the intervals of prediction and fitting used in this reduction, it was demonstrated that the analytic methods did not contribute significant error in the solutions. It is thought that the two error sources of fitting span, and system and site survey bias error effects can both be overcome if multiple orbits are used in the fitting procedures. The lever arm inherent in the long predictions will allow recognition or compensation for any initial misalignment of the injection conditions. Of course, care will have to be exercised to assure that the analytic model and techniques of prediction and fitting do not then become major error sources.

3.5 Satellite Line Crossings. Table 3-5 is a summary of the line crossing solutions and comparisons computed from the USA-2 satellite data. The results of the line crossing mode are commensurate with the theoretical results with two exceptions. All lines measured to the Herndon, Virginia site are long. The correlation of these results over all lines indicates that the Herndon site is, in all likelihood, mislocated. The rms errors shown in table 3-5 are misleading because they are computed from the differences between an analytic curve fit to the geodetic distance sums and the observation computed sums. A parabolic form is assumed in the curve fit. The geodetic distance sum however is not parabolic due to the earth's rotation. Even though the rms error is quite high in some lines, the fit is representative at the crossing. A direct comparison between the computed minimum crossing from the analytic fit and the measured data shows that the solution is accurate (to within one meter in all cases processed).

The strength of the line crossing solution in this experiment is the result of accurate ranging and the accurate determination of the satellite's distance from the earth's center. Use of a ground station to track the satellite during the crossing allows this high accuracy. The line crossings processed here represent the longest lines ever processed and clearly demonstrate the strength of the technique.

3.6 Large Quad 3-2 CORDEX Solution. In the large quad 3-2 simultaneous mode CORDEX solution, the base stations were located at San Diego, Austin, and Grand Forks. The Herndon site was chosen as the CORDEX site with its survey height assumed correct. The 3-2 CORDEX solution was then run using different orbits and geometries to see if any survey bias could be detected. The possibility of such a bias was suggested by the line crossing results. The results of the five solutions attempted are given in table 3-6. Because of the relatively short baseline obtained, the solutions show inconclusive results in latitude. That is, the standard deviation of the various latitude determinations from the mean exceeds the average latitude offset. The longitude, however, shows a rather consistent bias.

TABLE

GEODETIC SECOR USA-2 SATELLITE LINE CROSSING SOLUTIONS

LINE CROSSING RESULTS *							
ORBIT NO.	STATIONS	SECOR m	SURVEY m	SECOR-SURVEY m	RMS NOISE m	OBSERVED RSS m	THEORETICAL (1) RSS m
463	Austin Ft. Carson	1,137,573.2	1,137,559.8	13.4	5.6	14.5	25.8
532	Austin Ft. Carson	1,137,573.0	1,137,559.8	13.2	65.0	66.3	25.8
648	Austin San Diego	1,860,074.3	1,860,051.9	22.4	9.0	24.1	15.0
648	Stillwater San Diego	1,862,470.1	1,862,456.7	13.4	3.2	13.8	15.0
670	Stillwater San Diego	1,862,473.4	1,762,456.7	16.7	15.4	22.7	15.0
808	Stillwater San Diego	1,862,449.4	1,862,456.7	-7.3	12.3	14.3	15.0
1131	Stillwater San Diego	1,862,457.0	1,862,456.7	0.3	2.0	2.0	15.0
896	San Diego Herndon	3,628,320.3	3,628,265.0	55.3	0.5	55.3	9.6
1401	San Diego Herndon	3,628,315.4	3,628,265.0	50.5	3.5	50.6	9.6
1241	San Diego Herndon	3,628,300.5	3,628,265.0	35.5	13.1	37.8	9.6
1365	San Diego Herndon	3,628,296.3	3,628,265.0	31.3	5.0	31.7	9.6
1365	Larson Herndon	3,494,039.5	3,493,993.6	45.9	3.5	46.0	10.3
1365	Ft. Carson Herndon	2,374,063.3	2,374,034.5	28.8	7.7	29.8	13.8
1305	Austin Larson	2,641,159.4	2,641,164.4	-6.0	2.7	6.6	11.4
1305	G. Forks Larson	1,675,760.3	1,675,745.8	14.5	26.3	30.0	18.6
1305	San Diego G. Forks	2,387,105.5	2,387,098.2	7.3	1.7	7.5	12.0
1269	San Diego G. Forks	2,387,105.0	2,387,098.2	6.8	7.3	10.0	12.0
MEAN				20.1	10.8	27.2	14.3
STANDARD DEVIATION				18.0			

* Line Crossing Results based on International Spheroid

TABLE 3-6

GEODETIC SECOR USA-2 SATELLITE LARGE QUAD 3-2 CORDEX SOLUTIONS

Herndon Site Solutions (3-2 Simultaneous Mode) Large Quad								
Solution No.	Orbits Used For Solution	No. of Indiv. Comput.	SECOR - SURVEY			STANDARD DEVIATIONS		
			Lat M	Long M	RSS M	Lat M	Long M	RSS M
1	896	108	-111	138	177	4	1	4
2	1401	137	85	109	138	18	6	19
3	1401L - 896M	53	49	91	103	18	3	18
4	1365 - 1401	140	-30	142	145	5	3	6
5	1365	90	-71	114	134	16	9	18
	Mean		-15.6	118.8		12.2	4.4	
	Standard Deviation		73.0	19.0				

SECTION IV
RECOMMENDATIONS

4.1 Introduction. Techniques of data processing developed in the reduction of Geodetic SECOR USA-2 satellite data form the bases of a second generation set of solutions and operational procedures which may significantly influence future uses of the Geodetic SECOR equipment. Noted below is a tentative list of solutions and procedures that should be attempted before the present processing effort is terminated. Observational information from USA-2 Geodetic SECOR satellite tracking is thought to represent the most accurate, consistent, and extensive accumulation of satellite positional data ever taken. If further processing and extended solutions are not undertaken in the near future, present interest and experience will probably dissipate.

4.2 Extended Solutions.

4.2.1 3-N Solutions. The 3-N solution is an extension of the solutions discussed above in that all available data is included in a least squares solution for the CORDEX site position. This type of solution allows data from multiple satellite passes to be included and eliminates the necessity of using discrete solutions over limited spans of data.

The 3-N solution may be further enhanced by including weighting based upon geometry and an assumed error model plus observational noise estimates. This technique emphasizes data of low noise content in regions of good geometry. Preliminary solutions obtained with the 3-N solution indicate that stable solutions may be obtained where either all three coordinates of the CORDEX site are adjusted, or only the latitude and longitude are adjusted.

4.2.2 Analytic Calibration. An attempt should be made to utilize the overdeterminancy of the observations to adjust not only the coordinates of the CORDEX site, but also to adjust the calibration of the range data. This technique would help reduce the effect of calibration drifts (if any) in the satellite transponder. This type of adjustment is an extension of the technique used during the aircraft flight tests to establish range calibration.

4.2.3 Range Rate Solutions. Further solutions are possible using the computed range rate. The range rates could be used alone or in conjunction with the measured ranges. These solutions should be investigated to determine their relative merits.

4.3 Line Crossing Evaluation. An investigation should be made to determine the causes of the line length offsets which are evident in the data where Herndon was used as the CORDEX site. This investigation should include a network adjustment based upon the measured line lengths to determine if the errors might be attributed to survey offsets at one or more of the tracking sites.

4.4 Ionospheric Correction Evaluation. The ionospheric data obtained by using the dual frequency phase measurements should be further investigated. Of primary interest should be a comparison with data taken by other means (e.g., NBS ionosonde records) to determine the accuracy and consistency of the measurements. A secondary investigation should be made into better modeling of the ionosphere in order to account for predictable horizontal variations due to the solar zenith, magnetic latitude, etc. Such a model would allow a better ionospheric refraction correction to be made, and would enhance the accuracy of the solutions, particularly at lower elevation angles.

4.5 Orbital Accuracy. Further investigation should be made into the orbital prediction techniques. In particular, investigations of orbital prediction over larger portions of an orbit and also over multiple orbits should be made. These techniques are vital to the extension of the orbital mode operation and more automated techniques of data reduction.

4.6 Operational Orbital Data. The large quantities of data which can be taken by the operational Geodetic SECOR system require more sophisticated data processing techniques. Use of predicted orbital information could provide a valuable basis for such techniques. For example, data editing based upon orbital prediction could reduce the chances of the occurrence of offset edited data. Furthermore, in regions where only two trackers were locked or within line-of-sight, all range data available could be used in an adjustment program for the orbital elements.

APPENDIX A

CONSTANTS, UNITS, ROTATIONS, AND TRANSLATIONS

CONSTANTS . . . . Conversions of measurements and definitions of basic shapes and sizes are pertinent in data reduction procedures. The constants used in all processing and in earth shape and spin rate computations are given in table A-1.

TABLE A-1

BASIC CONSTANTS

QUANTITY	DIMENSION
ONE METER	3.28083333333 FEET
ONE DEGREE	0.01745329252 RADIANS
ONE NAUTICAL MILE	6076.10333333 FEET
ONE STATUTE MILE	5280.00000000 FEET
VACO VELOCITY OF LIGHT	983569220.000 FEET/SEC
π	3.1415926536
ONE SIDEREAL DAY	86164 MEAN SOLAR SECONDS
ONE MEAN SOLAR DAY	86400 MEAN SOLAR SECONDS
EARTH'S ANGULAR RATE (ω_e)	$\omega_e = \frac{2\pi \text{ RADIANS}}{1 \text{ SIDEREAL DAY}} = \frac{2\pi \text{ RADIANS}}{86164 \text{ MSS}}$
	ω_e = 0.0000729212351 RAD/SEC
KOZAI* MODEL OF EARTH	
PRINCIPAL GRAVITY	G = 32.14648177 FEET/SEC^2
SEMIMAJOR AXIS	a = 6378165 METERS
	a = 20925696.335 FEET
SEMIMINOR AXIS	b = 6356783.287 METERS
	b = 20855546.499 FEET
FLATTENING	f = 1/298.3

*Kozai, Yoshihide, "Numerical Results from Orbits," Smithsonian Institute Astrophysical Observatory Special Report No. 101.

TABLE A-1 (Cont'd)

BASIC CONSTANTS

QUANTITY	DIMENSION
INTERNATIONAL EARTH MODEL	
PRINCIPAL GRAVITY	G = 32.199 FEET/SEC^2
SEMIMAJOR AXIS	a = 6378388.0 METERS
	a = 20926427.961 FEET
SEMIMINOR	b = 6356911.946 METERS
	b = 20855968.607 FEET
FLATTENING	f = 1/297
1866 CLARK EARTH MODEL	
SEMIMAJOR AXIS	a = 6378206.4 METERS
	a = 20925832.162 FEET
SEMIMINOR AXIS	b = 6356583.8 METERS
	b = 20854892.015 FEET
FLATTENING	f = 1/294.978698

A sidereal day is the time for one rotation of the earth A mean solar day is an average of true solar days, where a true solar day is the time elapsed for successive intersection of an earth meridian with a sun reference. The mean solar day is defined as 24 hours and, correspondingly, 86400 seconds.

In trajectory predictions, the earth's sidereal period defines the earth's angular rate. Most local and universal time is expressed in mean solar time.

UNITS . . . . The formulae for development of two-body trajectory predictions and partial derivatives are conveniently represented when units of length and time are expressed in what is defined as canonical units. The values of canonical units for near earth two-body equations are given by:

1UL = one unit of length = a

1UV = one unit of velocity = $\sqrt{Ga}$

1UT = one unit of time = 1UL/1UV

where

G = 32.14648177 feet/sec^2, the principal gravity term in the Kozai earth model.

a = 20925696.335 feet, the earth's equatorial radius in the Kozai earth model.

ROTATIONS . . . . Assume a right-handed convention (i.e., the X axis perpendicular to Y where a 90° counterclockwise rotation of X rotates X into Y and the Z axis is normal to the XY plane) for all coordinate systems; then the following set of simple rotations will reduce the complexity of representation in each reference frame used in trajectory prediction and vehicle position and velocity computations. In all rotations, the angle of rotation is measured counterclockwise from the new axis to the former axis.

In figure A-1, a rotation about the Z axis that would rotate a vector in the primed into the unprimed system takes the standard matrix form

$$M = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

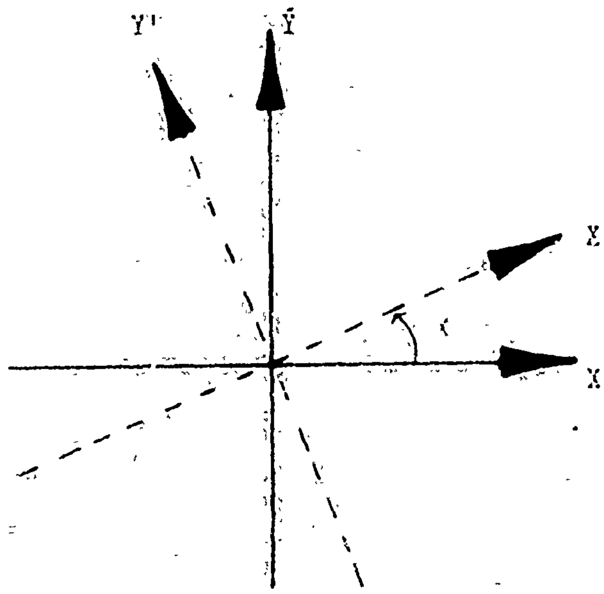

Figure A-1

Convention for Rotation Angles

Hence,

$$\bar{F} = M\bar{F}' \tag{2}$$

and because M will always be an orthogonal matrix (i.e., $M^T M = I$),

$$\bar{F}' = M^T \bar{F} \tag{3}$$

where the T superscript means transpose.

Adhering to the counterclockwise definition of angles and the right-handed convention stated above, then the following matrices can be formed to rotate vectors from some local earth surface system of coordinates into equatorial coordinates; hence,

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos B & -\sin B \\ 0 & \sin B & \cos B \end{bmatrix} \quad \text{about X to local vertical} \tag{4}$$

$$M_2 = \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{about Z to local east-north} \tag{5}$$

$$M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \Phi & -\cos \Phi \\ 0 & \cos \Phi & \sin \Phi \end{bmatrix} \quad \text{about local east to equatorial vertical} \tag{6}$$

$$M_4 = \begin{bmatrix} -\sin \lambda & -\cos \lambda & 0 \\ \cos \lambda & -\sin \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{about equatorial vertical to colinear equatorial} \tag{7}$$

and

$$M_L = M_2 M_1 = \text{local to east-north-up (ENU)} \tag{8}$$

$$M_G = M_4 M_3 = \text{east-north-up to equatorial} \tag{9}$$

$$M_{LG} = M_G M_L = M_4 M_3 M_2 M_1 = \text{local to equatorial} \tag{10}$$

where

Equatorial = axis in the equatorial plane through the center of mass of the earth and the Greenwich meridian, Y axis in the equatorial plane and 90° counterclockwise from X, and the Z axis along the axis of rotation of the earth.

Vertical = the plumb line or normal to the local lines of equipotential gravity (GEOID)

λ = geodetic or geocentric longitude measured counterclockwise from the Greenwich meridian

Φ = geodetic (geographic) latitude measured from the equatorial plane

A = azimuth angle offset between local and ENU coordinates

D = vertical misalignment between local and ENU coordinates.

When the M_{LG} matrix above has been computed, a vector $\bar{R}_L$ measured in a local reference system is rotated to equatorial by the single rotation

$$\bar{R}_{EQ} = M_{LG}\,\bar{R}_L \tag{11}$$

and M_{LG}^T will rotate from equatorial to local coordinates.

Rotation from equatorial to inertial is given by

$$M_{EI} = \begin{bmatrix} \cos\omega_e(t-t_o) & -\sin\omega_e(t-t_o) & 0 \\ \sin\omega_e(t-t_o) & \cos\omega_e(t-t_o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{equatorial to inertial} \tag{12}$$

where

inertial = space fixed system which is colinear with equatorial at t_o.

t_o = time of epoch or injection

t = time in trajectory

TRANSLATIONS . . . . All trajectory computations are performed in an inertial reference system. Measured and computed data may be recorded or desired in some arbitrary local system. With the above matrices, the rotations and translations of position and velocity vectors from a local system to inertial are given by

$$\bar{R}_{EQ} = [M_{LG}\,\bar{R}_L + \bar{R}_\epsilon + \bar{R}'_{R_e} + EGT \cdot \bar{L}_{F_e}] \tag{13}$$

$$\bar{R}_{IN} = M_{EI}\,\bar{R}_{EQ} \tag{14}$$

$$\bar{V}_{EQ} = M_{LG}\,\bar{V}_L \tag{15}$$

$$\bar{V}_{IN} = M_{EI}\,\bar{V}_{EQ} + \bar{V}_E \tag{16}$$

where

$$\bar{1}'_{R_e} = \begin{bmatrix} \cos\lambda & \cos\phi' \\ \sin\lambda & \cos\phi' \\ \sin\phi' & \end{bmatrix} \quad \text{unit vector in geocentric coordinates} \tag{17}$$

$$\bar{1}_{R_e} = \begin{bmatrix} \cos\lambda & \cos\phi \\ \sin\lambda & \cos\phi \\ \sin\phi & \end{bmatrix} \quad \text{unit vector in geodetic coordinates} \tag{18}$$

HGT = height of the local coordinate system above mean sea level

$\bar{V}_E$ = velocity components due to the earth's rotation

$$\bar{V}_E = \omega_e \begin{bmatrix} -Y_{EQ} \\ X_{EQ} \\ 0 \end{bmatrix} \tag{19}$$

ϕ' = geocentric latitude

$\bar{R}_{IN}$, $\bar{V}_{IN}$ = position and velocity vectors in inertial coordinates, respectively.

Position and velocity vectors in inertial coordinates are transformed to local coordinates by the following sequence of rotations and translations.

$$\bar{R}_{EQ} = M^T_{EI}\,\bar{R}_{IN} \tag{20}$$

$$\bar{R}_L = M^T_L\,[\bar{R}_{EQ} - R_e \cdot \bar{1}'_{R_e} - HGT \cdot \bar{1}_{R_e}] \tag{21}$$

$$\bar{V}_{EQ} = M^T_{EI}\,\bar{V}_{IN} - \bar{V}_e \tag{22}$$

$$\bar{V}_L = M^T_L\,\bar{V}_{EQ} \tag{23}$$

When it is necessary to translate position vectors from arbitrary local origins to some known reference point, such as a bench mark, before transforming into equatorial, inertial, etc., the following convention will eliminate sign and sequence errors.

$$\overline{R}_N = \overline{R}_O + \overline{R}_{OO} - \overline{R}_{NO} \qquad (24)$$

where

$\overline{R}_N$ = vector expressed in new (N) system

$\overline{R}_O$ = vector expressed in old (O) system

$\overline{R}_{OO}$ = origin of old system

$\overline{R}_{NO}$ = origin of new system

In summary--add the old and subtract the new.

APPENDIX

RANGE RESOLUTION

Cubic Corporation's DME equipments measure slant range by observing the phase shift of a CW signal. The maximum non-ambiguous range obtained from such a measurement is determined by the wavelength of the signal while the precision is determined by the precision of the phase measuring device. In order to have long range tracking capability and a high degree of precision, the phase shifts of signals at two or more different wavelengths are measured.

The data output by most DME systems corresponds to two or more digital range words. The scaling of these words is chosen (i.e., choice of frequencies) so that the bit weightings form a continuous but overlapping binary word. The overlap is chosen to provide redundant information for use in removing intra-channel bias and noise. The basic assumption is that the combination of intra-channel bias and noise will not exceed the overlap bits.

In order to illustrate a method of range resolution, a two-channel (i.e., two frequency) system is illustrated in Figure B-1. The range resolution algorithm will be shown for this arrangement for simplicity but the generalization to a multi-channel system or one with different length words should be obvious.

Bit Weighting (meters)	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	
	C_8	C_7	C_6	C_5	C_4	C_3	C_2	C_1	COARSE

Bit weighting (meters)	F_8	F_7	F_6	F_5	F_4	F_3	F_2	F_1	FINE
	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	

Figure B-1. Two-Channel Range Resolution

Range Resolution Algorithm

1. Subtract the overlap bits adding a one as shown to force a positive difference.

$$\begin{array}{rlll} & 1 & F_8\ F_7\ F_6 & \text{FINE OVERLAP} \\ - & 0 & C_3\ C_2\ C_1 & \text{COARSE OVERLAP} \\ \hline & b & X_3\ X_2\ X_1 & \text{DIFFERENCE} \end{array}$$

2. Add the difference to the coarse to form the corrected coarse word discarding the carry bit, K. Note that the bit X_3 is repeated to the left.

$$\begin{array}{rll} & C_8\ C_7\ C_6\ C_5\ C_4\ C_3\ C_2\ C_1 & \text{COARSE} \\ + & X_3\ X_3\ X_3\ X_3\ X_3\ X_3\ X_2\ X_1 & \text{DIFFERENCE} \\ \hline K & C_8^1\ C_7^1\ C_6^1\ C_5^1\ C_4^1\ C_3^1\ C_2^1\ C_1^1 & \text{CORRECTED COARSE} \end{array}$$

3. Combine the corrected coarse with the least significant bits of the fine word to obtain the resolved range:

$$C_8^1\ C_7^1\ C_6^1\ C_5^1\ C_4^1\ C_3^1\ C_2^1\ C_1^1\ F_5^1\ F_4^1\ F_3^1\ F_2^1\ F_1^1 \qquad \text{RESOLVED RANGE}$$

4. For another channel, say a VERY COARSE, the corrected COARSE now plays the part of the FINE and the VERY COARSE the role of the COARSE in steps one through three.

EXAMPLE 1

$$\begin{array}{lr} 1\,0\,0\,0\,1\,0\,0\,0 & \text{COARSE} \\ \underline{1\,1\,0\,0\,0\,1\,0\,1} & \text{FINE} \end{array}$$

$$\begin{array}{rlr} & 1\,1\,1\,0 & \text{FINE OVERLAP} \\ - & 0\,0\,0 & \text{COARSE OVERLAP} \\ \hline & 1\,1\,1\,0 & \text{DIFFERENCE} \end{array}$$

(Note X_3= 1)

1 0 0 0 1 0 0 0	COARSE
+ 1 1 1 1 1 1 1 0	DIFFERENCE
1 / 1 0 0 0 0 1 1 0	CORRECTED COARSE
1 0 0 0 0 1 1 0 0 0 1 0 1	RESOLVED RANGE

EXAMPLE 2

1 0 0 0 1 0 0 0	COARSE
0 0 1 0 0 1 0 1	FINE
1 0 0 1	FINE OVERLAP
- 0 0 0	COARSE OVERLAP
1 0 0 1	DIFFERENCE

(Note X_3= 0

1 0 0 0 1 0 0 0	COARSE
+ 0 0 0 0 0 0 0 1	DIFFERENCE
1 / 1 0 0 0 1 0 0 1	CORRECTED COARSE
1 0 0 0 1 0 0 1 0 0 1 0 1	RESOLVED RANGE

APPENDIX C

DATA EDITING

In DME and AMR Digital Processing Units, overlapping frequency or baseline channels with successively higher resolution are processed into a single word. When these overlapping words are combined, high noise levels, intrachannel bias, or spurious bits will occasionally give a disagreement in the channel overlap. Incorrect overlap can cause an ambiguity in the resolved word with a value dependent upon the bit weightings of the overlap. Ambiguities may simultaneously occur in more than one overlap position to give some combination of integral number of least significant ambiguities.

Editing data is the process of recognizing and, if possible, removing ambiguities or spurious samples. The three types of events which occur in the data and are cause for a decision during editing are:

1. Ambiguities
2. Noise
3. Bad data

Ambiguities can generally be removed from data if sufficient non-ambiguous data is available. Noise implies randomness and may be removed to some extent by smoothing. Bad data are meaningless data which cannot be recovered by removal of ambiguities. A limited number of bad data points may be removed by replacement based on a prediction with dynamics established from previous data.

The process of editing data must begin with some criteria for finding "good information" on which to start a testing procedure. One such criterion is to examine spans of data for an average second difference which is within

the limits of the dynamics of the vehicle. This minimizes the possibility that the span contains ambiguous or bad samples. A second criterion is to begin with an estimated data point and first difference. Other criteria may be dictated by the particular system being employed.

Once the criteria for "good information" are satisfied the basic editing procedure begins. In order to reduce the effects of the target's dynamics, the editing is done on the first differences. Thus only acceleration and higher order rates affect the data.

The fundamental decision whether a sample may be edited or not is made by comparing the measured first difference against a predicted first difference. The predicted first difference is computed from a span of previously edited data using a linear extrapolation.

The discrepancy between the measured and computed first differences is:

$$\epsilon_i = \{(\Delta U_i)_P - [U_i - (U_{i-1})_E]\}$$

P = predicted

E = edited

Since all measurements are subject to random errors (δ), ambiguities (nA_L), and bias (Δ), then

$$(\Delta U_i)_P = (\Delta U_i)_T + \bar{\delta}_i, \quad T = \text{true}$$

$$U_i = (U_i)_T + \delta_i + nA_L + \Delta$$

$$(U_i)_E = (U_i)_T + \delta_i + \Delta$$

$$\epsilon_i = [(\Delta U_i)_T + \bar{\delta}_i - (U_i)_T - \delta_i - nA_L - \Delta + (U_{i-1})_T + \delta_{i-1} + \Delta]$$

$$\epsilon_i = [-nA_L + (\bar{\delta}_i + \delta_{i-1} - \delta_i)]$$

where

$n = 0, \pm 1, \pm 2, \ldots$

and

$\bar{\delta}_i$, δ_i are random variables.

It is clear that if the random noise is known to be small compared to A_L, a noise tolerance gate may be used to determine the acceptability of the data sample. That is, whether $|\varepsilon_i + nA_L| < K_{NOISE}$. K_{NOISE} is chosen from some knowledge of the noise content of the data (e.g., the 3σ or 4σ value of random noise). Any sample not meeting this requirement would be assumed to be bad.

The details of finding $|\varepsilon_i + nA_L|$ depend upon the characteristics of the computer used. In any case, $n = \left(\frac{\varepsilon_i}{A_L}\right)$ Nearest Integer.

Once the basic decision is made as to the editability of the sample, a good sample is adjusted by $\pm nA_L$ and a bad sample is either replaced by a predicted value, or if too many successive samples have been found to be bad, a new search for "good information" is initiated.

A flow diagram for an editing procedure, figure C-1, illustrates the general approach and sequence of testing. A starting criteria is used which tests for continuity on the second differences. Symbols used in the flow diagram are defined below.

NOTATION	DEFINITION
$(U_1, U_2, \ldots, U_N)$	Data Span
$(\Delta U_1, \Delta U_2, \ldots, \Delta U_{N-1}) = (U_2 - U_1, U_3 - U_2, \ldots, U_N - U_{N-1})$	First differences
N	Number of samples in data span
M	Number of samples in starting span
$\varepsilon_i = [(\Delta U_i)_P - \Delta U_i]$	Residual
A_L	Least significant ambiguity
$n = (\frac{\varepsilon}{A_L})_{INTEGER}$	Number of ambiguities added (or removed) from data
K_{NOISE}	Noise gate
K_{START}	Maximum average second difference gate for starting

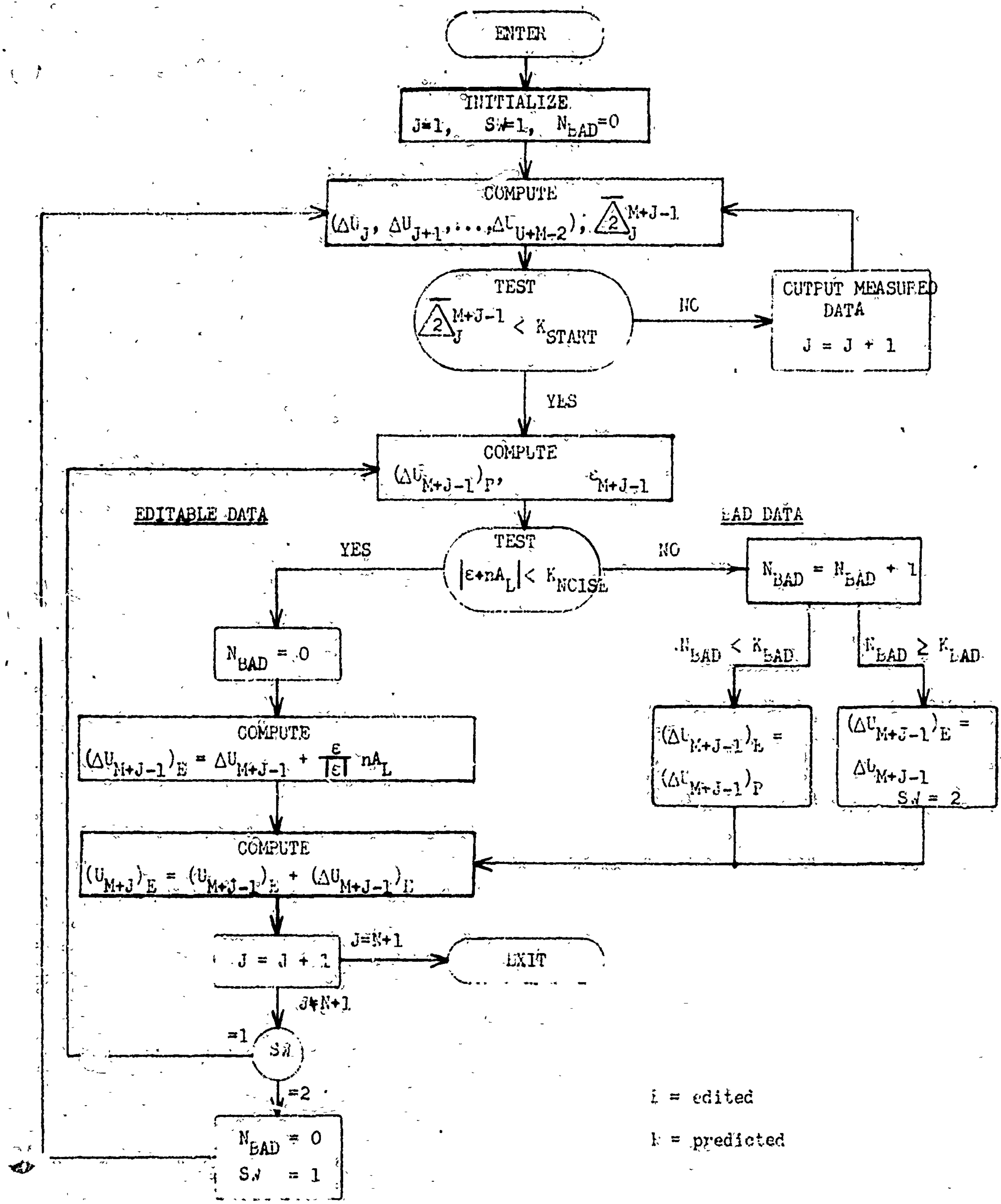

Figure C-1. Data Editing Flow Diagram

NOTATION	DEFINITION
N_{BAD}	Number of successive bad samples
K_{BAD}	Maximum allowable number of successive bad samples
$\overline{\triangle 2}_{\ell}^{m} = \frac{1}{m-\ell-1} \sum_{i=\ell}^{m-1} \left\lvert \Delta U_{i+1} - \Delta U_{i} \right\rvert$	Average second difference

The predicted ΔU's are determined by extrapolating the previous M edited first differences using a least squares polynomial fit.

Because of the finite memory available in computers, long tracks must be sectioned into blocks. In order to use the previous history of the data, the blocks are overlapped so that the starting procedure begins on previously edited data.

A sample of data editing is given in Figure C-2. One set of data has a span of ambiguous samples which are completely removed while the other set has a span of bad samples which are replaced by predicted values until the failure counter exceeds the limit (five in this case).

DCW:kkb

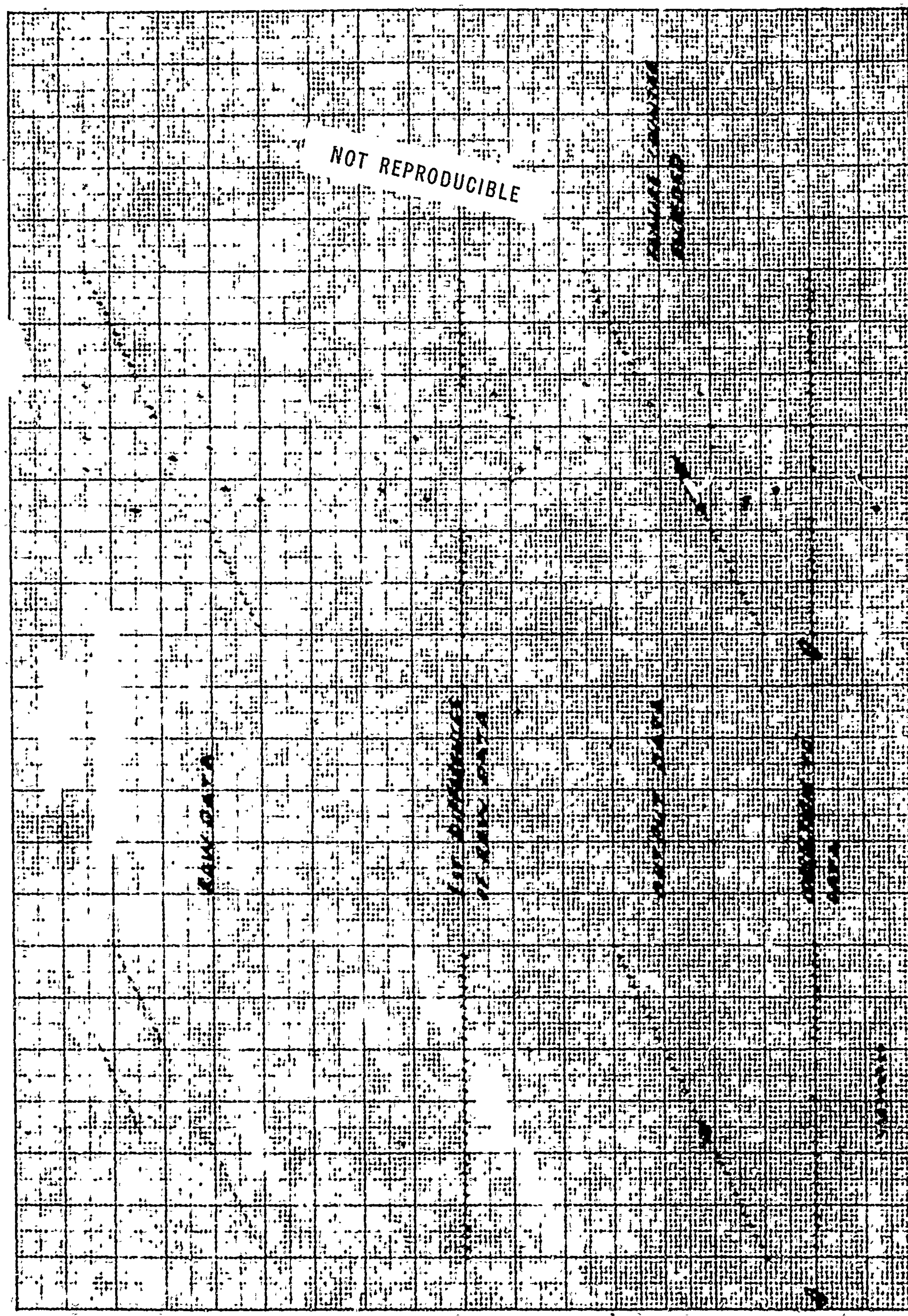

Figure C-2. Data Editing

APPENDIX

LEAST SQUARES MOVING SPAN COEFFICIENTS, SMOOTHING, AND DERIVATIVE COMPUTATION

If it is assumed that a set of data can be approximated by an arbitrary degree polynomial, then a set of least squares weighting coefficients can be precomputed and used to perform the curve fitting. The general form of the solution for position-to-position, position-to-velocity, etc., least squares smoothing is given by

$$U^{L}_{(\beta)} = \frac{1}{\Delta T^{L}} \left[W_1 U_1 + W_2 U_2 + \ldots + W_i (K, L, n, \beta) U_i \right] \tag{1}$$

where

U_i - the input data

$i = 1, 2, \ldots n$ -- number of equally spaced data points in input span (i is odd)

W_i - least squares weighting coefficients which yield $U^{L}_{(\beta)}$

ΔT = time interval between data samples

L - order of derivative (e.g., L = 1 for position to velocity)

K - degree of polynomial approximation

β = lead of output point or position of output point in input span with 21-point, mid-point, first degree zero order coefficients; therefore, $\beta = 11$, $n = 21$, $L = 0$, and $K = 1$.

$U^{L}_{(\beta)}$ = least squares fit at β point in input span.

Weighting coefficients* are given by

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*"Manual for Moving Polynomial Arc Smoothing," by J. K. Sterrett, Ballistics Research Laboratories, Nov. 1952.

$$W_r(K, L, n, \beta) = \sum_{v=L}^{K} \frac{P_{v,n}(i)\, P^L_{v,n}(\beta)}{S_{v,n}} \tag{2}$$

where

$$P_{v,n}(i) = \text{orthogonal polynomials of any kind} \tag{3}$$

$$S_{v,n} = \sum_{i=1}^{n} \left[P_{v,n}(i) \right]^2 \quad \text{the sum of the squares of the orthogonal polynomials} \tag{4}$$

$$P^L_{v,n}(\beta) = \frac{d^L}{d_i^L} P_{v,n}(i) \Big|_{i=\beta} \tag{5}$$

A set of orthogonal polynomials for $v = 0, 1, 2, 3$ and $L = 0, 1, 2$ are:

$$P^o_{o,n} = 1 \tag{6}$$

$$P^o_{1,n}(i) = \left(i - \frac{n+1}{2}\right) \tag{7}$$

$$P^o_{2,n}(i) = \left(i - \frac{n+1}{2}\right)^2 - \frac{n^2 - 1}{12} \tag{8}$$

$$P^o_{3,n}(i) = \frac{5}{6}\left[\left(i - \frac{n+1}{2}\right)^3 - \left(i - \frac{n+1}{2}\right)\left(\frac{3n^2 - 7}{20}\right)\right] \tag{9}$$

$$P^1_{o,n} = 0 \tag{10}$$

$$P^1_{1,n} = 1 \tag{11}$$

$$P^1_{2,n}(i) = 2\left(i - \frac{n+1}{2}\right) \tag{12}$$

$$P^1_{3,n}(i) = \frac{5}{6}\left[3\left(i - \frac{n+1}{2}\right)^2 - \frac{3n^2 - 7}{20}\right] \tag{13}$$

$$P^2_{o,n} = 0 \tag{14}$$

$$P^2_{1,n} = 0 \tag{15}$$

$$P^2_{2,n} = 2 \tag{16}$$

$$P^2_{3,n}(i) = 5\left(i - \frac{n+1}{2}\right) \tag{17}$$

Some typical, previously computed least squares weighting coefficients are tabulated in table D-1.

When equation (1) is used to fit sequential data samples having a variance σ_u^2, the variance of the mean or output sample is given by

$$\sigma^2_{u^L(\beta)} = \sum_{i=1}^{n} \frac{\left[W_i(K, L, n, \beta)\right]^2}{(\Delta T^L)^2} \sigma_u^2 \tag{18}$$

$$C = \left(\sum_{i=1}^{n} \left[W_i(K, L, n, \beta)\right]^2\right)^{\frac{1}{2}} \tag{19}$$

is defined as the mean reduction factor, C. This factor is the root sum square of n smoothing coefficients, and serves as an index of noise reduction of the output point due to polynomial smoothing. By using the mean reduction factor one can intelligently select the appropriate data span, degree of fit, and output point position to obtain optimum refinement of noise reduction. Tabulations of mean reduction factor for various input spans, three different degrees, and three output point positions are shown in tables D-2 and D-3 for position-to-position and position-to-velocity, respectively. In addition, figures D-1 and D-2 plot the variations in mean reduction factor values with variations in lead point (β) for a 25-point span and third degree fit for position-to-position and position-to-velocity, respectively. In the case of position-to-position smoothing, optimum output point position for first and third degree polynomial least squares fit is the mid-point indicated by absolute minimum value of C. For second degree smoothing, either

the one-quarter or three-quarter lead point position is ideal due to the filter symmetry. Further representations of this type of analytic filter are shown in figures D-3 through D-11 which illustrate the filter characteristic response curves to a unity-amplitude sine wave for the 25-, 51-, and 101-point span for the three degrees.

TABLE D-1

SMOOTHING COEFFICIENTS FOR 11 POINTS SPAN

SPAN N	ORDER L	LEAD B	DEGREE K	SMOOTHING COEFFICIENTS										
11	0	6	1	.09091	.09091	.09091	.09091	.09091	.09091	.09091	.09091	.09091	.09091	.09091
11	0	11	1	-.13636	-.09091	-.04545	0	.04545	.09091	.13636	.18182	.22727	.27273	.31818
11	1	6	1	-.04545	-.03636	-.02727	-.01818	-.00909	0	.00909	.01818	.02727	.03636	.04545
11	0	6	2	-.08392	.02098	.10256	.16084	.19580	.20746	.19580	.16084	.10256	.02098	-.08392
11	0	11	2	.12587	.01399	-.06294	-.10490	-.11189	-.08392	-.02098	.07692	.20979	.37762	.58042
11	1	6	2	-.04545	-.03636	-.02727	-.01818	-.00909	0	.00909	.01818	.02727	.03636	.04545
11	0	6	3	-.08392	.02098	.10256	.16084	.19580	.20746	.19580	.16084	.10256	.02098	-.08392
11	0	11	3	-.08392	.05594	.09091	.05594	-.01399	-.80392	-.11888	-.08392	.05594	.33566	.79021
11	1	6	3	.05828	-.05711	-.10334	-.09771	-.05750	0	.05750	.09771	.10334	.05711	-.05828

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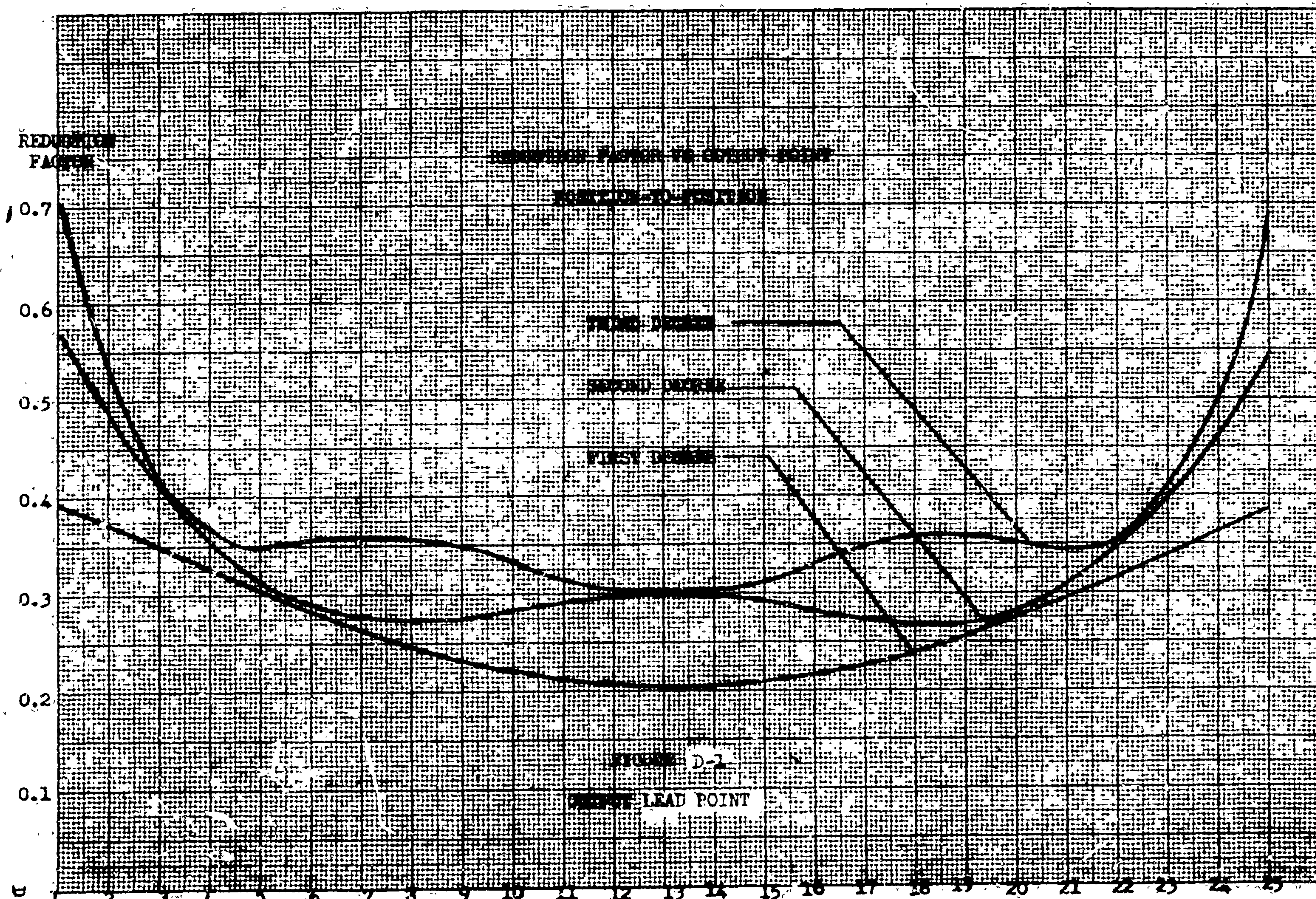
TABLE D-2

POSITION-TO-POSITION "MEAN REDUCTION FACTORS"					
SPAN N	ORDER L	LEAD β	FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
11	0	6	.30151	.45548	.45548
11	0	9	.415608	.41700	.53545
11	0	11	.56407	.76185	.88894
21	0	11	.21822	.32795	.32795
21	0	16	.28300	.29351	.39109
21	0	21	.42130	.59691	.73759
31	0	16	.17961	.26965	.26965
31	0	24	.24096	.24429	.31901
31	0	31	.35069	.50534	.63959
41	0	21	.15617	.23437	.23437
41	0	31	.20447	.21050	.27877
41	0	41	.30672	.44655	.57165
51	0	1	.27599	.40413	.52133
51	0	26	.14004	.21012	.21012
51	0	44	.22120	.23441	.24362
51	0	51	.27600	.40413	.52133
75	0	38	.11546	.17323	.17323
75	0	57	.15362	.15646	.20530
75	0	75	.22865	.33737	.43972
101	0	51	.09950	.14926	.14926
101	0	76	.13107	.13435	.17729
101	0	101	.19753	.29269	.38368
151	0	76	.08136	.12207	.12207
151	0	114	.10798	.11015	.14476
151	0	151	.16196	.24094	.31760

TABLE D-3

POSITION-TO-VELOCITY "MEAN REDUCTION FACTORS"					
SPAN N	ORDER L	LEAD β	FIRST DEGREE	SECOND DEGREE	THIRD DEGREE
11	1	6	.09535	.09535	.24572
11	1	9	.09535	.22594	.25446
11	1	11	.09535	.35446	.80949
21	1	11	.03604	.03604	.09082
21	1	16	.03604	.07587	.07676
21	1	21	.03604	.13831	.32737
31	1	16	.02008	.02008	.05039
31	1	24	.02008	.04496	.04755
31	1	31	.02008	.07805	.18770
41	1	21	.01320	.01320	.03307
41	1	31	.01320	.02824	.02883
41	1	41	.01320	.05165	.12531
51	1	1	.00951	.03738	.09120
51	1	26	.00951	.00951	.02381
51	1	44	.00951	.02771	.04279
51	1	51	.00951	.03738	.09120
75	1	38	.00533	.00533	.01334
75	1	57	.00533	.01175	.01225
75	1	75	.00533	.02108	.05181
101	1	51	.00341	.00341	.00854
101	1	76	.00341	.00738	.00759
101	1	101	.00341	.01353	.03339
151	1	76	.00187	.00187	.00467
151	1	114	.00187	.00409	.00425
151	1	151	.00187	.00742	.01840


0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
SECOND DEGREE
FIRST DEGREE
D-1
LEAD POINT

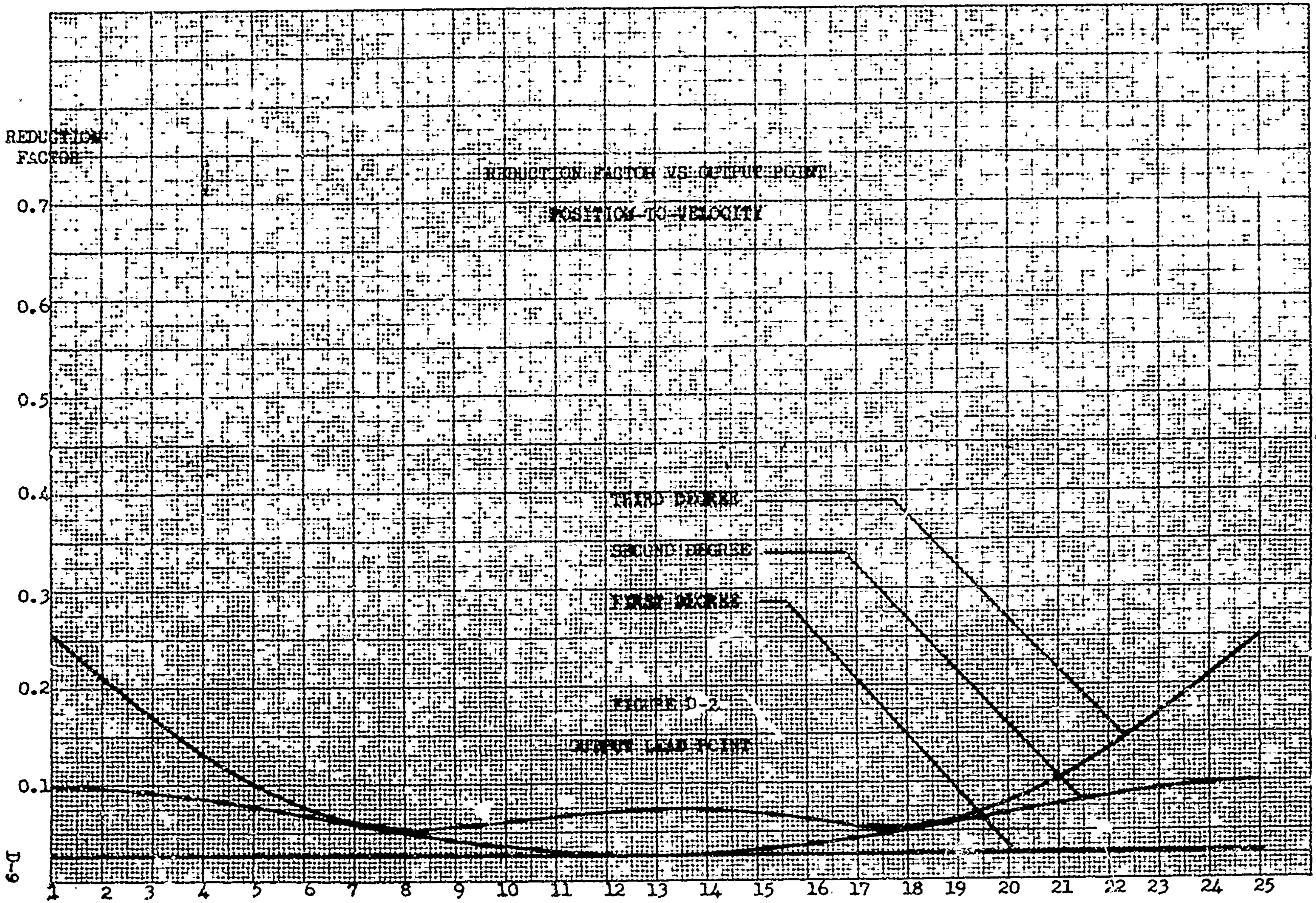


FIGURE D-2
OUTPUT LEAD POINT

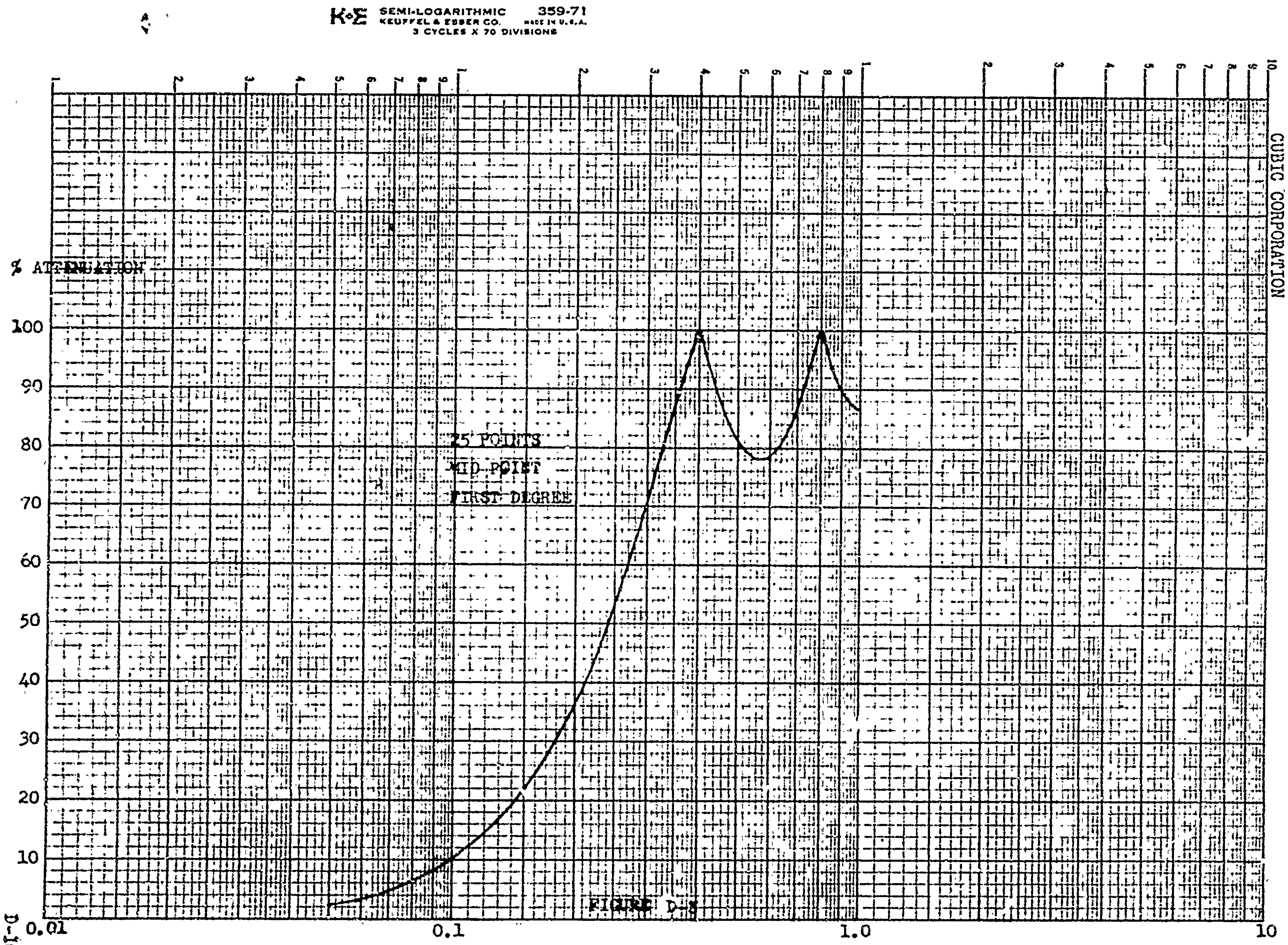
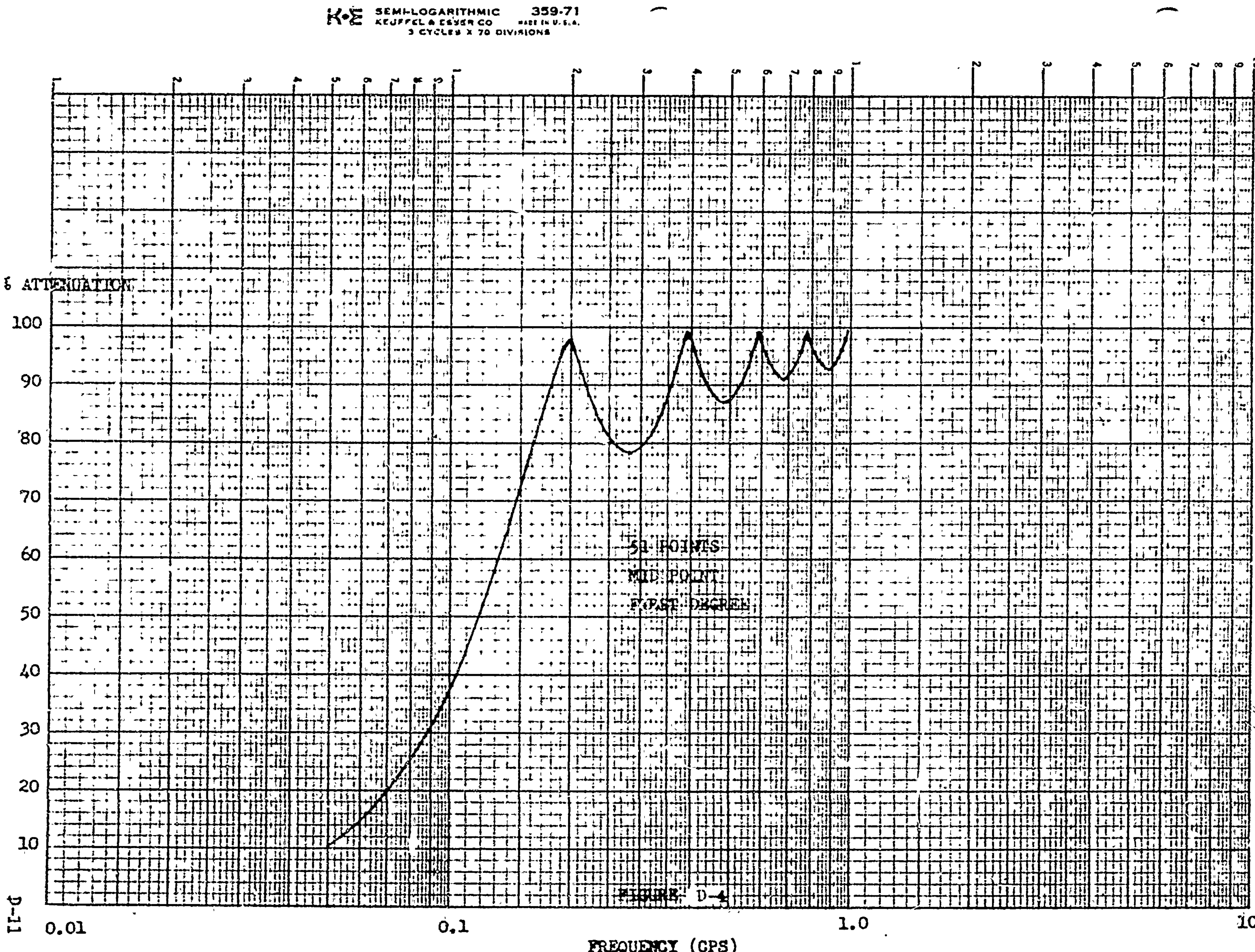

K+E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
CUBIC CORPORATION
% ATTENUATION
100
90
80
70
60
50
40
30
20
10
0.01
0.1
1.0
10
25 POINTS
MID POINT
FIRST DEGREE
FREQUENCY (CPS)

FIGURE D-8



K•E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
CUBIC CORPORATION
% ATTENUATION
100
90
80
70
60
50
40
30
20
10
51 POINTS
MID POINT
FIRST DEGREE
0.01
0.1
1.0
10
FREQUENCY (CPS)

FIGURE D-4

K&E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS

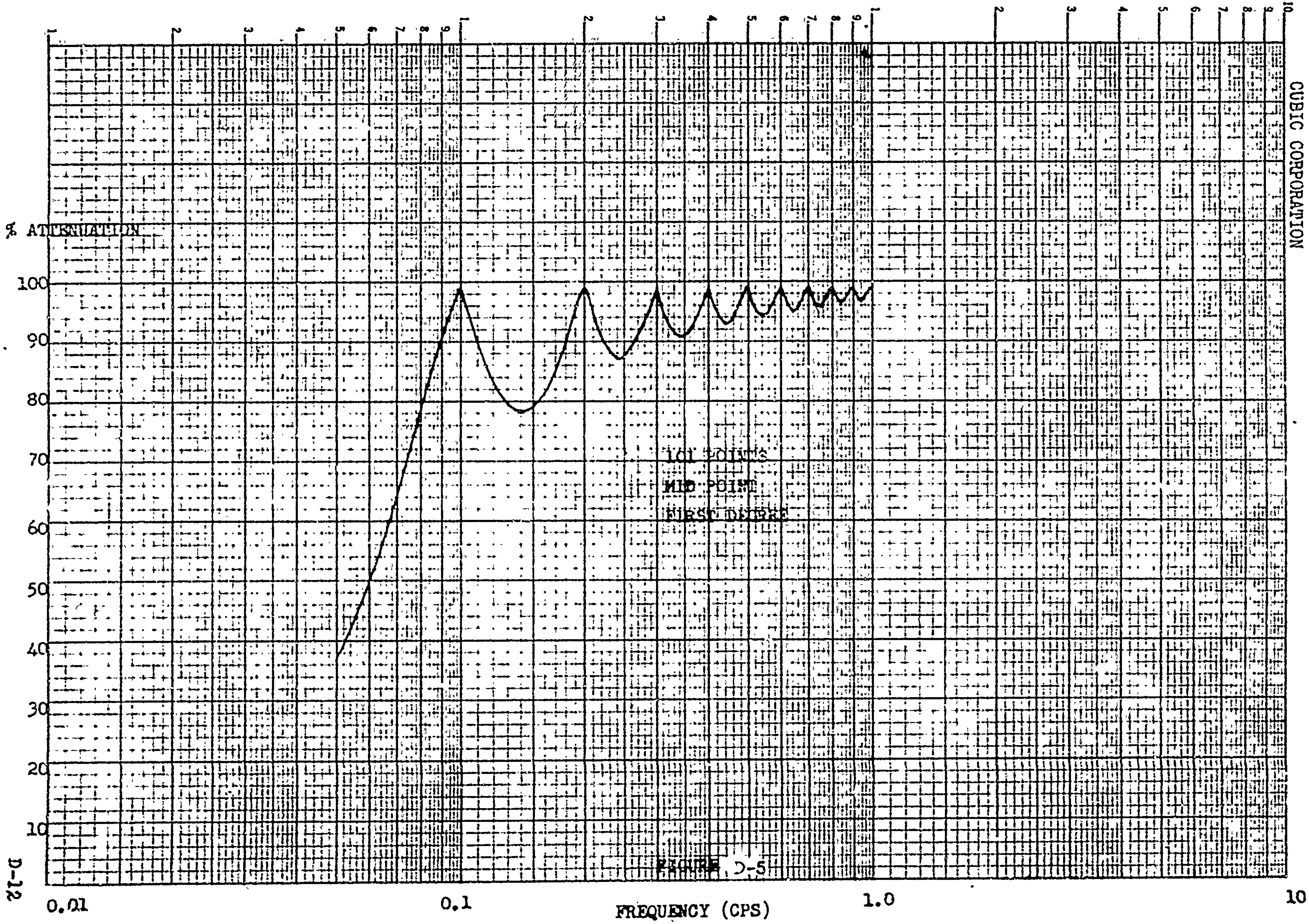
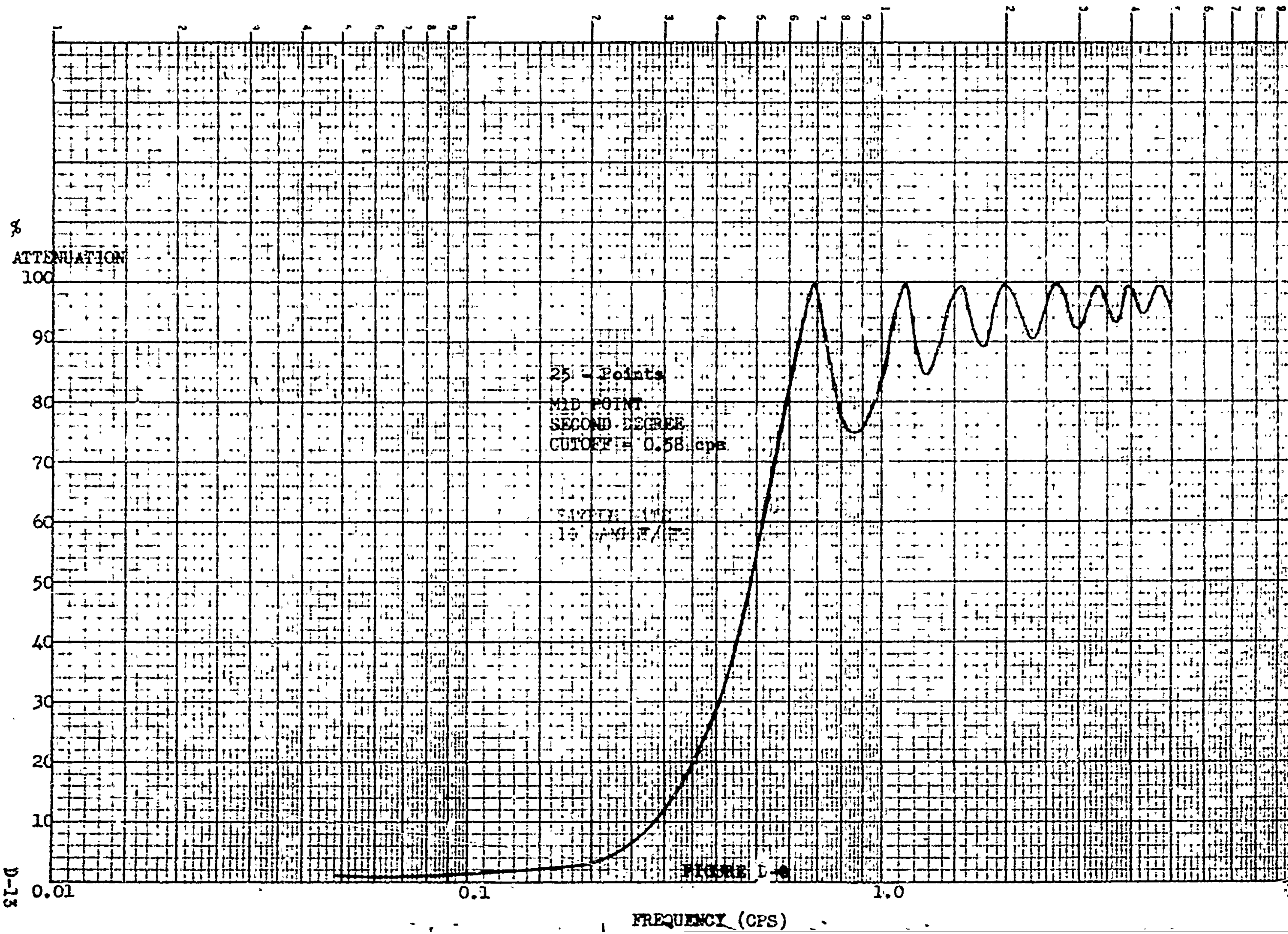


FIGURE D-5

CUBIC CORPORATION



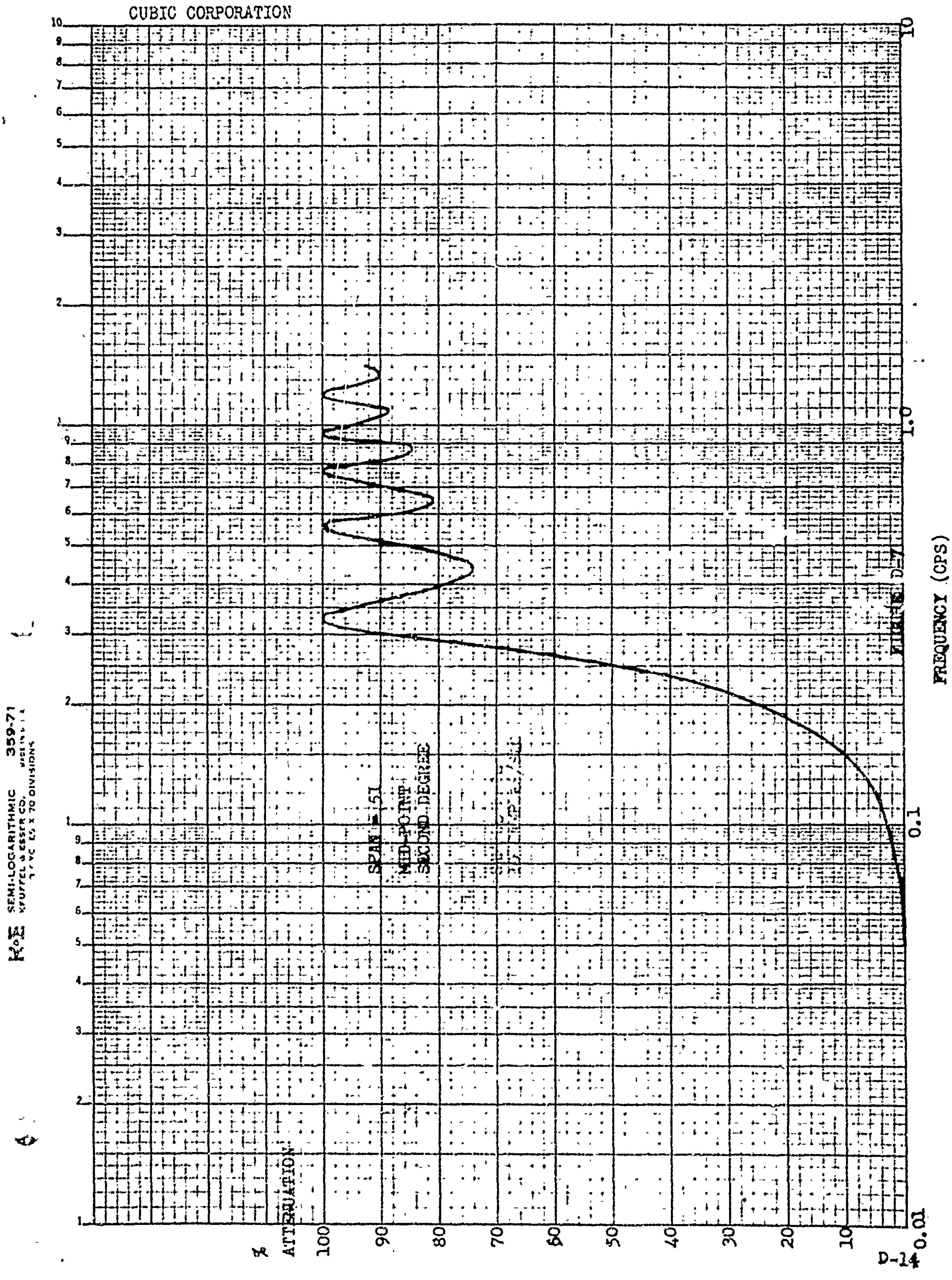

CUBIC CORPORATION
SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
SPAN = 51
MID-POINT
SECOND DEGREE
% ATTENUATION
100
90
80
70
60
50
40
30
20
10
0.01
0.1
1.0
10
FREQUENCY (CPS)

FIGURE D-7

CUBIC CORPORATION

K&E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS

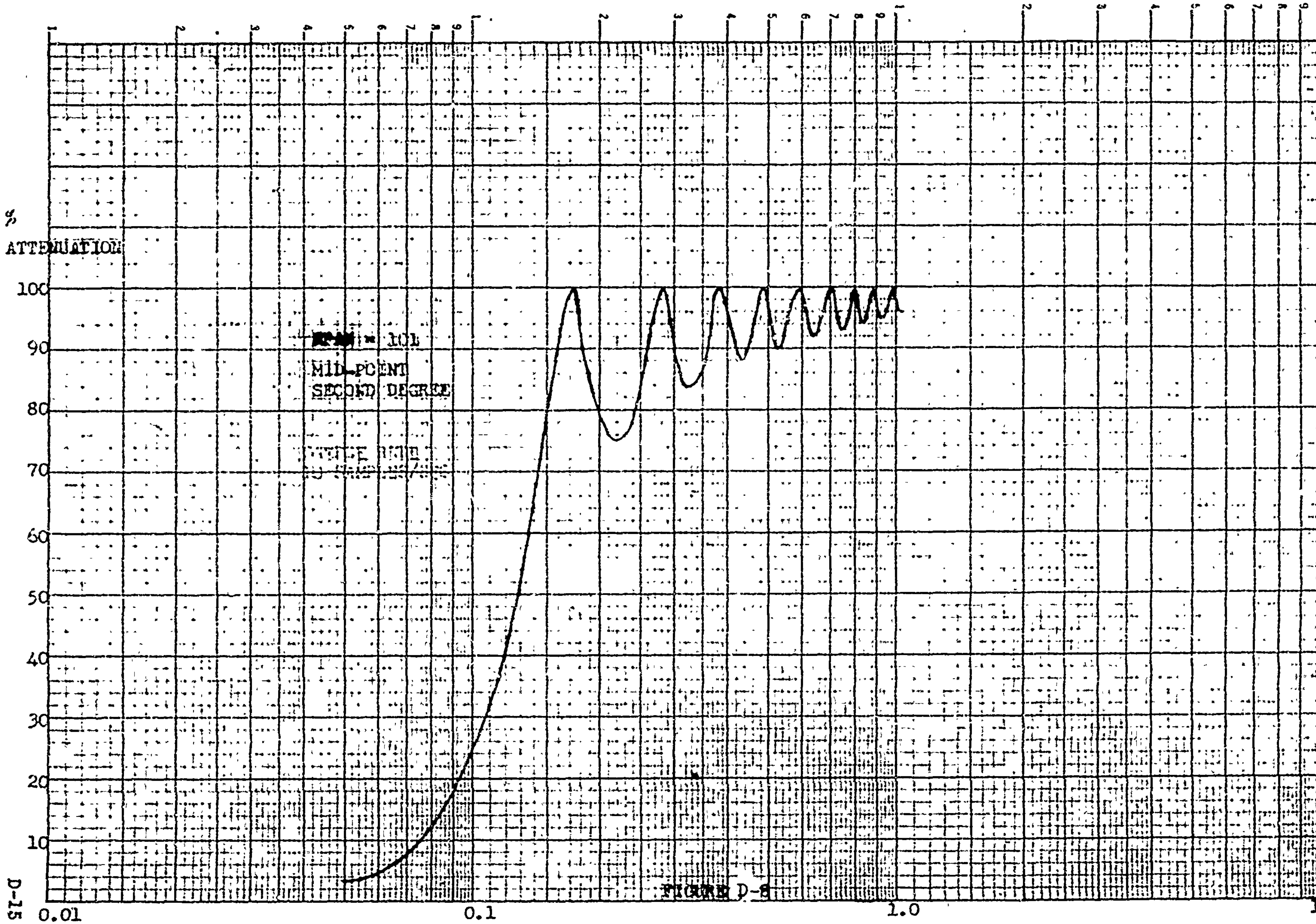
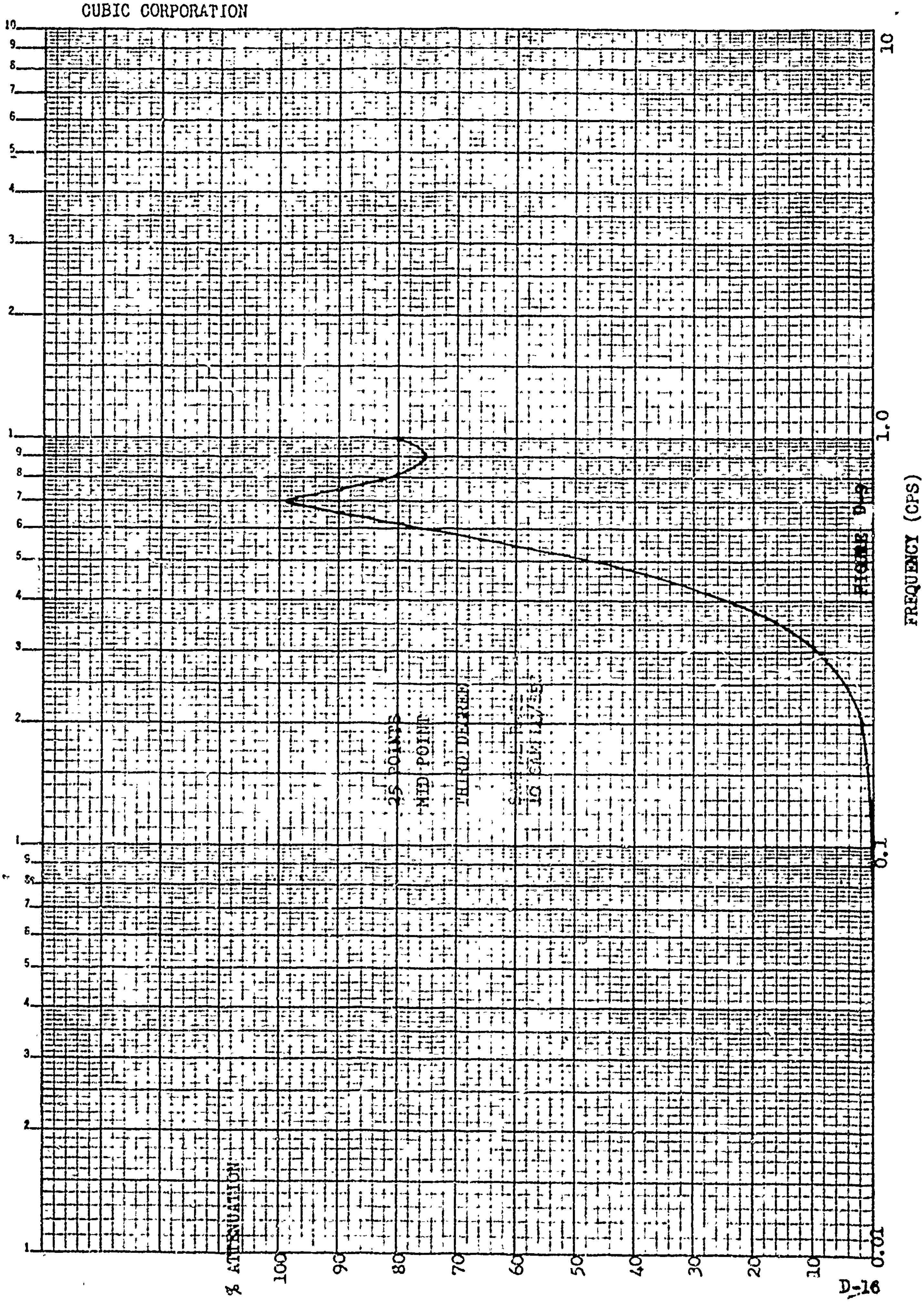


FIGURE D-8

CUBIC CORPORATION



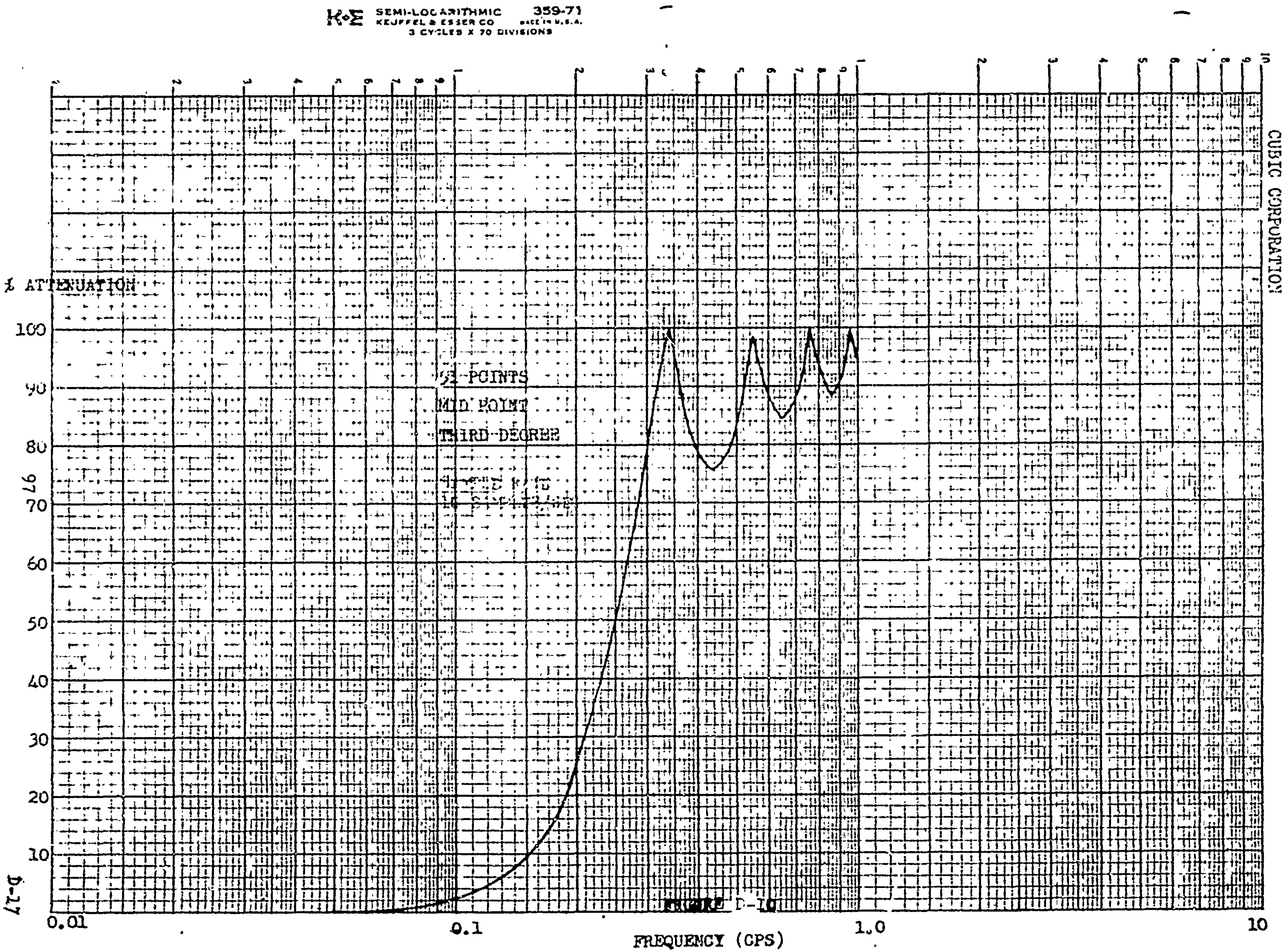

K&E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
% ATTENUATION
100
90
80
70
60
50
40
30
20
10
0.01
0.1
1.0
10
FREQUENCY (CPS)
32 POINTS
MID POINT
THIRD DEGREE

FIGURE D-10

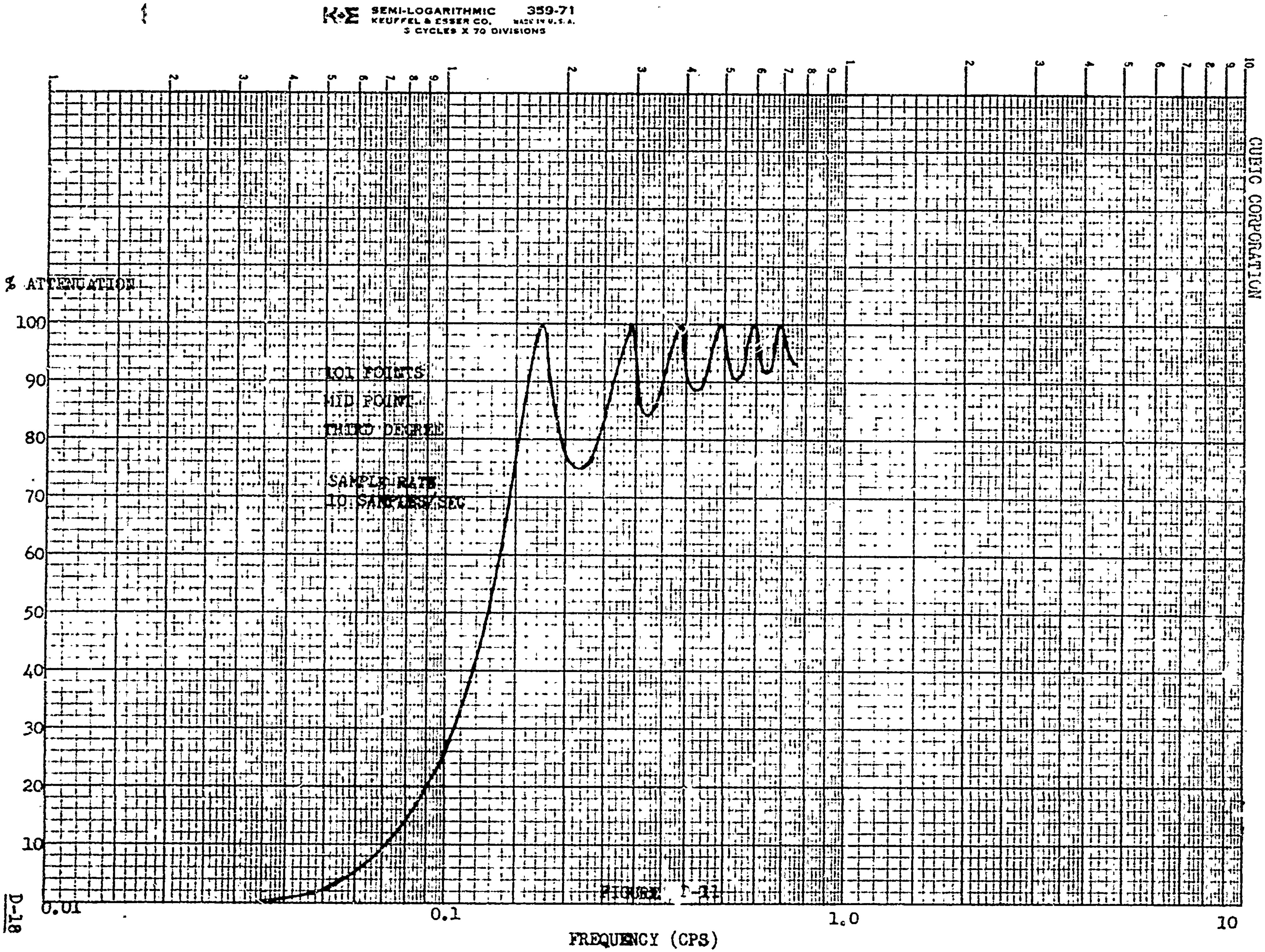

K+E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS
CUBIC CORPORATION
% ATTENUATION
100
90
80
70
60
50
40
30
20
10
101 POINTS
MID POINT
THIRD DEGREE
SAMPLE RATE
10 SAMPLES/SEC
0.01
0.1
1.0
10
FREQUENCY (CPS)

FIGURE I-31

APPENDIX E

EARTH-REFERENCED COORDINATE SYSTEMS

Equatorial Coordinate System (X_E, Y_E, Z_E)

The equatorial coordinate system is a right-handed Cartesian system with origin at the earth's center of mass. The positive Z_E axis is oriented along the earth's rotational axis toward the north pole. The X_E axis lies in the equatorial plane and passes through the prime meridian and the earth's center of mass. The positive Y_E axis lies in the equatorial plane and is 90° counterclockwise from X_E. **(See figure E-1.)**

Geodetic Coordinate System (φ, λ, h)

Geodetic coordinates are expressed in terms of spheroidal angles and the height above the spheroid. Longitude (λ) is the angle in the equatorial plane between the X_E axis and the projection of the radius vector. Longitude is measured positive in a counterclockwise direction from the positive X_E axis. Geodetic latitude is the angle subtended with the equatorial plane by the normal to the spheroid which passes through the point. The height is the distance of the point above (or below) the spheroid measured along the local normal. **(See figures E-1 and E-2.)**

Conversion between Geodetic and Equatorial Coordinates

Conversions between the geodetic and equatorial coordinate systems depend upon the spheroid constants used to represent the earth. As a matter of common definition, the earth is assumed to be an ellipsoid of revolution about the polar axis. This figure may be defined by specifying the semi-major and semi-minor axes, a and b respectively. The general equation for an ellipsoid is:

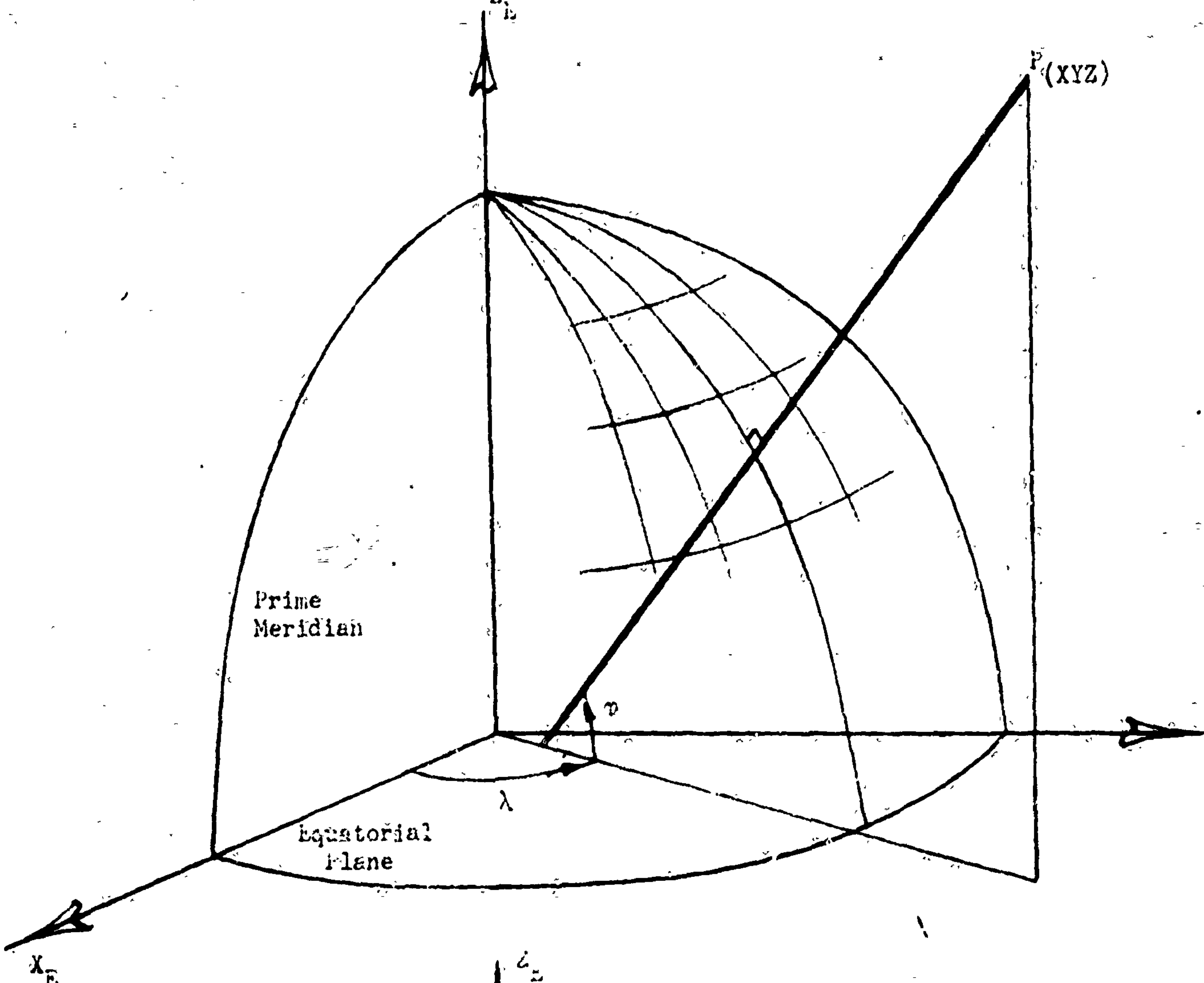


Figure E-1

Equatorial Coordinate System

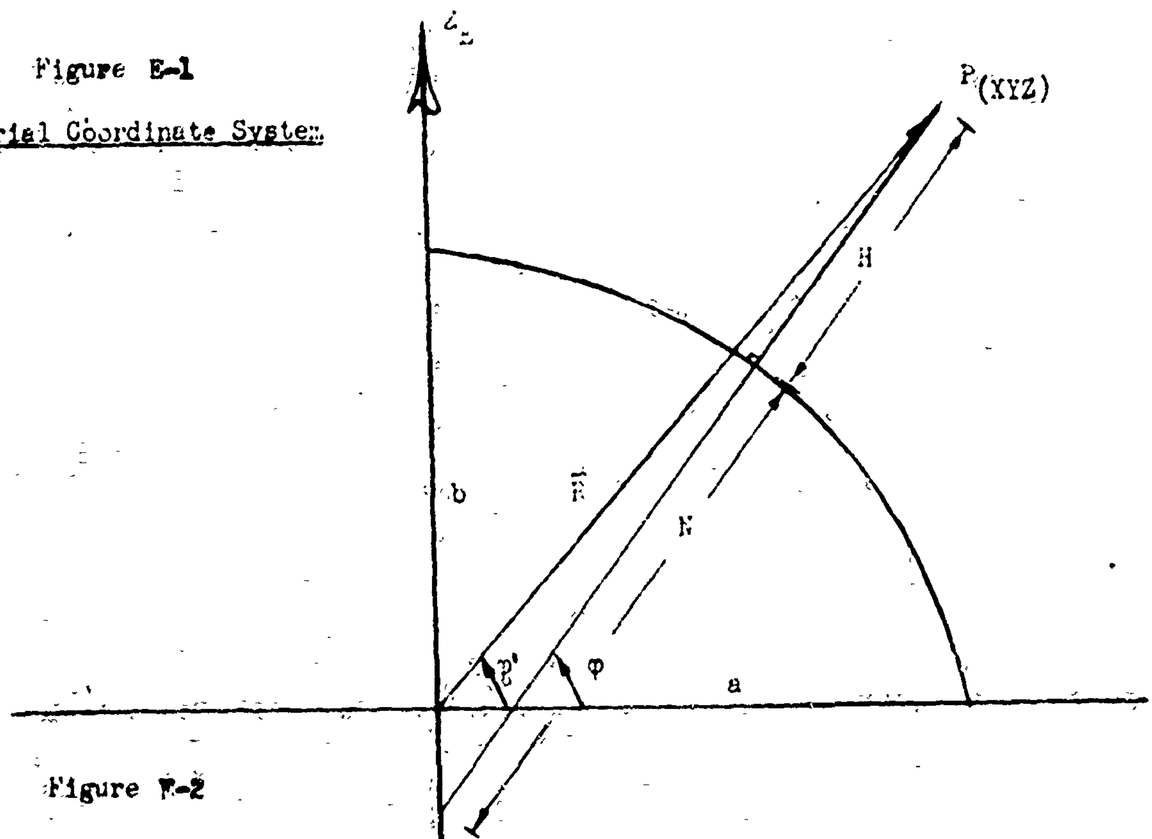


Figure E-2

Geodetic Coordinate System

$$\frac{X_E^2 + Y_E^2}{a^2} + \frac{Z_E^2}{b^2} = 1 \qquad (1)$$

Further parameters of the ellipsoid are defined as:

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad (2)$$

$$f = \frac{a - b}{a} \qquad (3)$$

where e = eccentricity

f = flattening

It is often necessary to convert the (XYZ) equatorial coordinates of a point into geodetic latitude, longitude, and height. The following equations give the conversion between geodetic and equatorial coordinates.[1]

It can be shown that

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}} \qquad (4)$$

$$X_E = (N + h) \cos \varphi \cos \lambda \qquad (5)$$

$$Y_E = (N + h) \cos \varphi \sin \lambda \qquad (6)$$

$$Z_E = [N(1 - e^2) + h] \sin \varphi \qquad (7)$$

Determination of geodetic longitude from the equatorial coordinates is given by

$$\lambda = \tan^{-1} \left(\frac{Y_E}{X_E} \right) \qquad (8)$$

with the appropriate quadrant selected from the signs of X_E and Y_E.

[1] Schreiter, J. B., The Need for Space Rectangular Coordinates in Modern Geodetic Operations, The Ohio State University Research Foundation, Project No. 378, Astia #ATI-00538.

Computation of geodetic latitude is not possible in closed form. An iterative solution for latitude is accomplished as follows:[2]

(1) estimate h and φ from:

$$h = +\sqrt{X_E^2 + Y_E^2 + Z_E^2} - b \tag{9}$$

$$\varphi = \sin^{-1}\left[\frac{Z_E}{\sqrt{X_E^2 + Y_E^2 + Z_E^2}}\right] \tag{10}$$

(2) calculate t:

$$t = \frac{(1 + k)Z_E - k(h \sin \varphi)}{r} \tag{11}$$

$$k = \frac{a^2 - b^2}{b^2} \tag{12}$$

$$r = +\sqrt{X_E^2 + Y_E^2} \tag{13}$$

(3) calculate φ:

$$\varphi = \tan^{-1}\left[\frac{(1 + k + t^2)Z_E + kt(t\sqrt{1 + k + t^2} - r)}{(1 + t^2)r}\right] \tag{14}$$

(4) calculate h:

$$h = \frac{r}{\cos \varphi} - N \tag{15}$$

Steps (2) through (4) are iterated to yield the desired result. Test cases run for heights up to 1000 n. miles indicate a convergence of $\tan \varphi$ to 10^{-8} in two iterations.

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[2] Schreiter, Ibid.

APPENDIX F

CARTESIAN POSITION, VELOCITY, ACCELERATION FROM RANGE AND RANGE RATE OBSERVATIONS

Three Range to Position (XYZ)

Let $(XYZ)_1$, $(XYZ)_2$, and $(XYZ)_3$ be the locations of three distance measuring equipment (i.e., DME) sites relative to some local coordinate system. Let (XYZ) be the unknown cartesian coordinates of the vehicle relative to this same coordinate system and R_1, R_2, R_3 are measured slant ranges to the vehicle.

The basic equations relating the measured quantities to the vehicle position are:

$$\begin{aligned} R_1^2 &= (x - X_1)^2 + (y - Y_1)^2 + (z - Z_1)^2 \\ R_2^2 &= (x - X_2)^2 + (y - Y_2)^2 + (z - Z_2)^2 \\ R_3^2 &= (x - X_3)^2 + (y - Y_3)^2 + (z - Z_3)^2 \end{aligned} \tag{1}$$

The solution of this set of equations may be obtained by eliminating unknowns by successive substitutions; however, the resulting solution is rather complex and has sign ambiguities due to the quadratic nature of the equations. A simpler solution is possible if a temporary coordinate system is used. This temporary system has its origin at one of the three trackers--say site one so that $X_1' = Y_1' = Z_1' = 0$. Axes of this system are oriented so that the $X' - Y'$ plane contains the other two trackers (i.e., $Z_2' = Z_3' = 0$). The orientation of the X' and Y' axes is arbitrary since the results will be transformed back to the original system. The transformation from the

unprimed to the primed system will be

$$\bar{R}' = T \cdot (\bar{R} - \bar{R}_1) \quad (2)$$

where:

$$\bar{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \bar{R}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}, \quad \bar{R}_1 = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad (3)$$

and T is the rotation matrix.

The T rotation matrix may be found from the composition of rotations about the x and y axes and requiring that $Z_2' = Z_3' = 0$, hence,

$$T = T_\alpha T_\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$T = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ -\sin\alpha\sin\beta & \cos\alpha & \sin\alpha\cos\beta \\ -\sin\beta\cos\alpha & -\sin\alpha & \cos\alpha\cos\beta \end{bmatrix} \quad (4)$$

Now,

$$Z_2' = -X_2\sin\beta\cos\alpha - Y_2\sin\alpha + Z_2\cos\alpha\cos\beta = 0 \quad (5)$$

$$Z_3' = -X_3\sin\beta\cos\alpha - Y_3\sin\alpha + Z_3\cos\alpha\cos\beta = 0 \quad (6)$$

and upon dividing equations (5) and (6) by $(-\cos\alpha\cos\beta)$,

$$X_2\tan\beta + Y_2\frac{\tan\alpha}{\cos\beta} - Z_2 = 0 \quad (7)$$

$$X_3\tan\beta + Y_3\frac{\tan\alpha}{\cos\beta} - Z_3 = 0 \quad (8)$$

Solving for tan β and tan α gives

$$\tan \beta = \left[\frac{Z_2Y_3 - Z_3Y_2}{X_2Y_3 - X_3Y_2}\right] \tag{9}$$

$$\tan \alpha = \left[\frac{X_2Z_3 - X_3Z_2}{X_2Y_3 - X_3Y_2}\right] \cos \beta. \tag{10}$$

With tan β and tan α known, the T rotation matrix can be computed.

When the primed coordinate system is used, the basic range equations become:

$$R_1^2 = x'^2 + y'^2 + z'^2 \tag{11}$$

$$R_2^2 = (x' - X_2')^2 + (y' - Y_2')^2 + z'^2 \tag{12}$$

$$R_3^2 = (x' - X_3')^2 + (y' - Y_3')^2 + z'^2 \tag{13}$$

Subtracting equation (11) from equations (12) and (13) yields

$$-\frac{1}{2}\left(R_2^2 - R_1^2 - r_2^2\right) = X_2'x' + Y_2'y'$$

$$-\frac{1}{2}\left(R_3^2 - R_1^2 - r_3^2\right) = X_3'x' + Y_3'y' \tag{14}$$

where: $r_i^2 = X_i'^2 + Y_i'^2$

Solving the linear system in x' and y' by Cramer's rule,

$$x' = \frac{1}{2\Delta}\begin{vmatrix} (R_1^2 - R_2^2 + r_2^2) & Y_2' \\ (R_1^2 - R_3^2 + r_3^2) & Y_3' \end{vmatrix} \tag{15}$$

$$y' = \frac{1}{2\Delta}\begin{vmatrix} X_2' & (R_1^2 - R_2^2 + r_2^2) \\ X_3' & (R_1^2 - R_3^2 + r_3^2) \end{vmatrix} \tag{16}$$

$$\Delta = \begin{vmatrix} X_2' & Y_2' \\ X_3' & Y_3' \end{vmatrix} \tag{17}$$

Expanding and grouping the constant terms gives

$$x' = K_1 R_1^2 + K_2 R_2^2 + K_3 R_3^2 + K_4 \tag{18}$$

$$y' = K_5 R_1^2 + K_6 R_2^2 + K_7 R_3^2 + K_8 \tag{19}$$

$$\begin{aligned} K_1 &= \frac{1}{2\Delta}(Y_3' - Y_2') & K_5 &= \frac{1}{2\Delta}(X_2' - X_3') \\ K_2 &= -\frac{1}{2\Delta}(Y_3') & K_6 &= \frac{1}{2\Delta}(X_3') \\ K_3 &= \frac{1}{2\Delta}(Y_2') & K_7 &= -\frac{1}{2\Delta}(X_2') \\ K_4 &= \frac{1}{2\Delta}(Y_3' r_2^2 - Y_2' r_3^2) & K_8 &= \frac{1}{2\Delta}(X_2' r_3^2 - X_3' r_2^2) \end{aligned} \tag{20}$$

and with $x'y'$ known, z' may be found by

$$z' = \pm\sqrt{R_1^2 - x'^2 - y'^2} \tag{21}$$

In most applications the sign of z' is easily determined. For example, with ground based trackers and an airborne vehicle, z' would be positive.

The solution in the primed coordinate system can be transformed into the unprimed system as follows:

$$\vec{r} = T^T \cdot \vec{r}' + \vec{r}_1 \tag{22}$$

where:

$$T^1 = T_\beta^T \cdot T_\alpha^T \tag{23}$$

Range Rate and Acceleration to $(\dot{X}\dot{Y}\dot{Z})$ and $(\ddot{X}\ddot{Y}\ddot{Z})$

Suppose that R_i, $\dot{R}_i$, and $\ddot{R}_i$ are determined from each of three known sites and it is desired to compute the velocity and acceleration in cartesian coordinates. The basic relationships at each site are given by

$$R_i^2 = (x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2 \tag{24}$$

$$R_i\dot{R}_i = [(x - X_i)\dot{x} + (y - Y_i)\dot{y} + (z - Z_i)\dot{z}] \tag{25}$$

$$R_i\ddot{R}_i + \dot{R}_i^2 = [(x - X_i)\ddot{x} + (y - Y_i)\ddot{y} + (z - Z_i)\ddot{z}] + \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \tag{26}$$

where: X_i, Y_i, Z_i are known site locations assumed to be fixed.

The quadratic set of equations (24) may be solved for x, y, z as described in the accompanying section. The set of equations resulting from (25) and (26) reduce to the linear forms:

$$\bar{V} = [C]^{-1} [\dot{R}] \tag{27}$$

and

$$\bar{A} = [C]^{-1} \{[\ddot{R}] + [r]\} \tag{28}$$

where:

$$\bar{V} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \tag{29}$$

$$[C] = \begin{bmatrix} (x - X_1)/R_1 & (y - Y_1)/R_1 & (z - Z_1)/R_1 \\ (x - X_2)/R_2 & (y - Y_2)/R_2 & (z - Z_2)/R_2 \\ (x - X_3)/R_3 & (y - Y_3)/R_3 & (z - Z_3)/R_3 \end{bmatrix} \tag{30}$$

$$[\dot{R}] = \begin{bmatrix} \dot{R}_1 \\ \dot{R}_2 \\ \dot{R}_3 \end{bmatrix}, \quad [\ddot{R}] = \begin{bmatrix} \ddot{R}_1 \\ \ddot{R}_2 \\ \ddot{R}_3 \end{bmatrix} \tag{31}$$

$$[r] = \begin{bmatrix} (\dot{R}_1^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2)/R_1 \\ (\dot{R}_2^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2)/R_2 \\ (\dot{R}_3^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2)/R_3 \end{bmatrix} \tag{32}$$

Two Ranges and Altitude to Position (XYZ)

Let $(XYZ)_1$ and $(XYZ)_2$ be the locations of two distance measuring equipment (i.e., DME) sites relative to some local coordinate system. Let (xyz) be the unknown cartesian coordinates of the vehicle relative to this same coordinate system. Furthermore, let R_1, R_2 be the measured slant ranges to the vehicle and let h denote the altitude of the vehicle.

The basic equations relating the measured quantities to the vehicle position are:

$$R_1^2 = (x - X_1)^2 + (y - Y_1)^2 + (z - Z_1)^2$$

$$R_2^2 = (x - X_2)^2 + (y - Y_2)^2 + (z - Z_2)^2$$

$$R_1^2 = (\rho_1 + h_1)^2 + (\rho + h)^2 - 2(\rho_1 + h_1)(\rho + h) \cos \psi$$

$$\text{where:} \quad \cos \psi = \frac{\rho_1 + h_1 + z}{\rho + h}$$

It is assumed that the altitude of the vehicle (h) is sufficiently small so that the local deflection of the normal does not introduce a significant error. (See figure F-1.)

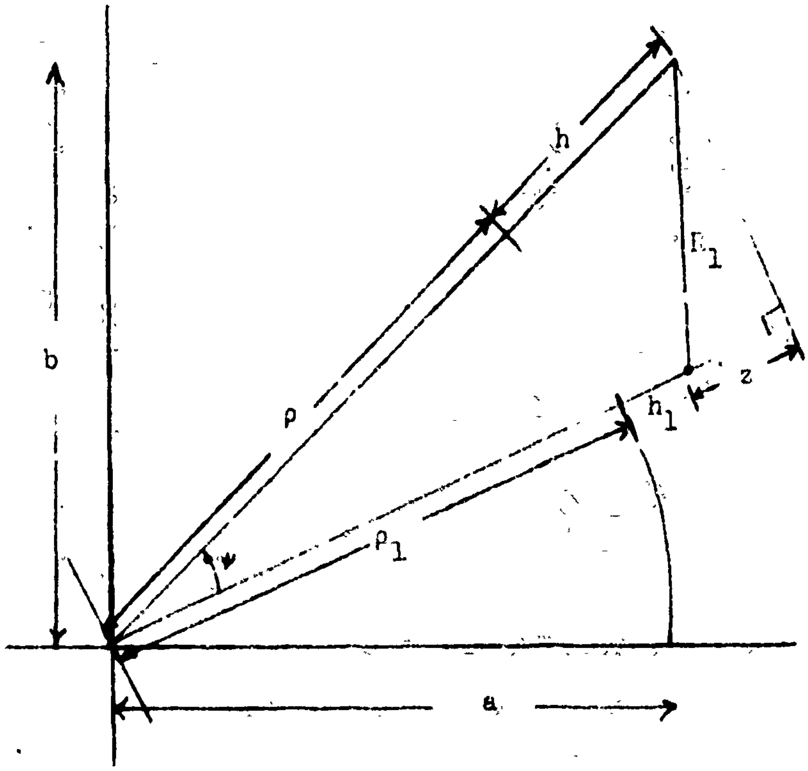


Figure F-1. Geometry for Two-Range and Altitude Solution

The above system of equations may be simplified by transforming to a temporary cartesian coordinate system. This system is denoted as the primed system and is defined as follows: (1) center at one of the tracking sites (site 1); (2) z' axis along the radius vector from the center of the earth; (3) the y' axis is normal to the z' axis and oriented such that $X_2' = 0$; (4) the x' axis completes the right-handed cartesian coordinate system.

The transformation to the primed system is given by:

$$\bar{r}' = T\cdot(\bar{r} - \bar{R}_1)$$

$$\text{where:}\quad \bar{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix},\quad \bar{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\quad \bar{R}_1 = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = \tan^{-1}\left(-\frac{X_2}{Y_2}\right)$$

The above set of equations in the primed system is:

$$R_1^2 = x'^2 + y'^2 + z'^2$$

$$R_2^2 = x'^2 + (y' - Y_2')^2 + (z' - Z_2')^2$$

$$R_1^2 = (\rho_1 + h_1)^2 + (\rho + h)^2 - 2(\rho_1 + h_1)(\rho_1 + h_1 + z')$$

This set of equations is not soluble in closed form since the radius of the earth (ρ) at the latitude of the vehicle is not known. An iterative solution is used where initially it is assumed that $\rho = \rho_1$. The resulting solution may be expressed in latitude, longitude, and height relative to some spheroid. Using this latitude, a new approximation of ρ may be computed and compared with the previous value. The solution is iterated until $/\rho^{(i+1)} - \rho^{(i)}/ <$ LIMIT, where LIMIT is some specified convergence limit.

The solution of the above equations for a given ρ proceeds as follows:

(1) The third equation is solved for z':

$$z' = \frac{(\rho + h)^2 - (\rho_1 + h_1)^2 - R_1^2}{2(\rho_1 + h_1)}$$

(2) Subtracting the first two equations:

$$R_2^2 - R_1^2 = -2Y_2' \; y' - 2Z_2' \; z' + Y_2'^2 + Z_2'^2$$

(3) Solving for y':

$$y' = \frac{R_1^2 - R_2^2 + Y_2'^2 + Z_2'^2 - 2Z_2' \; z'}{2Y_2'}$$

(4) Solving the first equation for x':

$$x' = \pm\sqrt{R_1^2 - y'^2 - z'^2}$$

The solution in the primed system for a given ρ is complete except for the sign of x'. This sign ambiguity results from the quadratic nature of the basic set of equations and must be resolved by a knowledge of on which side of the baseline the vehicle lies.

The solution in the primed system $(\bar{r}')$ may now be transformed back into the original coordinate system by:

$$\bar{r} = T^T \cdot \bar{r}' + \bar{R}_1$$

The latitude of the vehicle must now be determined from $\bar{r}$ by using the relationship between the unprimed coordinate system and the equatorial system.

The relationship between the radius (ρ) and the latitude ($\emptyset$) is given by:

$$\rho = \left(b^2 + \frac{a^2 e^2 \cos \varphi}{1 - e^2 \sin^2 \varphi} \right)^{\frac{1}{2}}$$

where a, b, e are the semi-major and semi-minor axes and eccentricity of the reference spheroid.

APPENDIX G

ANALYTIC TROPOSPHERIC REFRACTION CORRECTION

Empirical formulae for refraction correction may be computed quickly and require a minimum amount of meteorological data for their use. Because of these features, empirical formulae are useful when high accuracy is not the primary requirement. Given here are empirical refraction correction formulae that require only the index of refraction at the surface, maximum range adjustment at zenith and zero elevation angle. The tropospheric refraction correction is accurate to about 10 per cent of the correction.

The retardation in range due to propagation in the lower atmosphere is given approximately by:

$$\Delta R = \frac{K_1 \left(1 - e^{-ZR}\right)}{\sin E_o + K_2 \cos E_o} \qquad (1)$$

where $K_1 \simeq$ 2.6 meters.......the tropospheric refraction at zenith

$K_2 \simeq$ 0.0236...........a control constant

Z = 1/(22500 ft).....a control constant

R = slant range in feet

E_o = incident elevation angle

Angular bending in the vertical plane is given by

$$\Delta E = \left[1 - \frac{1}{ZR}\left(1 - e^{-ZR}\right)\right] 2\gamma(n_o - 1)(t + t^3 + t^5) \qquad (2)$$

where γ = 8.0.... a control constant

$$t = (1 + \gamma^2 \tan^2 E_o)^{1/2} - \gamma \tan E_o \qquad (3)$$

$n_o \cong$ 1.000360.... the surface index of refraction

Corrected range and elevation angles are given by

$$R' = R - \Delta R$$
$$E = E_o - \Delta E \qquad (4)$$

Figures 1 and 2 illustrate typical values of ΔR and ΔE as a function of elevation angle when the vehicle is above 75 km altitude. For comparison, values for ΔR and ΔE using the CRPL Exponential Reference Atmosphere * plotted (circled points) in figures G-1 and G-2.

* Bean, B.R. and Thayer, G.D., CRPL EXPONENTIAL ATMOSPHERE, National Bureau of Standards Monograph 4, October 29, 1959.

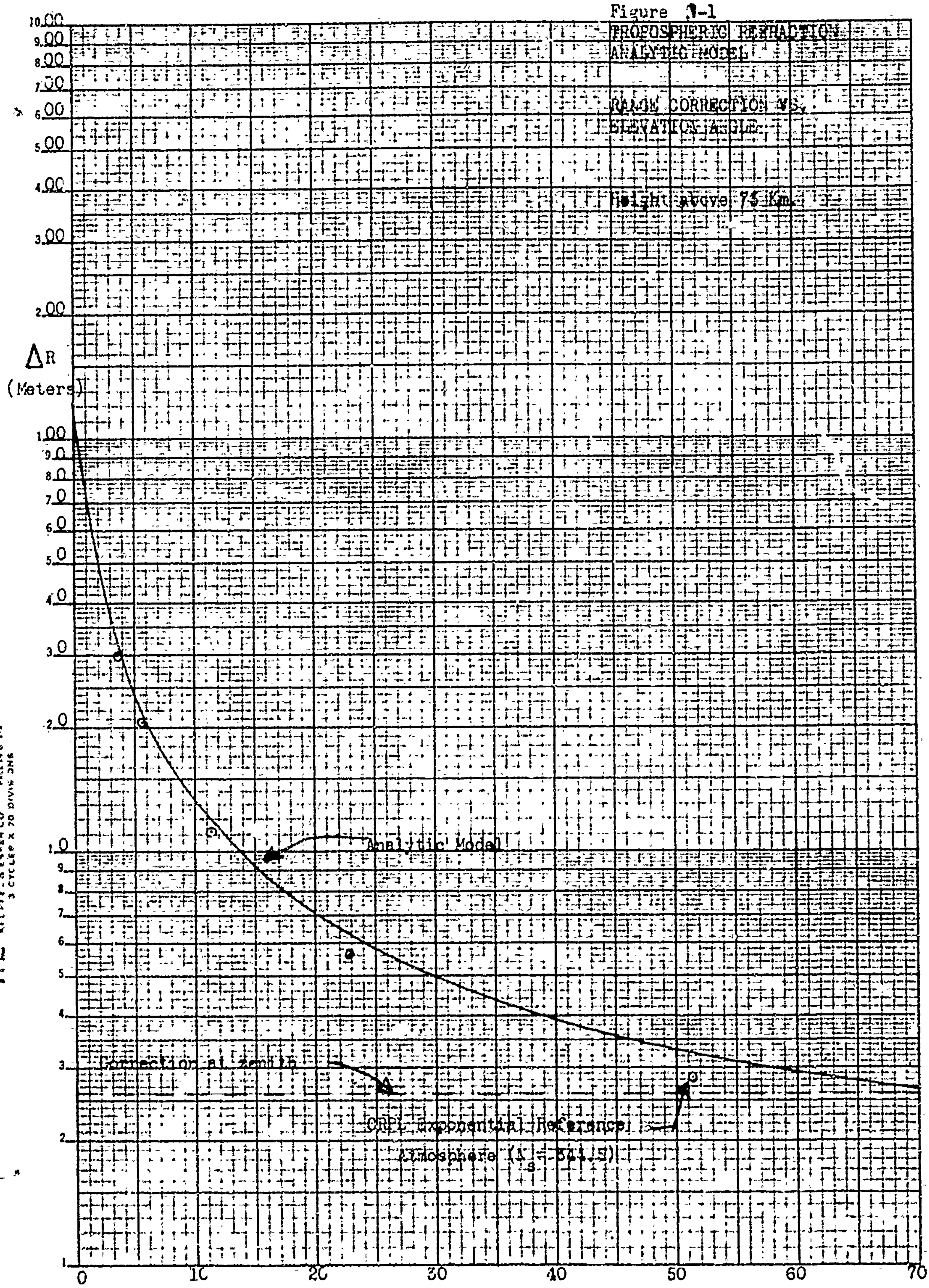


Figure 1-1
TROPOSPHERIC REFRACTION
ANALYTIC MODEL
RANGE CORRECTION VS.
ELEVATION ANGLE
Height above 75 Km.
ΔR
(Meters)
Analytic Model
Correction at zenith
CRPL Exponential Reference
Atmosphere
ELEVATION ANGLE (DEGREES)
SEMI-LOGARITHMIC 359-71
3 CYCLES X 70 DIVISIONS

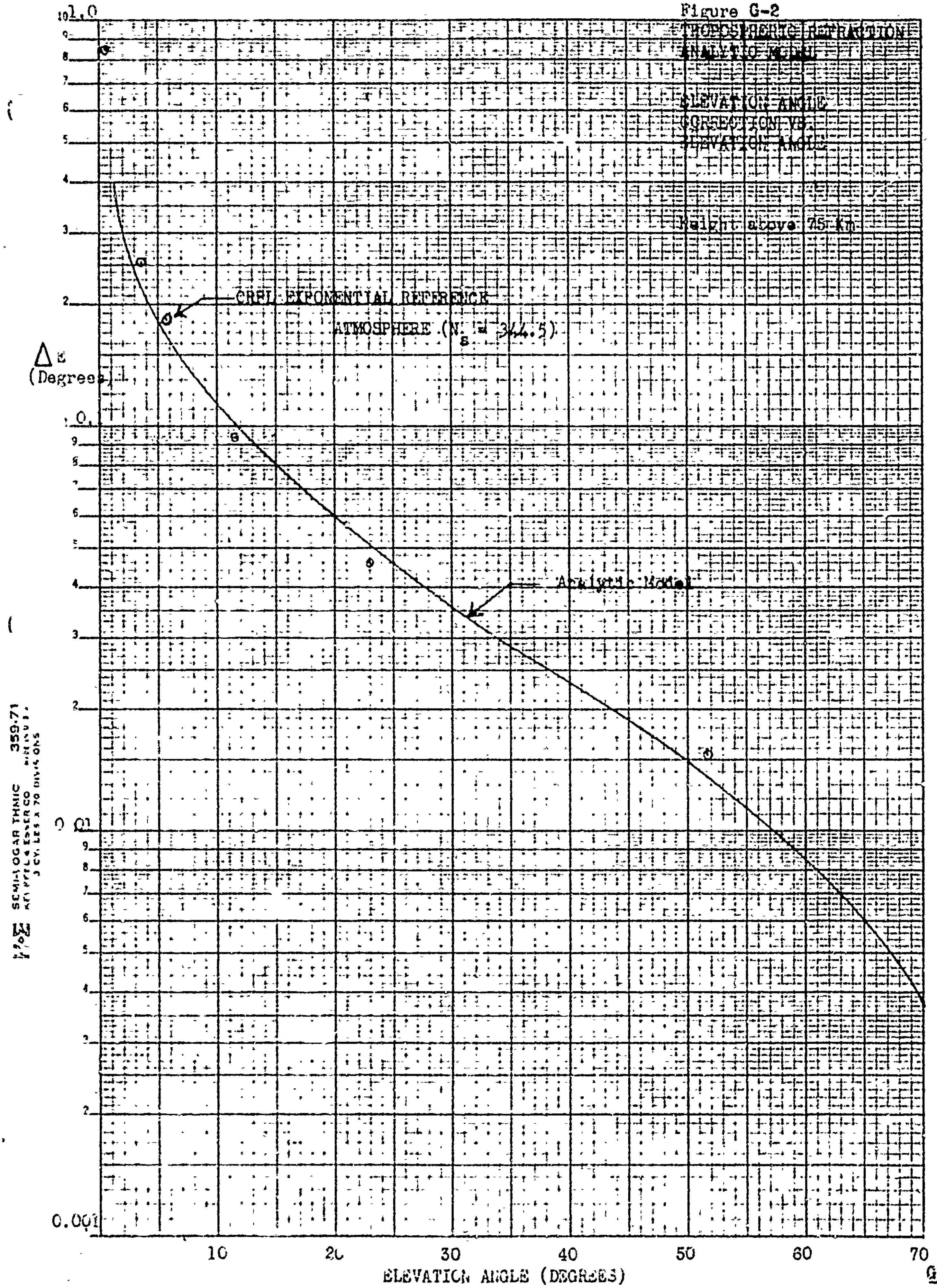


Figure G-2
TROPOSPHERIC REFRACTION
ANALYTIC MODEL
ELEVATION ANGLE
CORRECTION VS.
ELEVATION ANGLE
Height above 75 Km
CRPL EXPONENTIAL REFERENCE
ATMOSPHERE (N_s = 344.5)
Analytic Model
Δε
(Degrees)
1.0
0.1
0.01
0.001
10
20
30
40
50
60
70
ELEVATION ANGLE (DEGREES)
SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO.
3 CYCLES X 70 DIVISIONS

APPENDIX H

ANALYTIC IONOSPHERIC CORRECTION

The correction for the range retardation due to ionospheric effects is given by:

$$\Delta R = \frac{40.3\ H_o N_o}{\delta f^2(\sin E + K \cos E)} \left\{ \tan^{-1} \delta - \tan^{-1}\left[\delta\left(\frac{H_o - H}{H_o}\right)\right]\right\} \left(1 - e^{-HT}\right)$$

$N_o \times 10^{12}$ = maximum electron density of the F_2 layer in (electrons/meter^3)

f = carrier frequency in Mc/Sec.

H = height of vehicle in meters

H_o = height of N_o in meters

E = elevation angle

K = an empirical constant

$\delta = \left(\frac{2H_o}{H_U - H_L}\right)$ = control constant

H_U, H_L = upper and lower heights in meters of the half values of the electron density profile

T = 1/(50,000 meters)

The corrected range is given by:

$$R_C = R_M - \Delta R$$

The general form* of this correction may be deduced by assuming:

1. the refractive index for the ionosphere is given by:

$$n(f) \cong 1 - \frac{40.3\ N(H)}{f^2},$$

---

* "Processing and Analysis of Azusa MK II Data," General Dynamics Astronautics Technical Report No. AE 60-0017, by A. Saastad and F. C. Forbes, 10 June 1960.

2. the altitude dependence of the electron density is given by:

$$N(H) = \frac{N_o}{1 + \alpha^2 \left(\frac{H - H_o}{H_o}\right)^2};$$

3. the flat earth approximation, and horizontally stratified ionosphere.

The form of N(H) model is shown in figure H-1 and a comparison with a measured profile is shown in figure H-2.

The range error due to ionospheric effects may be written:

$$\Delta R = + \frac{c}{\omega}(\Delta\phi)_1 = - \frac{40.3}{f^2} \int_o^r N(H)dr$$

Substituting for N(H):

$$\Delta R = - \frac{40.3\, N_o}{f^2 \sin E} \int_o^H \frac{dH}{1 + \alpha^2 \left(\frac{H - H_o}{H_o}\right)^2}$$

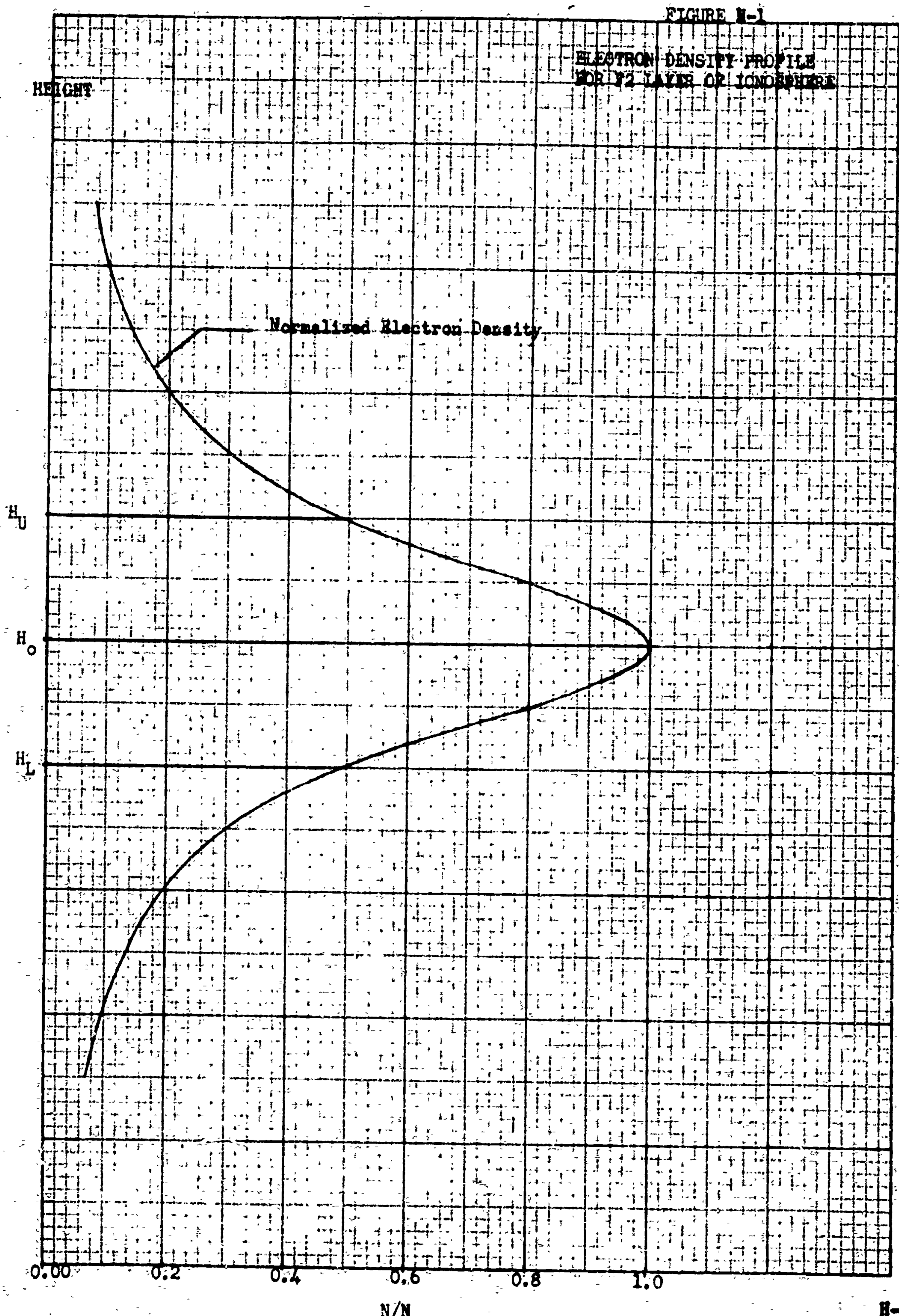
Integration yields:

$$\Delta R = \frac{40.3\, N_o\, H}{\alpha f^2 \sin E} \left\{ \tan^{-1}(\alpha) - \tan^{-1}\left[\left(\frac{H_o - H}{H_o}\right)\alpha\right]\right\}$$

This expression is the same form as the final range correction formula. The additional term in the denominator and the exponential term are added to agree more closely to experimental results at low altitudes and low elevation angles.

FIGURE H-1

ELECTRON DENSITY PROFILE
FOR F2 LAYER OF IONOSPHERE



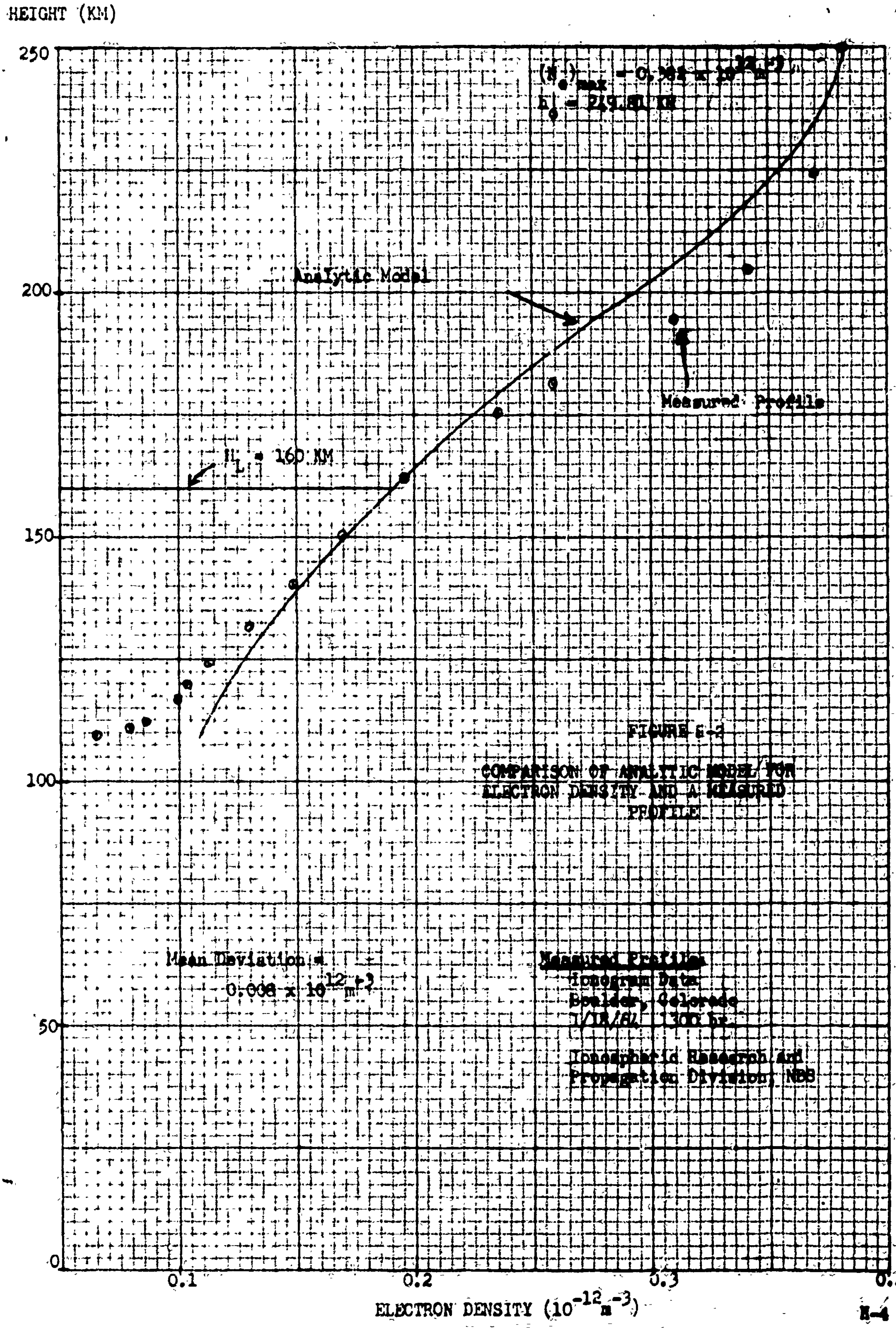


FIGURE E-2

COMPARISON OF ANALYTIC MODEL FOR ELECTRON DENSITY AND A MEASURED PROFILE

The parameters of the model generally vary as a function of the local time, sunspot activity, latitude, and various other factors. In order to fit the assumed model for $N(H)$ to existing conditions, two parameters are adjusted using the measured range corrections from dual frequency range measurements. The two parameters adjusted are N_o and K plus a calibration constant.

The normal equation associated with the i^{th} observation is:

$$\underset{(1\times1)}{[\varepsilon_i]} = \underset{(1\times3)}{[Q_i]}\underset{(3\times1)}{[D]}$$

where: $[\varepsilon_i] = (\Delta R_M - \Delta R_c + K_{cal})_i$

$$[Q_i] = \begin{bmatrix} \dfrac{\partial \Delta R}{\partial K_{cal}} & \dfrac{\partial \Delta R}{\partial N_o} & \dfrac{\partial \Delta R}{\partial K} \end{bmatrix}_i$$

$$[D] = \begin{bmatrix} \Delta K_{cal} \\ \Delta N_o \\ \Delta K \end{bmatrix}$$

The partial derivatives of $[Q_i]$ are:

$$\frac{\partial \Delta R}{\partial K_{cal}} = -1$$

$$\frac{\partial \Delta R}{\partial N_o} = \frac{\Delta R_c}{N_o}$$

$$\frac{\partial \Delta R}{\partial K} = \frac{\Delta R_c}{\sin E + K \cos E}$$

ΔR_c = range correction calculated from model.

ΔR_M = range correction from dual frequency range measurements.

K_{cal} = calibration constant for ΔR_M.

The least squares solution for $[D]$ with n observations is given by:

$$[D] = \left\{ \sum_{i=1}^{n} [Q_i]^T [Q_i] \right\}^{-1} \left\{ \sum_{i=1}^{n} [Q_i]^T [\varepsilon_i] \right\}$$

The adjustments to K_{cal}, N_o, K are added and the process iterated until the adjustments become small.

APPENDIX I

DUAL FREQUENCY IONOSPHERIC CORRECTION

The Geodetic SECOR system has a built-in method for determining the phase shift of the signal due to interaction with the ionosphere. In the simplified derivation of the correction technique given below, the following assumptions are made:

1. The carrier frequencies are much greater than the gyromagnetic frequency and the effective electron collision frequency.

2. The modulation frequencies are sufficiently close to the carrier frequency so that the differential phase shifts may be ignored.

A similar derivation may be performed without using these assumptions, but the mathematical complexity is much greater and the results are not significantly changed for frequencies used by the system.

The Geodetic SECOR ground stations transmit a single carrier frequency to the satellite transponder. The transponder generates two return signals which are initially phase coherent with the signal received at the satellite. The carrier frequencies used in the Geodetic SECOR system are:

$$\begin{aligned} f_1 &= f = 420.0 \text{ mc/sec} \\ f_2 &= f + \Delta f = 449.0 \text{ mc/sec} \\ f_3 &= \gamma f = 224.5 \text{ mc/sec} \end{aligned} \tag{1}$$

The corresponding frequencies expressed in radian/sec are related to the above by:

$$\omega = 2\pi f \times 10^6. \tag{2}$$

Under assumptions (1) and (2), the effective index of refraction for the ionosphere may be written:

$$n(\omega) = 1 - \frac{\eta N(r)}{\omega^2}, \quad \eta = \frac{2\pi e^2}{m} \tag{3}$$

e = electron charge = 4.8×10^{-10} esu

m = electron mass = 9.1×10^{-28} gm

$N(r)$ = electron density (electrons/cc)

The total phase shifts referenced to ω along each one-way path may be written as:

$$\Delta\varphi_1 = \frac{\omega}{c} \int_0^r n(\omega)\, dr \tag{4}$$

$$\Delta\varphi_2 = \frac{\omega}{c} \int_0^r n(\omega + \Delta\omega)\, dr$$

$$\Delta\varphi_3 = \frac{\omega}{c} \int_0^r n(\alpha\omega)\, dr$$

Then the total two-way phase shifts are given by:

$$\Delta\varphi_{12} = \Delta\varphi_1 + \Delta\varphi_2 = \frac{\omega}{c} \int_0^r [n(\omega) + n(\omega + \Delta\omega)]\, dr \tag{5}$$

$$\Delta\varphi_{13} = \Delta\varphi_1 + \Delta\varphi_3 = \frac{\omega}{c} \int_0^r [n(\omega) + n(\alpha\omega)]\, dr$$

Substituting the functional form of n:

$$\Delta\varphi_{12} = 2\frac{\omega}{c} \int_0^r dr - \frac{\omega}{c} \int_0^r \left[\frac{\eta N(r)}{\omega^2} + \frac{\eta N(r)}{(\omega + \Delta\omega)^2}\right] dr \tag{6}$$

$$\Delta\varphi_{13} = 2\frac{\omega}{c} \int_0^r dr - \frac{\omega}{c} \int_0^r \left[\frac{\eta N(r)}{\omega^2} + \frac{\eta N(r)}{\alpha^2\omega^2}\right] dr$$

In equations (6) the first term is the phase shift which would be observed in the absence of the ionosphere. Then the phase shifts along each path due to ionospheric effects are given by:

$$(\Delta\varphi_{12})_i = -\frac{n}{\alpha\omega}\left[1 + \frac{1}{(1 + \frac{\Delta\omega}{\omega})^2}\right]\int_0^r N(r)\,dr \tag{7}$$

$$(\Delta\varphi_{13})_i = -\frac{n}{\alpha\omega}\left[1 + \frac{1}{\alpha^2}\right]\int_0^r N(r)\,dr$$

Now let:

$$\beta = (1 + \frac{\Delta\omega}{\omega}) \tag{8}$$

$$I = \int_0^r N(r)\,dr = \text{integrated electron density}$$

Then:

$$(\Delta\varphi_{12})_i = -\frac{n}{\alpha\omega}(1 + \frac{1}{\beta^2})\,I \tag{9}$$

$$(\Delta\varphi_{13})_i = -\frac{n}{\alpha\omega}(1 + \frac{1}{\alpha^2})\,I$$

Now the relative phase shift $(\Delta\varphi_{13} - \Delta\varphi_{12})$ scaled to a one-way range difference (K) is determined:

$$K = \frac{c}{2\omega}(\Delta\varphi_{13} - \Delta\varphi_{12}) = \frac{c}{2\omega}\left[(\Delta\varphi_{13})_i - (\Delta\varphi_{12})_i\right] \tag{10}$$

$$= -\frac{n}{2\omega^2}\left[1 + \frac{1}{\alpha^2} - 1 - \frac{1}{\beta^2}\right] I$$

Solving for I:

$$I = \frac{\frac{2\omega^2}{n} K}{(\frac{1}{\alpha^2} - \frac{1}{\beta^2})} \tag{11}$$

Having determined I, the range error due to ionospheric effects is given by:

$$(\Delta R_{12})_i = \frac{c}{\omega}(\Delta\varphi_{12})_i = \frac{\left(1 + \frac{1}{\beta^2}\right)}{\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)} K \tag{12}$$

For the Geodetic SECOR system:

$$\alpha = 0.534 \tag{13}$$

$$\beta = 1.067$$

$$K = V - VPIC$$

so that $(\Delta R_{12})_i = 0.715\ K$ and the corrected range is:

$$R_c = R_m - (\Delta R_{12})_i . \tag{14}$$

APPENDIX J

TRANSIT TIME CORRECTION

The purpose of the transit time correction is to associate with a set of simultaneous range data a meaningful time. That is, a time which represents the "measured time."

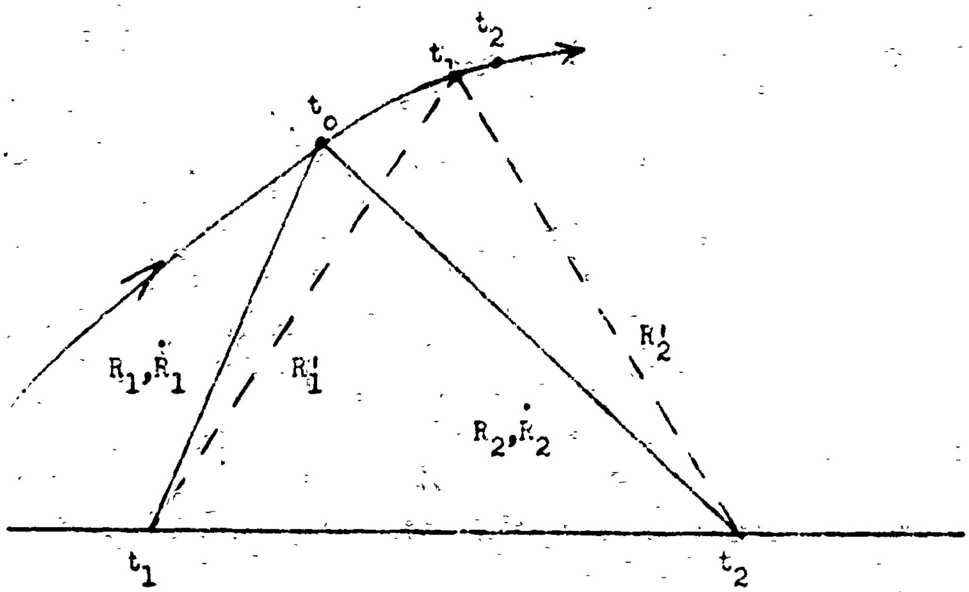


Figure J-1

At time t_o (figure J-1) the read pulse is sent from the satellite transponder. Due to the finite velocity of propagation C, the two stations will record the range at times t_1 and t_2, respectively. Assuming C is constant over the two paths;

$$t_1 = t_o + \frac{R_1}{C}, \quad t_2 = t_o + \frac{R_2}{C} \tag{1}$$

It is clear that the times and/or the ranges must be adjusted if the time reference is to correspond to the vehicle position.

One approach would be to use the measured ranges and adjust the times. In the simplified figure above, $\Delta t_1 = -R_1/C$ and $\Delta t_2 = -R_2/C$. The resulting times $t_1 + \Delta t_1$ and $t_2 + \Delta t_2$ would be equal and correspond to t_o. The resulting time scale, however, would be non-linear and vary with geometry (i.e., with changes in the ranges).

A second approach is to adjust the measured ranges to correspond to one of the recorded times. Suppose the time at station 1 is to be used as the reference. The measured ranges, however, correspond to time t_o. Using the Taylor's expansion about t_o;

$$R_1(t_1) = R_1(t_o) + \dot{R}_1(t_o)(t_1 - t_o) + \frac{1}{2}\ddot{R}_1(t_o)(t_1 - t_o)^2 + \ldots \tag{2}$$

Using the relation between t_1 and t_o above;

$$R_1(t_1) = R_1(t_o) + \dot{R}_1(t_o)\frac{R_1(t_o)}{C} + \frac{1}{2}\ddot{R}_1(t_o)\frac{R_1^2(t_o)}{C^2} + \ldots \tag{3}$$

For the second range;

$$R_2(t_1) = R_2(t_o) + \dot{R}_2(t_o)(t_1-t_o) + \frac{1}{2}\ddot{R}_2(t_o)(t_1 - t_o)^2 + \ldots \tag{4}$$

$$R_2(t_1) = R_2(t_o) + \dot{R}_2(t_o)\frac{R_1(t_o)}{C} + \frac{1}{2}\ddot{R}_2(t_o)\frac{R_1^2(t_o)}{C^2} + \ldots$$

The range adjustments are:

$$\Delta R_1 = \frac{\dot{R}_1 R_1}{C} + \frac{\ddot{R}_1 R_1^{\,2}}{2C^2} + \ldots \tag{5}$$

$$\Delta R_2 = \frac{\dot{R}_2 R_1}{C} + \frac{\ddot{R}_2 R_1^{\,2}}{2C^2} + \ldots$$

In general.

$$\Delta R = \frac{\dot{R}_i R_1}{C} + \frac{\ddot{R}_i R_1^{\,2}}{2C^2} + \ldots \tag{6}$$

The acceleration term will always be less than 0.002 meters (in **magnitude**): since,

$$\Delta R^{(2)}_{MAX} = \left| \frac{\ddot{R}_{MAX}\, R^2_{MAX}}{2C^2} \right| \tag{7}$$

?00 N.M. satellite:

$$R_{MAX} \cong 2.5 \times 10^6 \text{ meters} \tag{8}$$

$$\ddot{R}_{MAX} \cong 50 \text{ m/sec}^2$$

$$C \cong 3 \times 10^8 \text{ m/sec}$$

So,

$$\Delta R^{(2)}_{MAX} \cong 0.002 \text{ meters}$$

which may be neglected with respect to $\Delta R^{(1)}$. Thus the final range adjustments are:

$$\Delta R_1 = \frac{\dot{R}_1 R_1}{C} \tag{9}$$

$$\Delta R_2 = \frac{\dot{R}_2 R_1}{C}$$

APPENDIX K

TRAJECTORY FITTING TO POSITION AND/OR VELOCITY DATA

Injection vectors $(\overline{R}_o, \overline{V}_o)$ are initial conditions at some time (t_o) of a vehicle in freefall. When $\overline{R}_o, \overline{V}_o$ are known as a function of position and velocity and all accelerations affecting the vehicle's motion are sufficiently well behaved as to be representable as functions of position and velocity also, then the position, velocity, and these accelerations can be predicted as functions of time (t.). Trajectory fitting consists of establishing adjustments to $\overline{R}_o \overline{V}_o$ such that the trajectory computed from these injection vectors will satisfy some fitting criterion. A least-squares fit would be satisfied when the sum of the squares of the difference between computed and measured data (i.e., residuals) at each point along the trajectory was minimum.

To formulate a least-squares trajectory-fitting procedure, assume that the cartesian position and velocity vectors $(\overline{R}_o, \overline{V}_o)$ of the vehicle are known at points along the trajectory. A set of condition equations can then be formed to give

$$
\begin{array}{l}
\frac{\partial X}{\partial X_o}\Delta X_o + \frac{\partial X}{\partial Y_o}\Delta Y_o + \frac{\partial X}{\partial Z_o}\Delta Z_o + \frac{\partial X}{\partial \dot{X}_o}\Delta \dot{X}_o + \frac{\partial X}{\partial \dot{Y}_o}\Delta \dot{Y}_o + \frac{\partial X}{\partial \dot{Z}_o}\Delta \dot{Z}_o - (X_m - X_c) = v_X \\
\frac{\partial Y}{\partial X_o}\Delta X_o + \frac{\partial Y}{\partial Y_o}\Delta Y_o + \quad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad - (Y_m - Y_c) = v_Y \\
\frac{\partial Z}{\partial Z_o}\Delta X_o + \quad \vdots \\
\frac{\partial \dot{X}}{\partial X_o}\Delta X_o + \quad \vdots \\
\frac{\partial \dot{Y}}{\partial X_o}\Delta X_o + \quad \vdots \\
\frac{\partial \dot{Z}}{\partial X_o}\Delta X_o + \quad \cdot \qquad \cdot \qquad \cdot \qquad \frac{\partial \dot{Z}}{\partial \dot{Z}_o}\Delta \dot{Z}_o - (\dot{Z}_m - \dot{Z}_c) = v_{\dot{Z}}
\end{array}
\qquad (1)
$$

where $(X_m - X_c)$, etc. are discrepancy vectors ξ and v_X, $v_{\dot{X}}$, etc. are the residuals to be minimized,

or in matrix form

$$M\Delta - \xi = v \tag{2}$$

where

$$M = \begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix} \tag{3}$$

= two-body partial derivatives of vehicle's position and velocity with respect to the injection vectors, $\bar{R}_o$ and $\bar{V}_o$

$$Q_1 = \begin{bmatrix} \frac{\partial X}{\partial X_o} & \frac{\partial X}{\partial Y_o} & \frac{\partial X}{\partial Z_o} \\ \frac{\partial Y}{\partial X_o} & \frac{\partial Y}{\partial Y_o} & \frac{\partial Y}{\partial Z_o} \\ \frac{\partial Z}{\partial X_o} & \frac{\partial Z}{\partial Y_o} & \frac{\partial Z}{\partial Z_o} \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} \frac{\partial X}{\partial \dot{X}_o} & \frac{\partial X}{\partial \dot{Y}_o} & \frac{\partial X}{\partial \dot{Z}_o} \\ \frac{\partial Y}{\partial \dot{X}_o} & \frac{\partial Y}{\partial \dot{Y}_o} & \frac{\partial Y}{\partial \dot{Z}_o} \\ \frac{\partial Z}{\partial \dot{X}_o} & \frac{\partial Z}{\partial \dot{Y}_o} & \frac{\partial Z}{\partial \dot{Z}_o} \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \frac{\partial \dot{X}}{\partial X_o} & \frac{\partial \dot{X}}{\partial Y_o} & \frac{\partial \dot{X}}{\partial Z_o} \\ \frac{\partial \dot{Y}}{\partial X_o} & \frac{\partial \dot{Y}}{\partial Y_o} & \frac{\partial \dot{Y}}{\partial Z_o} \\ \frac{\partial \dot{Z}}{\partial X_o} & \frac{\partial \dot{Z}}{\partial Y_o} & \frac{\partial \dot{Z}}{\partial Z_o} \end{bmatrix}, \qquad Q_4 = \begin{bmatrix} \frac{\partial \dot{X}}{\partial \dot{X}_o} & \frac{\partial \dot{X}}{\partial \dot{Y}_o} & \frac{\partial \dot{X}}{\partial \dot{Z}_o} \\ \frac{\partial \dot{Y}}{\partial \dot{X}_o} & \frac{\partial \dot{Y}}{\partial \dot{Y}_o} & \frac{\partial \dot{Y}}{\partial \dot{Z}_o} \\ \frac{\partial \dot{Z}}{\partial \dot{X}_o} & \frac{\partial \dot{Z}}{\partial \dot{Y}_o} & \frac{\partial \dot{Z}}{\partial \dot{Z}_o} \end{bmatrix} \tag{4}$$

$$\Delta = \begin{bmatrix} \Delta\bar{R}_o \\ \Delta\bar{V}_o \end{bmatrix} = \begin{bmatrix} \Delta X_o \\ \Delta Y_o \\ \Delta Z_o \\ --- \\ \Delta\dot{X}_o \\ \Delta\dot{Y}_o \\ \Delta\dot{Z}_o \end{bmatrix} \tag{5}$$

= adjustments to injection vectors

$$\xi = \begin{bmatrix} \bar{R}_m - \bar{R}_c \\ \bar{V}_m - \bar{V}_c \end{bmatrix} = \begin{bmatrix} X_m - X_c \\ Y_m - Y_c \\ Z_m - Z_c \\ \hline \dot{X}_m - \dot{X}_c \\ \dot{Y}_m - \dot{Y}_c \\ \dot{Z}_m - \dot{Z}_c \end{bmatrix} = \text{discrepancy vectors of position and velocity} \qquad (6)$$

$$V = \text{residuals} = \begin{bmatrix} v_X \\ v_Y \\ v_Z \\ v_{\dot{X}} \\ v_{\dot{Y}} \\ v_{\dot{Z}} \end{bmatrix}$$

$\bar{R}_m$, $\bar{V}_m$ = measured position and velocity vectors of the vehicle, respectively, at time t

$\bar{R}_c$, $\bar{V}_c$ = predicted position and velocity vectors of the vehicle, respectively, at time t.

FITTING TO POSITION AND VELOCITY VECTORS

Consider the least squares solution for the injection vector adjustments where unity weighting is assumed, then the solution based on (n) vehicle positions and velocities has the form*

$$\underset{(1\text{x}6)}{\Delta} = [\underset{(6\text{x}6)}{\sum_{i=1}^{n} (M^T M)_i}]^{-1} \underset{(6\text{x}1)}{\sum_{i=1}^{n} (M^T \xi)_i} \qquad (7)$$

A weighted least squares solution will be given by

$$\Delta = [\sum_{i=1}^{n} (M^T \sigma^{-1} M)_i]^{-1} \sum_{i=1}^{n} (M^T \sigma^{-1} \xi) \qquad (8)$$

where

$$\sigma = [\sum_{j=1}^{m} (B^T \sigma_o^{-1} B)_j]$$

* Refer to appendix L, "Least Squares Adjustment."

$$\sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{X\dot{X}} & \sigma_{X\dot{Y}} & \sigma_{X\dot{Z}} \\ \sigma_{YX} & \sigma_Y^2 & & & & \sigma_{Y\dot{Z}} \\ \vdots & \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & \vdots & & \vdots \\ \vdots & \vdots & & & \vdots & \vdots \\ \sigma_{\dot{Z}X} & \sigma_{\dot{Z}Y} & \cdot & \cdot & \cdot & \sigma_{\dot{Z}}^2 \end{bmatrix} = \text{covariance matrix of vehicle position and velocity vectors} \tag{9}$$

$\sigma_{o_j}^{-1}$ = inverse variance of jth observation used in computing the $\bar{R}_m \bar{V}_m$ vectors at time t

B^T = matrix of partial derivatives of the observations with respect to the $\bar{R}_m \bar{V}_m$ vectors at time t.

As an example, when ranging is an observation used in computing $\bar{R}_m \bar{V}_m$, then

$$B^T = \begin{bmatrix} \frac{\partial R}{\partial X} & \frac{\partial R}{\partial Y} & \frac{\partial R}{\partial Z} & \frac{\partial R}{\partial \dot{X}} & \frac{\partial R}{\partial \dot{Y}} & \frac{\partial R}{\partial \dot{Z}} \end{bmatrix} \tag{10}$$

and

$$\sigma_o^{-1} = \frac{1}{\sigma_R^2} \tag{11}$$

FITTING TO POSITION OR VELOCITY VECTORS

If the trajectory is to be adjusted only to measured positions or only velocity components, then for a position fit, the M matrix of equations (7) and (8) has the form

$$M = \overset{(3x6)}{[Q_1 \; Q_2]} \tag{12}$$

and for a velocity fit

$$M = \overset{(3x6)}{[Q_3 \; Q_4]} \tag{13}$$

Constraining the adjustment of injection vectors to only the $\bar{R}_o$ elements or $\bar{V}_o$ will reduce the M matrix to

$$M = \overset{(3x3)}{Q_1} \tag{14}$$

for $\bar{R}_o$ position adjustment and

$$M = \overset{(3x3)}{Q_4} \tag{15}$$

for an adjustment to only $\bar{V}_o$.

When the observational data or the adjustment is constrained, the weighting matrix and the discrepancy vectors must correspondingly be modified.

FITTING TO OBSERVATIONAL DATA

It is not essential that the cartesian coordinates of the vehicle be used in a fitting procedure. A fitting directly to observational data can be formulated by the condition equations of (1) in the form

$$\begin{gathered}
\frac{\partial OBS_1}{\partial X_o}\Delta X_o + \frac{\partial OBS_2}{\partial Y_o}\Delta Y_o + \ldots + \frac{\partial OBS_1}{\partial \dot{Z}_o}\Delta \dot{Z}_o - (OBS_1 - OBS_c) = V_1 \\
\frac{\partial OBS_2}{\partial X_o}\Delta X_o + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\vdots \qquad\qquad\qquad\qquad\qquad \vdots \qquad \vdots \\
\frac{\partial OBS_n}{\partial X_o}\Delta X_o + \cdot \quad \cdot \quad \cdot + \frac{\partial OBS_n}{\partial \dot{Z}_o}\Delta \dot{Z}_o - (OBS_n - OBS_c) = V_n
\end{gathered} \tag{16}$$

where OBS = arbitrary observation.

For example, a condition equation for a slant range observation which is to be used in adjusting $\bar{R}_o$ and $\bar{V}_o$ will be

$$\frac{\partial R_1}{\partial X_o}\Delta X_o + \frac{\partial R_1}{\partial Y_o}\Delta Y_o + \frac{\partial R_1}{\partial Z_o}\Delta Z_o + \frac{\partial R_1}{\partial \dot{X}_o}\Delta \dot{X}_o + \frac{\partial R_1}{\partial \dot{Y}_o}\Delta \dot{Y}_o + \frac{\partial R_1}{\partial \dot{Z}_o}\Delta \dot{Z}_o - (R_{1_m} - R_{1_c}) = V_{R_1} \tag{17}$$

A system of equations similar to (17) can be formed with each range or other independent observation taken to a vehicle. Weighting the solution or constraining the adjustments follows the schemes shown above for vector fitting.

COMMENTS REGARDING TRAJECTORY FITTING

The assumption of linearity implicit in the condition equations (1) is not sufficiently correct to avoid using an iterative solution in the least squares fitting procedure. This condition usually means that all measured data used in a fitting procedure be stored or retained for the successive iterations of the adjustment. In most problems, the amount of data used in a solution is less significant than the relative independence of the observations (i.e., the effective geometric variability of sampling).

The above methods of trajectory fitting have proven very accurate using two-body partial derivatives (for near earth trajectories) to form the condition equations of (1). Precision methods of trajectory prediction, which take into account all significant perturbative accelerations, are used to compute the discrepancy vectors (i.e., the computed $\bar{R}_c$ $\bar{V}_c$). Two-body partials are representative enough to establish convergence of position

and velocity vectors to within a hundredth of a foot and a thousandth of a foot per second in three iterations for most trajectory fitting spans of up to 400 seconds of data. This does not imply that the trajectory is known to this accuracy, which is of course determined by the measured data quality and accuracy.

When the adjustments of injection vectors are in any way constrained, then it is accepted that any unsolved for adjustment will be distributed in the solution in such a way as to satisfy the imposed least-squares criterion. However, in any practical problem, not all observational biases, parameter offsets, etc., can be either anticipated or modeled adequately to allow for their solution. A natural constraining is therefore implicit in any adjustment. The problem is to define the compromise between constraining and adjusting. Over-constraining will cause an unnatural distribution of error. Over adjustment will reduce the strength of a solution by adding unknowns, which will in turn increase the amount of correlation between elements in the adjustment and result in a useless sympathetic adjustment. An evaluation of residuals over a large sampling space, combined with careful analysis of the adjustment based on experience, is the most practical way of verifying an adjustment scheme.

APPENDIX L

LEAST SQUARES ADJUSTMENTS

Observations refer to quantities measured directly by some equipment. Parameters are computed from observations. An example of independent observations would be three ranges measured to some vehicle when the ranging equipment are located at different sites. The cartesian XYZ coordinates of the vehicle would be parameters which could be computed from the independent observations.

Any observation will consist of the true value and some aggregation of error terms. Error terms will include noise, cyclic, and bias. Errors are grouped into categories customarily based on their frequency. As a matter of definition, cyclic errors are perturbations in the data with a time occurrence period greater than 10 and less than 100 seconds. Noise will be considered as spurious data with period less than 10 seconds. Biases are essentially constant or can be represented by some analytic form composed of constant terms. A measured observation could be expressed mathematically by

$$U_o = U_T + \Delta U_o + v \qquad (1)$$

where

U_o = measured observation

U_T = true value

ΔU_o = aggregation of biases

v = residual (noise and cyclic error)

Similarly, a calculated approximation to the observation is formed by

$$U_C = U_T - \Delta U_C \tag{2}$$

where

U_C = approximation to U_T

ΔU_C = unknown adjustment to U_C due to approximations

Equations (1) and (2) can be differenced to form the condition equation

$$\Delta U_o - \Delta U_C + v - (U_o - U_C) = \xi \tag{3}$$

where ξ = discrepancy vector

Equation (3) may not be amenable to solution if terms higher than the first degree are carried. The equation, however, can be linearized by expanding it in a Taylor's series and deleting all second and higher power terms. Expanding equation (3) by taking partials with respect to each free variable gives

$$\Sigma \frac{\partial U_o}{\partial b} \Delta b + \Sigma \frac{\partial U_C}{\partial P} \Delta P + v = \xi \tag{4}$$

where

$\frac{\partial U_o}{\partial b}$ = partial derivatives of the observations with respect to observational biases

$\frac{\partial U_C}{\partial P}$ = partial derivatives of the observations with respect to each free parameter adjustment

That is, ΔU_o and ΔU_C are assumed to have the form:

$$\Delta U_o = \Sigma \frac{\partial U_o}{\partial b} \Delta b$$

$$\Delta U_C = \Sigma \frac{\partial U_C}{\partial P} \Delta P$$

Δb = observational bias adjustments

ΔP = parameter adjustments

As a matter of convenience, equation (4) can be written in matrix notation, thus,

$$v + A\Delta b + B\Delta P = \xi \qquad (5)$$

where

$$A = \begin{bmatrix} \frac{\partial U_o}{\partial b_1} & \frac{\partial U_o}{\partial b_2} & \cdots & \frac{\partial U_o}{\partial b_n} \end{bmatrix}$$ = partials of observations with respect to bias adjustments

$$B = \begin{bmatrix} \frac{\partial U_C}{\partial P_1} & \frac{\partial U_C}{\partial P_2} & \cdots & \frac{\partial U_C}{\partial P_m} \end{bmatrix}$$ = partials of observations with respect to parameter adjustments

$$\Delta b = \begin{bmatrix} \Delta b_1 \\ \Delta b_2 \\ \vdots \\ \Delta b_n \end{bmatrix}$$ = adjustments to observational biases

$$\Delta P = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_m \end{bmatrix}$$ = adjustments to parameters

In order to simplify the general representation of the solution for adjustments, equation (5) is further reduced to yield

$$v + M\Delta = \xi$$

where

$$\underset{3x(n+m)}{M} = \begin{bmatrix} A & B \end{bmatrix}$$

$$\underset{(n+m)x1}{\Delta} = \begin{bmatrix} \Delta b \\ \Delta P \end{bmatrix}$$

A linearized condition equation similar to (6) can be formed for each observation which is to enter into a solution. When more independent observations exist than unknowns in the solution, the assemblage of condition equations must be solved to satisfy some statistical or convergence criterion. Assuming an overdetermined set of condition equations exist, then the composite set are expressed as

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_S \end{bmatrix} + \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_S \end{bmatrix} \Delta - \begin{bmatrix} \xi_1 \\ \xi_2 \\ \\ \xi_S \end{bmatrix} = 0 \qquad (7)$$

and upon condensing

$$V + Q\Delta - \xi = 0 \qquad (8)$$

It is assumed initially that the system of equations (7) will remain linear over the sampling space. The condition equations are also approximations in that second and higher order derivative terms are deleted from the expansion in equation (4). When the first order approximations of derivatives are not sufficiently representative, it becomes necessary to iterate the solution to equation (7). In the iterative procedure, computed adjustments are sequentially added to the initial estimates of the parameters and the observations.

To obtain a solution for the system of equations given by (7), some criterion must be established. A general least squares solution can be derived from equation (7) and the multivariate normal distribution function for a set of statistically independent observations. The multivariate probability density for statistically independent observations is given by

$$f(r_1, r_2, \ldots, r_S) = \prod_{i=1}^{S} \frac{1}{\sqrt{2\pi}(\sigma_{r_i})} \, EXP^{-\frac{1}{2}\left(\frac{r - \bar{r}}{\sigma_r}\right)_i^2} \qquad (9)$$

where

$r_1, r_2 \ldots r_S$: a set of statistically independent observations or samples

$\bar{r}$ = sample mean

$\prod_{i=1}^{S}$ = take products from i=1 to S

$\sigma_{r_i}^2$ = variance of observations

In the adjustment of data, the problem is to establish the means or parameters of a distribution so that the probability density function is maximized. To maximize the probability, the exponents of the density function in equation (9) must be minimized. Rewriting the exponent to be minimized gives

$$G = \sum_{i=1}^{S} \left(\frac{r - \tilde{r}}{\sigma_r} \right)_i^2 \tag{10}$$

or in matrix form

$$G = V^T \sigma^{-1} V \tag{11}$$

where

$$\sigma = \overset{S \times S}{\begin{bmatrix} \sigma_{r_1}^2 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \sigma_{r_2}^2 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \sigma_{r_S}^2 \end{bmatrix}} = \text{matrix of variance of observations} \tag{12}$$

$$V = \begin{bmatrix} r_1 - \tilde{r}_1 \\ r_2 - \tilde{r}_2 \\ \cdot \\ \cdot \\ \cdot \\ r_S - \tilde{r}_S \end{bmatrix} = \text{matrix of residuals} \tag{13}$$

T, -1 = refer to matrix transpose and inverse, respectively.

$\sigma_{r_S}^2$ = sample variance

From equation (10) it is seen that the solution which maximizes the density function (9) is the solution which makes the weighted sum of the squares of the residuals a minimum, where the weighting is the inverse variance of the observations.

With the matrix of observational variance given by equation (12), then from the two equations in two unknowns, V and Δ, namely

$$V + Q\Delta - \varepsilon = 0$$

and

$$V^T \sigma^{-1} V = 0$$

a solution* for Δ is given by

$$\Delta = [Q^T \sigma^{-1} Q]^{-1} \; Q^T \sigma^{-1} \varepsilon \qquad (14)$$

Equation (14) is a general form of the weighted least squares adjustment. A common form of the solution is that where unity weighting is assumed. Under the conditions of unity weighting, the solution reduces to

$$\Delta = [Q^T Q]^{-1} Q^T \varepsilon \qquad (15)$$

where

$$Q^T = [M_1^T \; M_2^T \; . \; . \; . \; . \; M_S^T] \qquad (16)$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_S \end{bmatrix} \qquad (17)$$

---

* See pages 112-114 "RCS Data Technical Report No. 39," by D. C. Brown, 20 August 1957.

It can also be shown that when the minimization condition specified by equation (10) is satisfied, then the accuracy of the final adjustments to the observations and parameters will be given by the matrix of covariance given by

$$N^{-1} = [Q^T \sigma^{-1} Q]^{-1} \tag{18}$$

For the case where the parameters being adjusted are the cartesian coordinates of some vehicle, then N^{-1} has the form

$$N^{-1} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{YX} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{ZX} & \sigma_{ZY} & \sigma_Z^2 \end{bmatrix} = \text{matrix of variance and covariance of the adjustment [illegible] rs and [illegible]} \tag{19}$$

The square roots of the diagonal elements of the inverse normal equations given by (18) are the projections of the error volume on the axis of the respective coordinate axis being considered and are customarily considered the standard deviations of the solution components. The covariance matrix (18) therefore defines the solution distribution when an overdetermined set of observations have been used in the solution adjustment.

It should be emphasized that satisfaction of equation (10) is paramount before the distribution given by (18) is valid. In many solutions, conditions can exist which preclude satisfactory convergence to a meaningful adjustment. Any analysis based on the characteristics of the inverse normal equations will therefore have little merit. If elements in the adjustment are highly correlated (i.e., appear similar and mathematically inseparable) or if insufficient analytic variation exists in the observations,

then a stable solution may not be possible. The adjustment may even satisfy the residual minimization criterion over some limited sampling space. The end condition is that the minimum criterion be satisfied over the entire sample space, which should, in turn, be adequate to insure validity of results.

APPENDIX M

NUMERICAL INTEGRATION OF TRAJECTORY EQUATIONS--
RECTIFICATION AND PREDICTION INTERVALS

NUMERICAL INTEGRATION . . . . In the most elementary two-body trajectory predictions, it is possible to develop and use a closed analytic form of the prediction equations. When precise trajectories are to be computed, all accelerations, including perturbation terms such as aerodynamic drag and lift, earth's asymmetric gravity field, etc., must be integrated. Closed form analytic expressions which describe a body's motion in a complex perturbation environment do not exist. The nature of perturbations usually dictates that they be represented in a series form which in turn will not have explicit integrals. Successive numerical approximations or numerical integrations therefore are conventionally used to evaluate at least some portion of the dynamic equations used in precise trajectory computations.

There are no absolutely preferred methods of performing numerical integration. The method used is usually specified by the characteristics of the functions being evaluated and the associated available information which may aid the solution. A popular numerical integration procedure used in trajectory prediction computations is that termed Runge[1] - Kutta - Gill Linear Differential Solver. This method has been used and tested, but is not described herein.

In practical applications where the high-order derivative is assumed linear, it was demonstrated that equivalent results could be achieved by a

1 "Mathematical Methods for Digital Computers," A. Ralston, H. Will, published by John Wiley and Sons, Inc., New York, 1960, pages 110-120

less involved procedure based on expanding the dynamic function in a Taylor's series about an initial point. Successively incrementing and re-establishing initial conditions of the expansion yields the desired integration. Hence the dynamic equations are written in the power series;

$$S_1 = S_o + V_o t_1 + \frac{a_o t_1^2}{2} + \frac{\dot{a}_o t_1^3}{6} + \ldots \quad (1)$$

$$V_1 = V_o + a_o t_1 + \frac{\dot{a}_o t_1^2}{2} + \ldots \quad (2)$$

where S_o, V_o, a_o, $\dot{a}_o$ = the initial position, velocity, acceleration, and accelerosity at time t = 0.

The position and velocity at some later time t_2 will be given by

$$S_2 = S_1 + V_1 t_2 + \frac{a_1 t_2^2}{2} + \frac{\dot{a}_1 t_2^3}{6} + \ldots \quad (3)$$

$$V_2 = V_1 + a_1 t_2 + \frac{\dot{a}_1 t_2^2}{2} + \ldots \quad (4)$$

and so on.

When the above method is used to perform numerical integration of the dynamic equations of a vehicle, it is assumed that the acceleration and at least the accelerosity term (i.e., rate of change of acceleration) can be evaluated at each initial position. Positions and velocities are carried forward in the sequential computation. In simulations where thrusts are applied to the vehicle which are not predictable from the environment (predictable accelerations are gravity, drag, etc.), then these terms must be incremented and carried forward in the computation of position and velocity.

RECTIFICATION AND PREDICTION INTERVALS . . . . The rectification interval is the time over which a prediction is made before initial conditions are re-established. Prediction interval is the total time over which a prediction is to be carried.

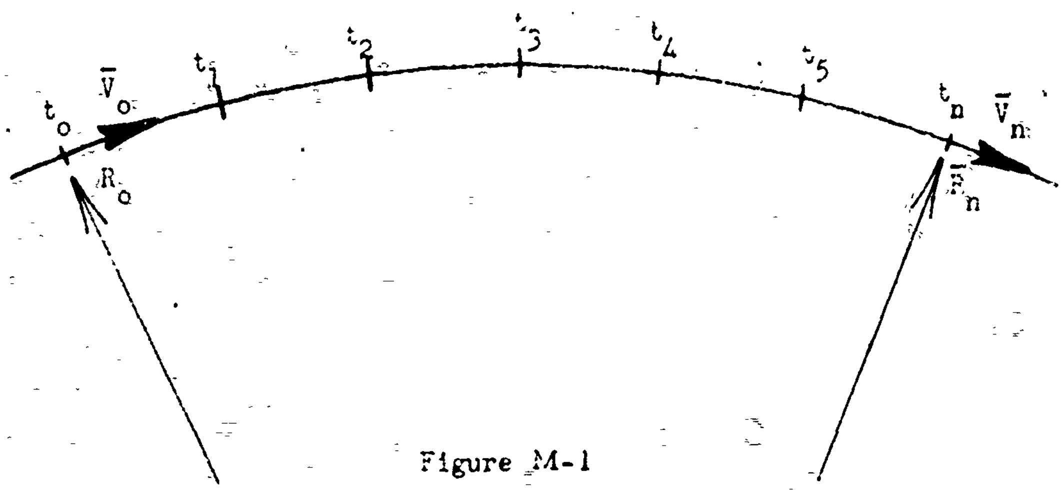


Figure M-1

Rectification and Prediction Intervals

From figure M-1 the rectification and prediction intervals are, respectively

$$RI = (t_{i+1} - t_i) \tag{5}$$

$$PI = (t_n - t_o) \tag{6}$$

$$RI \quad PI/n \tag{7}$$

where n = the number of time segments used to form the prediction interval

The equation of motion given as equations (1) and (2) above describe the physical environment over only a limited region, thus, there is a requirement to rectify the prediction equations frequently. Accuracy tolerances and the manner in which the equations of motion are used will regulate the selection

of a rectification interval. For example, when the Encke's method of prediction is used (i.e., numerical integration of perturbation terms only and analytic closed form computation of reference trajectory), the acceleration and higher order terms are small and the rectification interval can be increased with little degradation in accuracy.

APPENDIX N

ENCKE'S AND COWELL'S METHODS OF TRAJECTORY PREDICTION

Consider the total accelerations acting on a vehicle which is not under powered flight to be composed of a primary term with a summation of generally lesser perturbation term; hence,

$$\bar{A} = \bar{G} + \sum \bar{G}_p \qquad (1)$$

where

$\bar{A}$ = components of total acceleration

$\bar{G}$ = principal components of acceleration which are due to the earth's symmetric mass

$\sum \bar{G}_p$ = perturbative accelerations due to earth's asymmetric mass distribution, atmospheric drag, aerodynamic lift, etc.

To preserve continuity, the components of acceleration are taken to be projected along the XYZ axes of an inertial coordinate system which is coincident with the equatorial earth-fixed coordinates at epoch (i.e., the X_{EQ} axis in the equatorial plane through the earth's center of mass and the Greenwich meridian, Y_{EQ} axis in the equatorial plane and 90° counterclockwise from X_{EQ}, and Z_{EQ} axis along the polar rotational axis).

If the accelerations given by equation (1) can be predicted as a function of time and position, and the position and velocity vectors of the vehicle are known at an initial time t_o, then the trajectory can be computed as a function of time. It should be remembered that initial considerations and the dynamics are assumed known.

ENCKE'S METHOD . . . . The method of predicting a reference trajectory based on two-body motion and then adding adjustments computed by numerically integrating perturbation terms is referred to as Encke's.

A technique similar to Encke's has been developed and tested. The peculiarities of the technique are described in the following. (See figure N-1.)

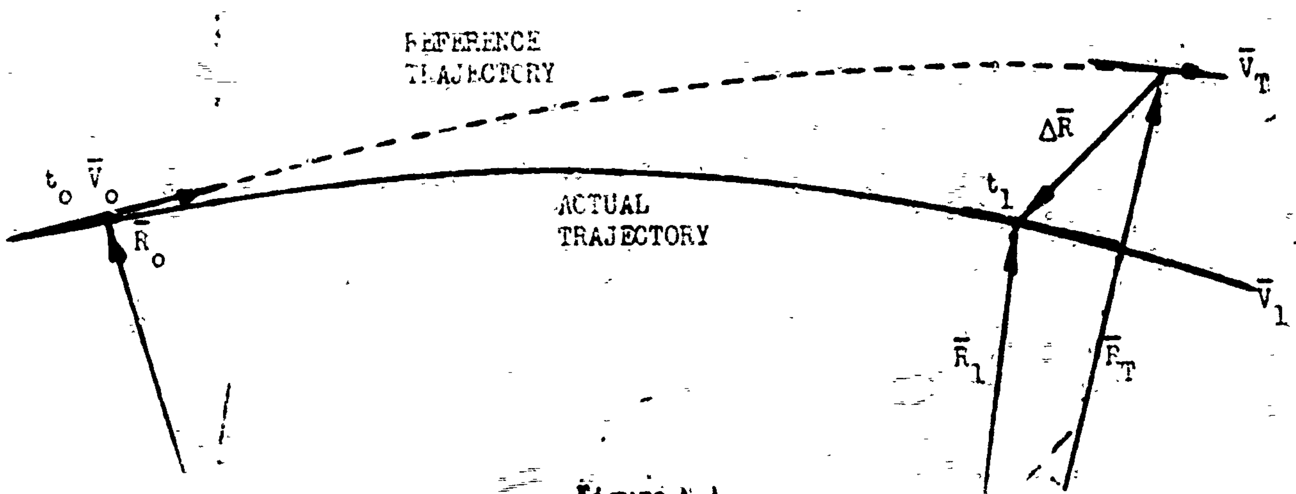


Figure N-1

Encke's Reference Orbit with Perturbations

If $\bar{R}_o \bar{V}_o$ are the position and velocity vectors expressed in inertial coordinates at epoch time t_o and the value of G of equation (1) (with the associate semimajor axis of the earth) is used to define the canonical units of length and time, then two-body* position and velocity predictions at time t_1 are given by

$$\bar{R}_T = \bar{R}_o f + \bar{V}_o g \quad (2)$$

$$\bar{V}_T = \bar{R}_o \dot{f} + \bar{V}_o \dot{g} \quad (3)$$

where f, g, $\dot{f}$, $\dot{g}$ = two-body integration constants

* Refer to appendix O. Two-Body Trajectory Prediction

The actual position and velocity vectors at t_1 will now be

$$\bar{R}_1 = \bar{R}_T + \Delta\bar{R} \tag{4}$$

$$\bar{V}_1 = \bar{V}_T + \Delta\bar{V} \tag{5}$$

where $\Delta\bar{R}$, $\Delta\bar{V}$ = adjustments due to perturbations.

When the $\bar{R}_o\bar{V}_o$ are initial or rectified points along the trajectory, then

$$\Delta\bar{R} = \frac{\left(\sum\bar{G}_{p_o}\right)}{2}\Delta t^2 + \frac{\left(\sum\dot{\bar{G}}_{p_o}\right)}{6}\Delta t^3 + \dots \tag{6}$$

$$\Delta\bar{V} = \left(\sum\bar{G}_{p_o}\right)\Delta t + \frac{\left(\sum\dot{\bar{G}}_{p_o}\right)}{2}\Delta t^2 + \dots \tag{7}$$

where

$\sum\bar{G}_{p_o}$ = perturbative accelerations at $\bar{R}_o$

$$\sum\dot{\bar{G}}_{p_o} \simeq \frac{\sum\bar{G}_{p_1} - \sum\bar{G}_{p_o}}{\Delta t} \tag{8}$$

$\sum\bar{G}_{p_1}$ = perturbative accelerations at $\bar{R}_1$

Δt = the rectification interval

Δt = $t_1 - t_o$

Because all perturbation accelerations are a function of position and velocity, the $\bar{R}_o\bar{V}_o$ are used to compute $\sum\bar{G}_{p_o}$ and correspondingly $\bar{R}\,\bar{V}$ are used to compute $\sum\bar{G}_{p_1}$. The assumption of linearity in $\sum\dot{\bar{G}}_{p_o}$ and that higher order terms in the series of (6) and (7) are negligible will cause error build up. How much error occurs is related to the rectification interval. Results of a simulation which demonstrates the influence of rectification interval are shown on figures N-3 and N-4. It can be seen

that for a Δt of 20 sec which is larger than that used in the SECOR orbital mode, the total expected divergence should be less than one meter.

COWELL'S METHOD . . . . Numerical integration of the total accelerations and velocities is identified as Cowell's method of trajectory prediction. One variation of Cowell's method which has been developed and tested is presented here. Starting with initial composite accelerations given by equation (1) and the initial position and velocity vectors, $\bar{R}_o \bar{V}_o$, we have

$$\bar{R}_1 = \bar{R}_o + \bar{V}_o \Delta t + \frac{\left(\bar{G}_o + \sum \bar{G}_{p_o}\right)}{2} \Delta t^2 + \frac{\left(\dot{\bar{G}}_o + \sum \dot{\bar{G}}_{p_o}\right)}{6} \Delta t^3 + \ldots \quad (9)$$

$$\bar{V}_1 = \bar{V}_o + \left(\bar{G}_o + \sum \bar{G}_{p_o}\right) \Delta t + \frac{\left(\dot{\bar{G}}_o + \sum \dot{\bar{G}}_{p_o}\right)}{2} \Delta t^2 + \ldots . \quad (10)$$

where $\dot{\bar{G}}_o = \dfrac{\bar{G}_1 - \bar{G}_o}{\Delta t}$

The rate of change of $(\bar{G}_o + \sum \bar{G}_{p_o})$ in equations (9, and (10) is computed in the same fashion as shown in equation (8) above. In the technique discussed here, the first estimates of $\bar{R}_1 \bar{V}_1$ used to compute $(\bar{G}_1 + \sum \bar{G}_{p_1})$ are obtained from a two-body prediction. An iteration of the total numerical integration would suffice. When $\bar{R}_1 \bar{V}_1$ have been computed as described above, $\bar{R}_1 \bar{V}_1$ become initial conditions for the next sequential prediction and so on. Derivative terms above $\dot{\bar{G}}_o$ have been deleted from the above power series.

COMPARISON OF ENCKE'S, COWELL'S, AND TWO-BODY METHOD OF TRAJECTORY PREDICTIONS . . . . The advantage of one prediction method over another is of interest only in that limits of applicability of each are defined by comparison. It is obvious from the above formulation that Cowell's

method is subject to numerical approximation error to a greater extent than is Encke's. This situation is due to the large magnitude of the terms carried in the numerical integration and the lack of physical representation of terms over large time intervals. However, when a vehicle is in powered flight, the reference trajectory computed by Encke's method will be of little use and numerical integration of total accelerations, including thrusts, is preferable.

T O-BODY METHOD. . . In some applications, simple two-body predictions may be more representative than those given by either Cowell's or Encke's methods, both of which require numerical integration. If sufficient care is not taken to perform numerical integration accurately and to use reasonable rectification intervals, then the build-up of error will exceed the total effect of perturbations and useless computation results.

Because accuracy, computing time, and rectification interval are directly related in trajectory computations, simulations were performed to give quantitative comparison of the three methods mentioned above. Actual measured data from two distinctly different type trajectories was used to compute trajectories for the comparison. A nearly vertical trajectory with apogee of approximately 400 miles (obtained from Thor launchings at Johnston Island nuclear tests of 1962) and a nearly circular orbit with apogee of approximately 460 miles (USA-2 Geodetic SECOR satellite of 1964) were used in the simulation. Figure N-2 is a plot of the range and elevation angles versus time for the two vehicles.

The satellite orbit closely follows lines of equal gravity while the Thor trajectory experiences a dramatic variation in gravity.

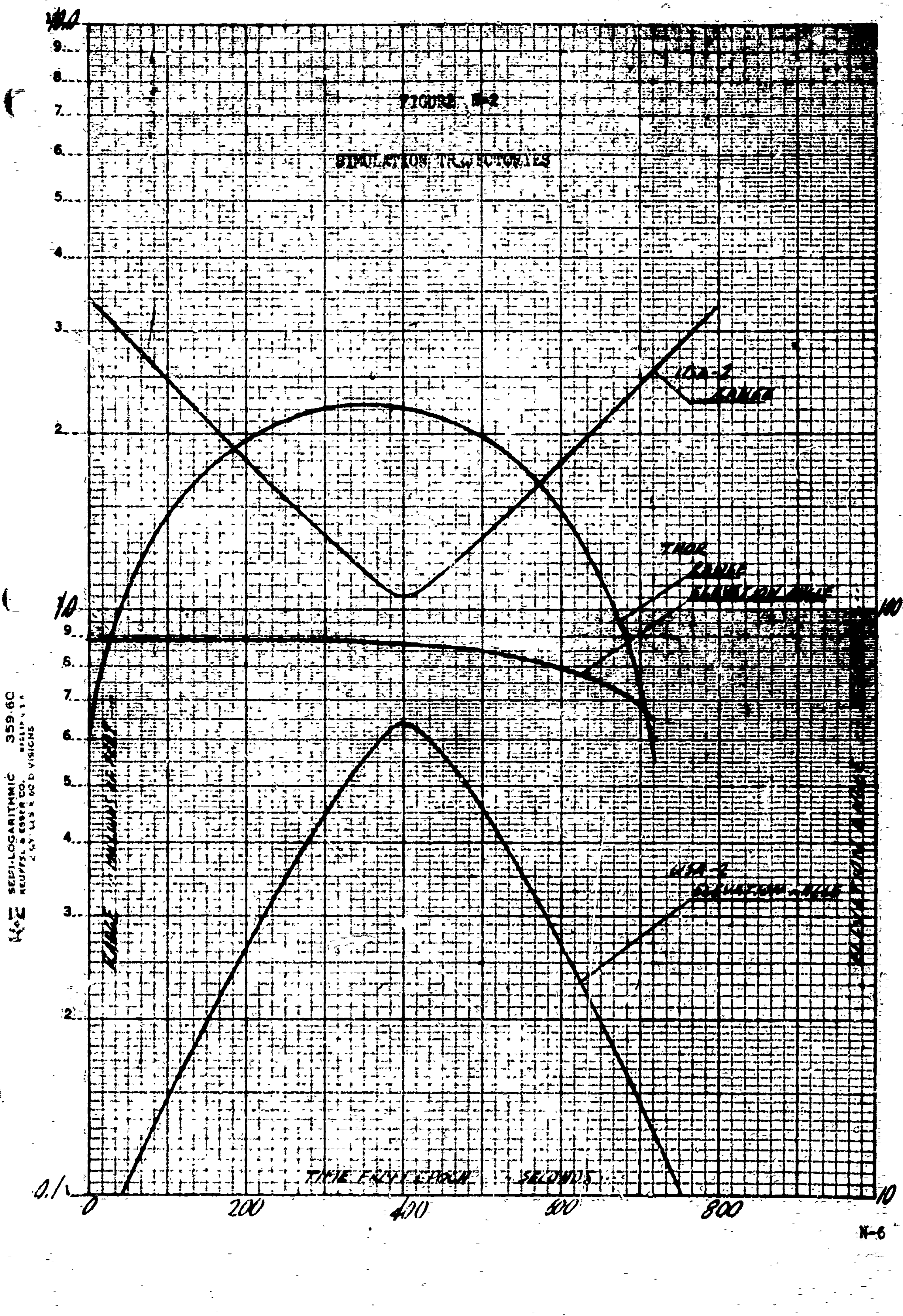


FIGURE N-2
SIMULATION TRAJECTORIES
USA-2
RANGE
THOR
RANGE
ELEVATION ANGLE
USA-2
ELEVATION ANGLE
TIME FROM EPOCH SECONDS
0
200
400
600
800
0.1
10
100
SEMI-LOGARITHMIC 359-60
KEUFFEL & ESSER CO.

Atmospheric drag is also minimized for the satellite and conversely will become a predominant deceleration for the Thor during reentry.

Figures N-3 through 6 illustrate the build up of error due entirely to analytic approximations using the Encke and Cowell methods of prediction. The amount of error is controlled by the rectification interval, or the interval over which the prediction is carried without re-establishing initial conditions. To avoid masking the prediction errors with other sources of error such as tracking equipment, gravity model, atmospheric drag, etc., the measured data was used in a least square fitting procedure to establish injection vectors. A test trajectory was then computed using a rectification interval of one second. The predicted and measured trajectories had agreement of about ±20 feet in each position component over the 800 second span used in the simulation.

After the "standard" had been computed, a set of predictions were made varying only the rectification interval. The separation of the predicted trajectory from the "standard" was attributed to analytic approximations which are inherent in the numerical integration schemes. Rectification intervals were reduced to 0.1 seconds and no separation from the "standard" was encountered. It was necessary to develop a very accurate solution of Kepler's equation of mean motion before the errors could be reduced to the values shown for Encke's method and the two-body predictions.

A review of Figures N-3 through N-8 demonstrates that Encke's method produced significantly better accuracies than the Cowell's. Computing time for the two procedures is about equal due to the use of two-body in both schemes. Computing time per rectification could be reduced in the

Cowell's method; however, this method would require many more rectifications to achieve equivalent results. Comparison of Encke's and Cowell's results with the two-body predictions indicates that large rectification intervals can reduce accuracy more than the total effect of perturbations. For example, if a 100-second rectification interval is used, the arithmetic error after 800 seconds will be 58' and 7400' for Encke's and Cowell's, respectively, and 9600' of total perturbation offset for the USA-2 satellite orbit. With the Thor trajectory after only 500 seconds of prediction with a 100-second rectification interval, Encke's and Cowell's had errors of 38' and 9000', respectively, and 4700' of perturbation offset.

The problem of analytic approximation error quickly becomes the dominant consideration in predictions. When orbits are to be computed over several day periods, care must be exercised if results are to bear any resemblance to actual conditions.

Time scales on all figures are in seconds with respect to an arbitrary epoch time, which is taken for convenience at some time in freefall. The "effect of perturbations" shown on Figures N-7 and N-8 is simply the offset difference between the two-body trajectory and the precise trajectory computed by Encke's method carrying all significant perturbations. Perturbations included the first nine zonal harmonics of the earth's gravity and atmospheric drag and lift. The effect of drag and lift was undetectable over 800 seconds of the USA-2 satellite orbit.

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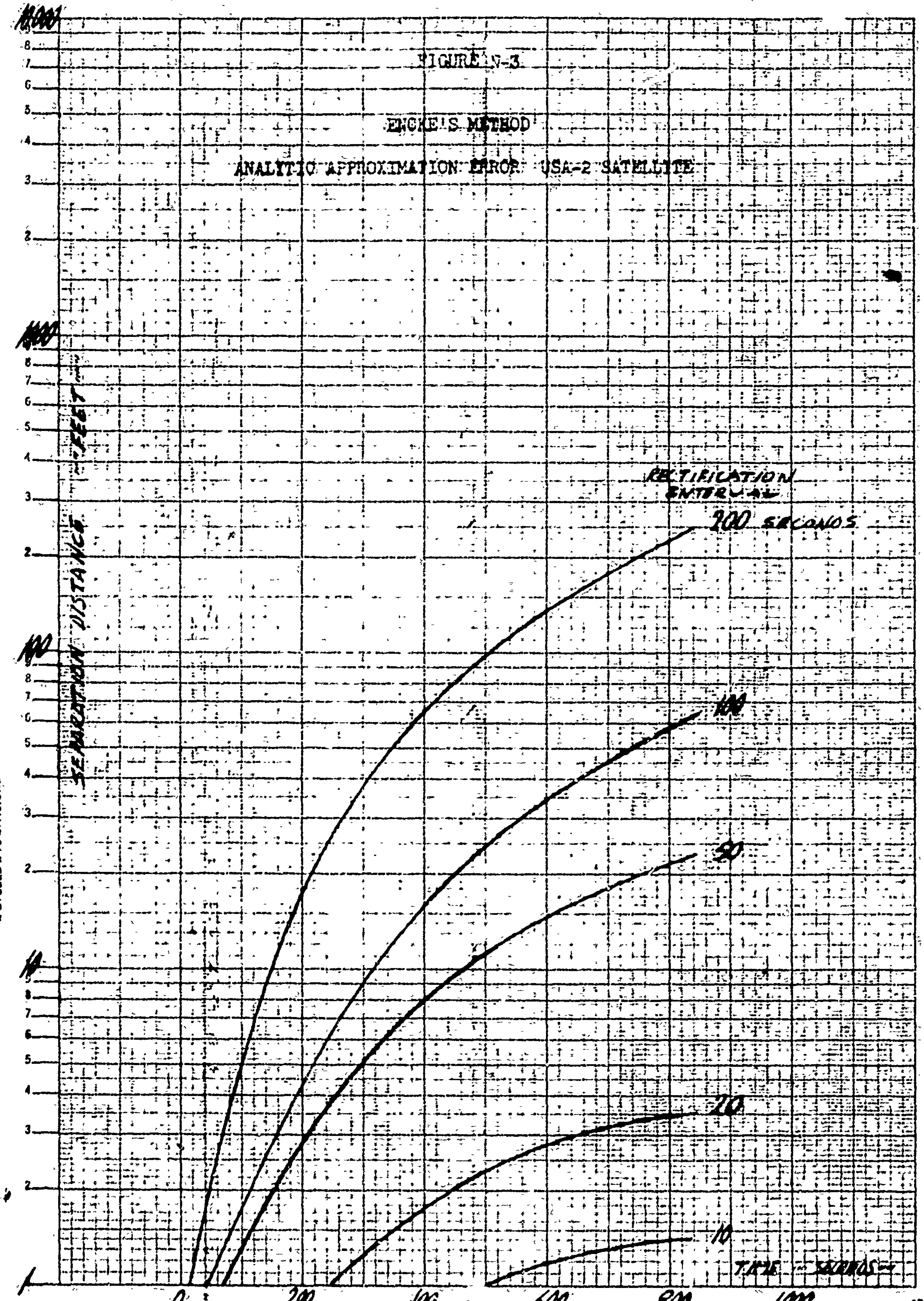


FIGURE N-3
ENCKE'S METHOD
ANALYTIC APPROXIMATION ERROR USA-2 SATELLITE
SEPARATION DISTANCE – FEET
RECTIFICATION INTERVAL
200 SECONDS
100
50
20
10
TIME – SECONDS
0
200
400
600
800
1000
1
10
100
1000
10000

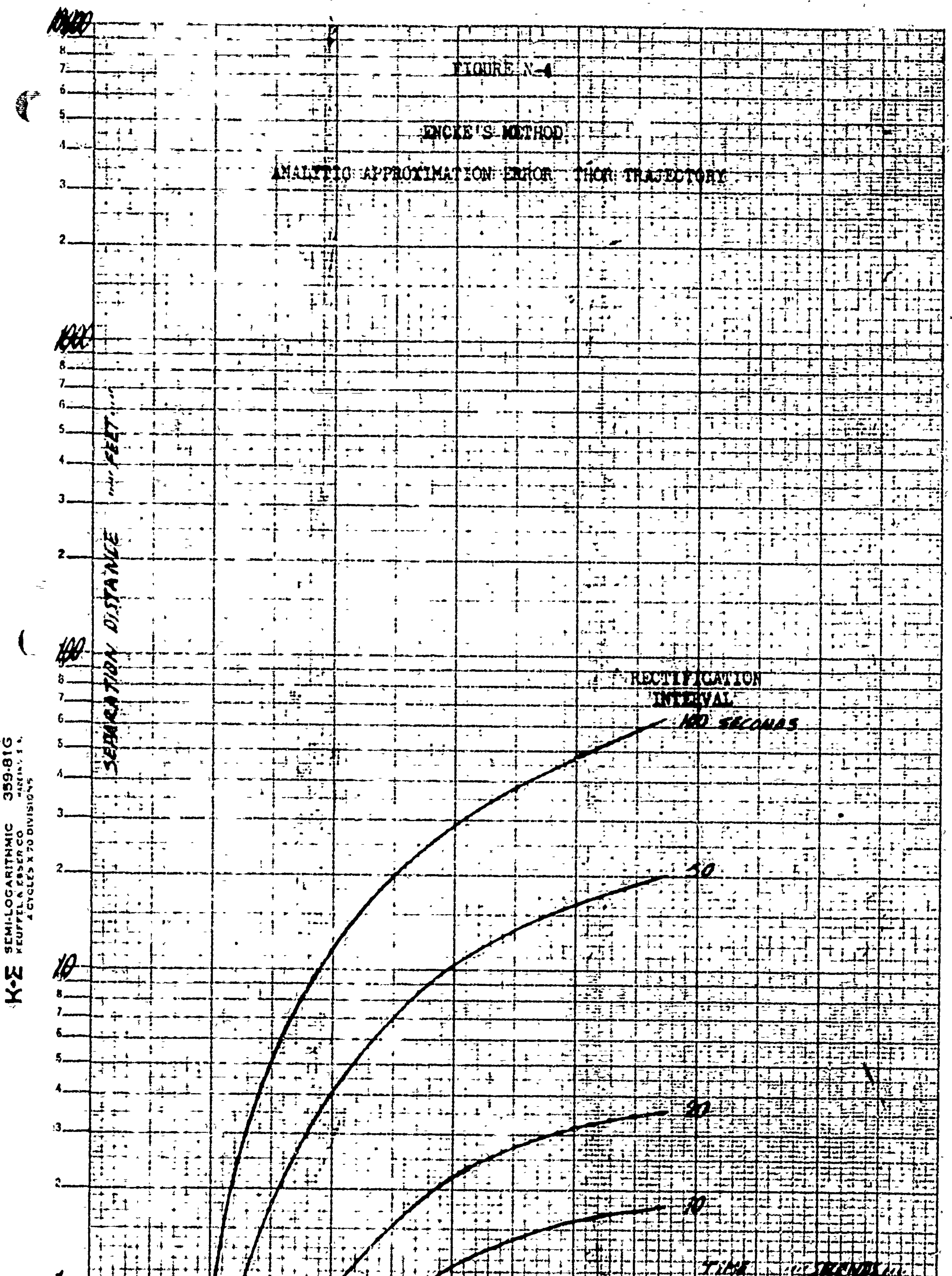
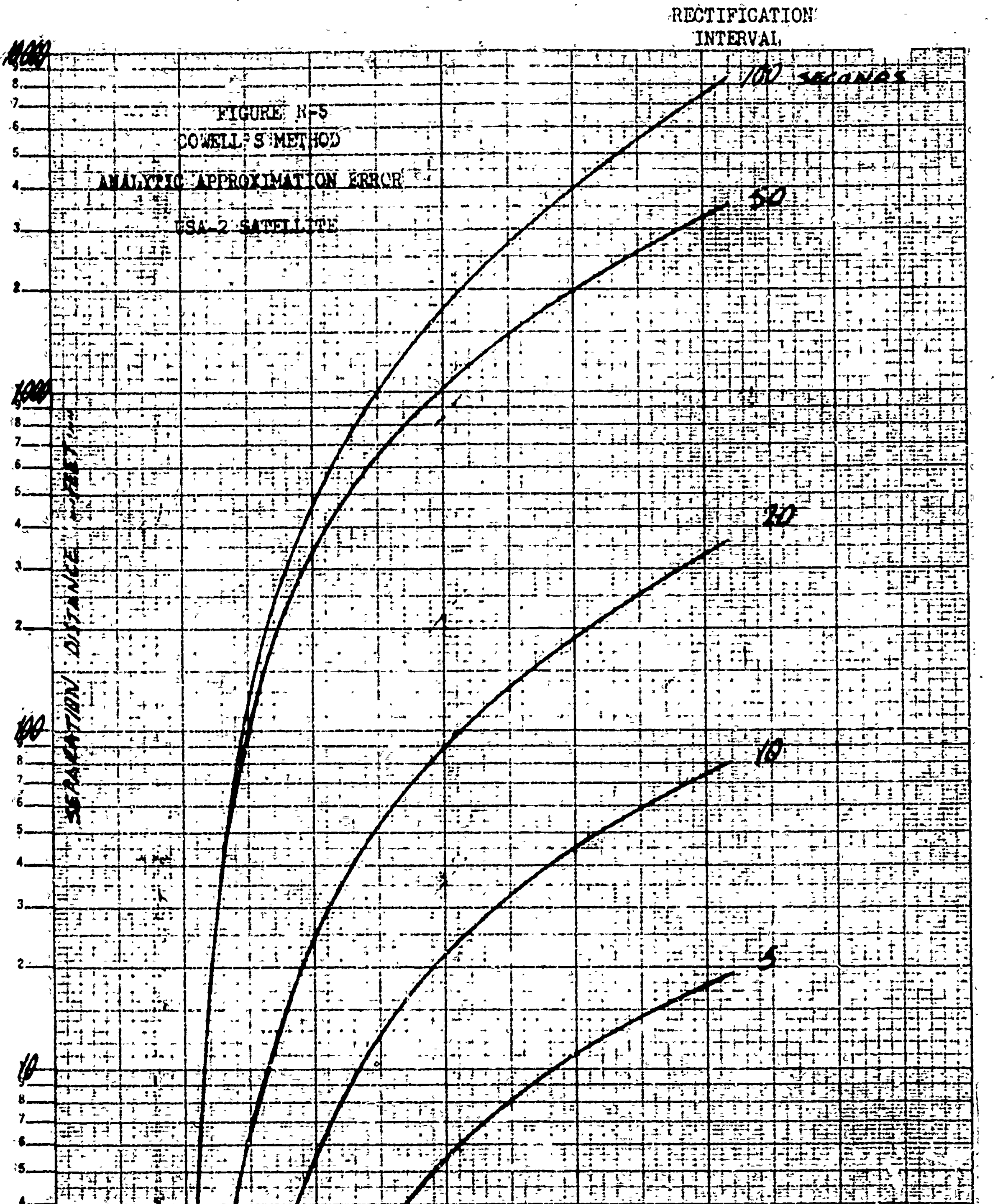
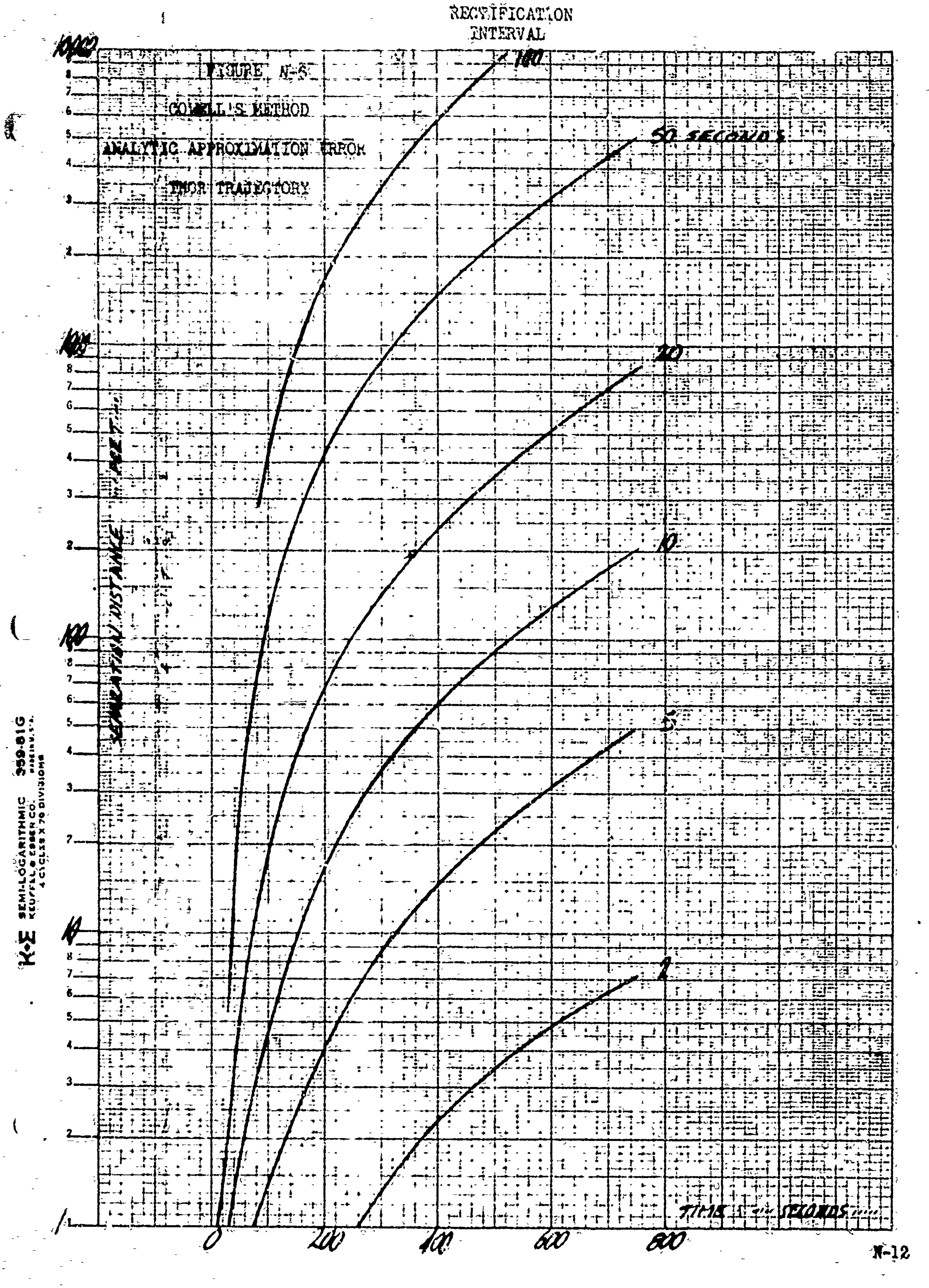
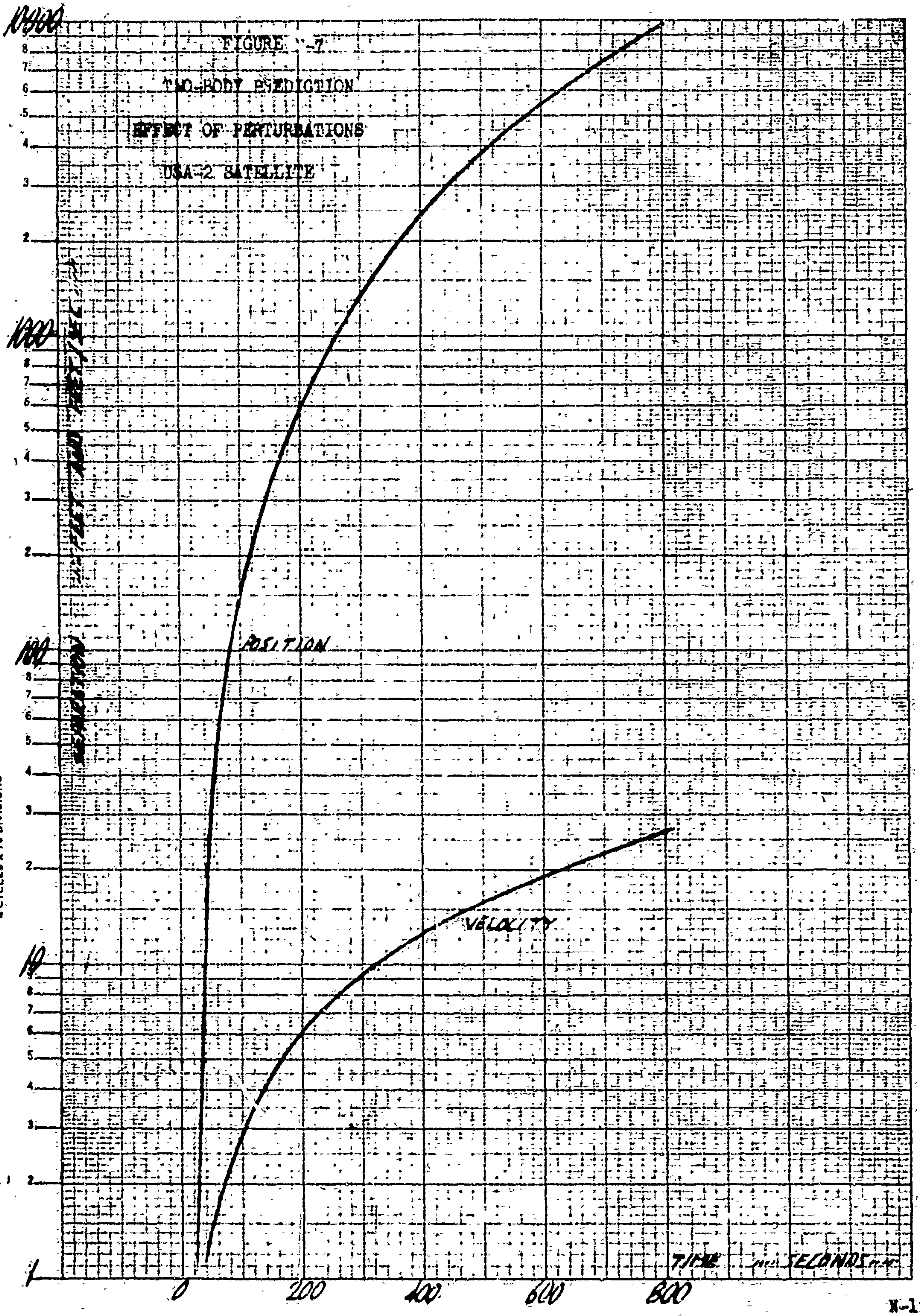


FIGURE N-4
ENCKE'S METHOD
ANALYTIC APPROXIMATION ERROR THOR TRAJECTORY
SEPARATION DISTANCE ... FEET ...
RECTIFICATION INTERVAL
100 SECONDS
50
20
10
TIME ... SECONDS
0
200
400
600
800
10000
1000
100
10
1
K+E SEMI-LOGARITHMIC 359-81G
KEUFFEL & ESSER CO.
4 CYCLES X 70 DIVISIONS



RECTIFICATION INTERVAL
FIGURE N-5
COWELL'S METHOD
ANALYTIC APPROXIMATION ERROR
USA-2 SATELLITE
100 SECONDS
50
20
10
5
2
SEPARATION DISTANCE - FEET
TIME - SECONDS
200
400
600
800
1000
K•E SEMI-LOGARITHMIC 359-81G
KEUFFEL & ESSER CO.
4 CYCLES X 70 DIVISIONS


RECTIFICATION
INTERVAL
FIGURE N-8
COWELL'S METHOD
ANALYTIC APPROXIMATION ERROR
TMOR TRAJECTORY
100
50 SECONDS
20
10
5
2
SEPARATION DISTANCE FEET
TIME SECONDS
0
200
400
600
800
1000
100
10
1
K·E SEMI-LOGARITHMIC 359-81G
KEUFFEL & ESSER CO.
4 CYCLES X 70 DIVISIONS



K+E SEMI-LOGARITHMIC 359-81G KEUFFEL & ESSER CO. 4 CYCLES X 70 DIVISIONS

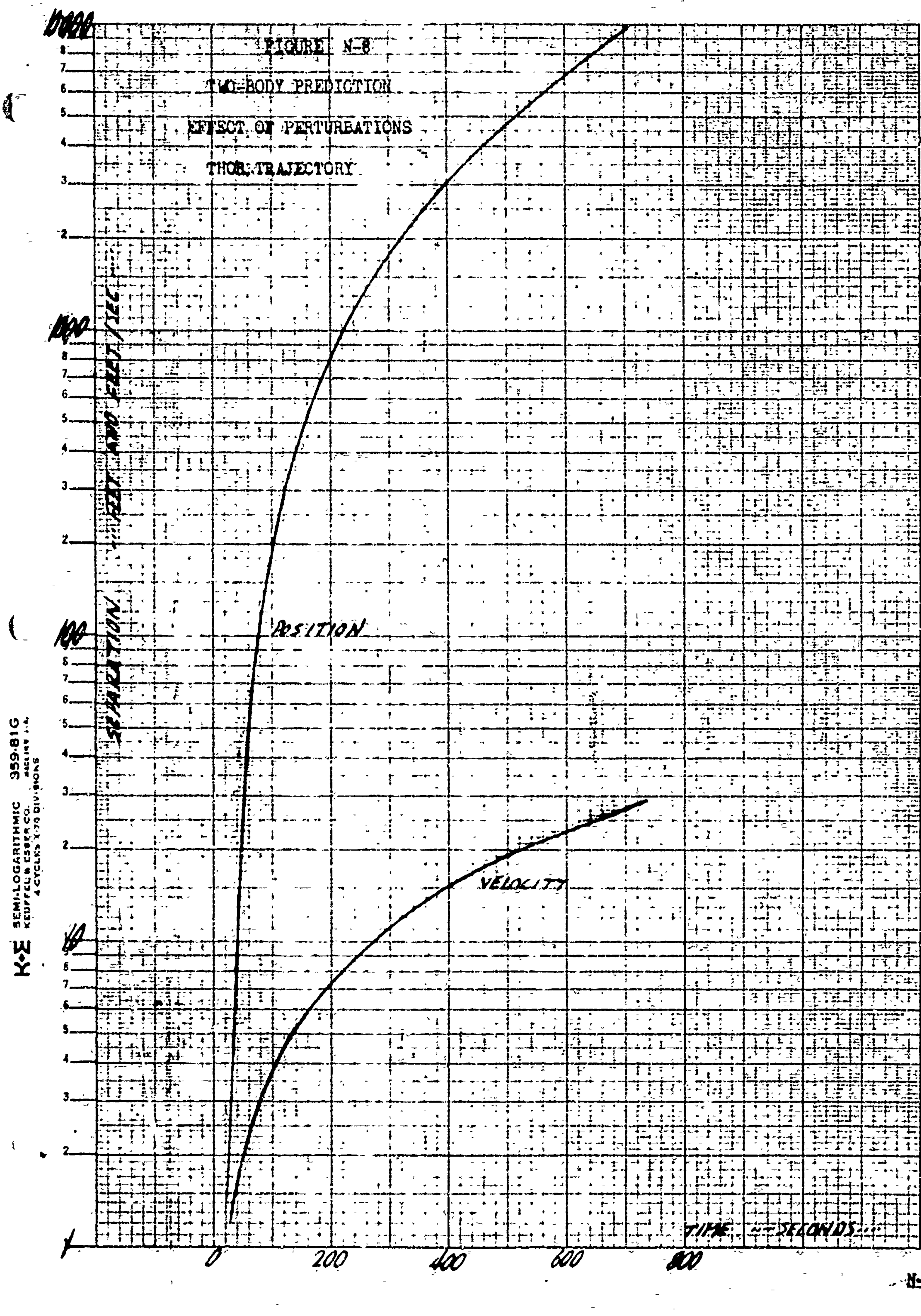


FIGURE N-8
TWO-BODY PREDICTION
EFFECT OF PERTURBATIONS
THOR TRAJECTORY
SEPARATION -- FEET AND FEET/SEC
POSITION
VELOCITY
TIME -- SECONDS
10000
1000
100
10
1
0
200
400
600
800
K+E SEMI-LOGARITHMIC 359-81G
KEUFFEL & ESSER CO.
4 CYCLES X 70 DIVISIONS

CUBIC CORPORATION

APPENDIX

TWO-BODY TRAJECTORY PREDICTION, PARTIAL DERIVATIVES, SOLUTION OF KEPLERS EQUATION, AND INERTIAL TO EQUATORIAL ROTATIONS

TRAJECTORY PREDICTION . . . . If the position and velocity vectors are known at one time for a body in freefall, a "closed expression" can be developed* to predict the two-body motion of the body versus time. When all quantities are expressed in canonical units and the position and velocity vectors are expressed in inertial coordinates (i.e., with origin at the dynamic center, which is the center of mass of the earth for near earth trajectories and orbits), predicted positions and velocities** for some time, t, following epoch (time t_o) are given by

$$\bar{R} = f\,\bar{R}_o + g\,\bar{V}_o \tag{1}$$

$$\bar{V} = \dot{f}\,\bar{R}_o + \dot{g}\,\bar{V}_o \tag{2}$$

where

$\bar{R}_o$, $\bar{V}_o$ = the position and velocity vectors at injection, respectively

$$f = a(\cos \Delta E - \beta_o)/R_o \tag{3}$$

$$g = a^{3/2}(\sin \Delta E - \delta + \delta_o) \tag{4}$$

$$\dot{f} = -a^{3/2}(\sin \Delta E)/(R\,R_o) \tag{5}$$

$$\dot{g} = a(\cos \Delta E - \beta)/R \tag{6}$$

---

* "Position, Velocities, Ephemerides, Referred to the Dynamical Center," Astrodynamical Report No. 7, by Samual Herrick, Dept. of Astronomy, University of California of Los Angeles and Aeronutronics, July, 1960.

** Elliptic motion assumed in this set of equations

$$R_o = (\bar{R}_o \cdot \bar{R}_o)^{1/2} \tag{7}$$

$$V_o^2 = (\bar{V}_o \cdot \bar{V}_o) \tag{8}$$

$$1/a = 2/R_o - V_o^2 \tag{9}$$

$$\delta_o = (\bar{R}_o \cdot \bar{V}_o)/a^{1/2} = e \sin E_o \tag{10}$$

$$\beta_o = 1 - R_o/a = e \cos E_o \tag{11}$$

$$E_o = \tan^{-1} (\delta_o/\beta_o) \text{ with quadrant test} \tag{12}$$

$$E_o = \text{eccentric anomaly at time } t_o$$

$$e = (\delta_o^2 + \beta_o^2)^{1/2} = \text{eccentricity of ellipse} \tag{13}$$

$$1/a^{3/2} = n = \text{mean motion} \tag{14}$$

$$\bar{R}_o = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} \qquad \bar{V}_o = \begin{bmatrix} \dot{X}_o \\ \dot{Y}_o \\ \dot{Z}_o \end{bmatrix} = \text{injection vectors} \tag{15}$$

$$\Delta E = E - E_o \tag{16}$$

$$E = \text{eccentric anomaly at time } t$$

$$R = a(1 - \beta) \tag{17}$$

$$\beta = e \cos E \tag{18}$$

$$\delta = e \sin E \tag{19}$$

Once $\bar{R}$ and $\bar{V}$ are computed by equations (1) and (2) they can be transformed into any desired coordinate system. The above equations are valid in an inertial reference frame. (See Figure C-1.)

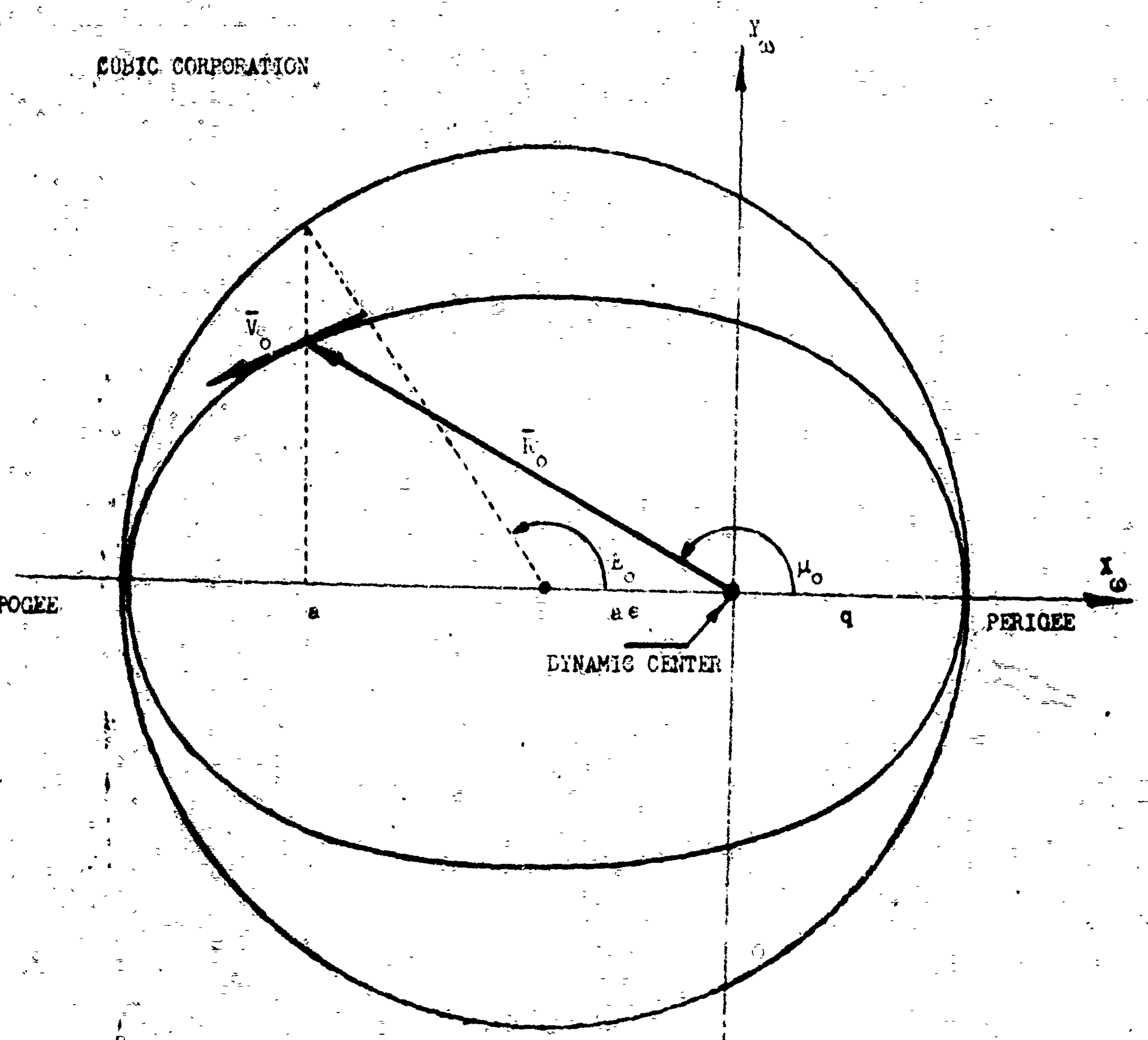


Figure -1

Two-Body Geometries and Notation

t_o = Time of epoch or injection

$\bar{R}_o$ = Position vector at injection

$\bar{V}_o$ = Velocity vector at injection

E_o = Eccentric anomaly at injection

μ_o = True anomaly at injection

a = Semimajor axis of ellipse

e = Eccentricity of ellipse

q = a(1 - e) = perigee distance

X_ω, Y_ω = Coordinate axes referred to orbital plane and perigee

SOLUTION OF KEPLER'S EQUATION FOR ΔE . . . . When $(t - t_o)$ is given, an extremely efficient iteration (less than six iterations for accuracies of ± 0.1 μ radian) can be set up to solve the transcendental function called Kepler's equation of two-body motion; namely,

$$n(t - t_o) = M - M_o = (E - E_o) - e \sin E + e \sin E_o \tag{20}$$

$$\Delta M = \Delta E - e \sin (E_o + \Delta E) + \delta_o \tag{21}$$

where M is the mean anomaly.

A common form of the solution for ΔE given in the literature is to iterate equation 21 where the first approximation of ΔE is given by

$$\Delta E_1 = \Delta M - \delta_o + e \sin (E_o + \Delta M) \tag{22}$$

and succeeding iterations take the form

$$\Delta E_{i+1} = \Delta M - \delta_o + e \sin (E_o + \Delta E_i) \tag{23}$$

Convergence of the iteration is established when

$$\left| \Delta E_{i+1} - \Delta E_i \right| \geq K \approx 0 \tag{24}$$

where K = a precomputed accuracy limit.

The results of one such classic iteration are shown on figure 0-2. Differences of the ΔE_i and the final ΔE are plotted versus the number of iterations. The solution oscillates systematically and converges slowly to a fixed value. In the example shown on figure 0-2, more than 170 iterations were required for $K = 1.10^{-7}$.

CUBIC CORPORATION

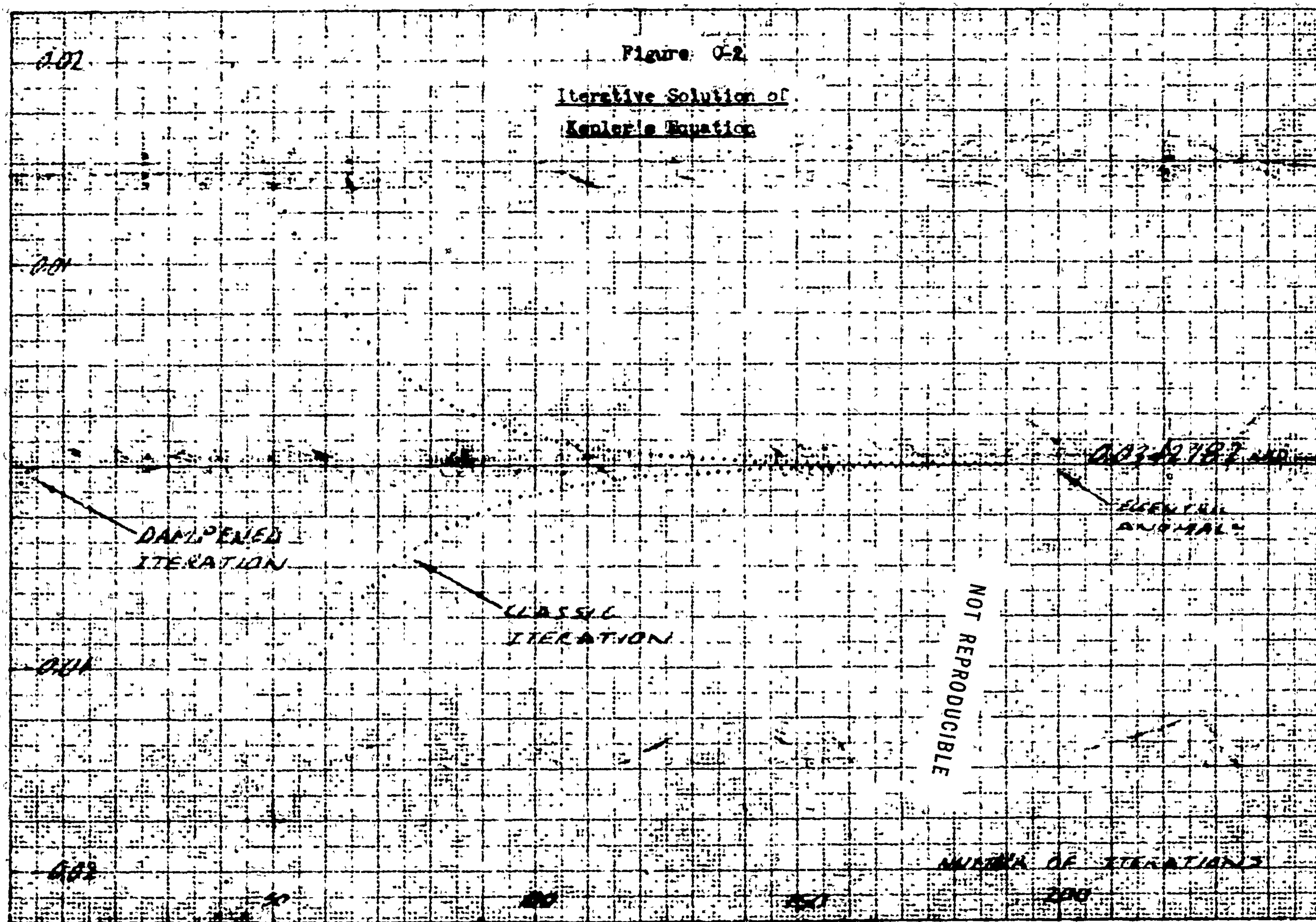


Figure C-2

Iterative Solution of Kepler's Equation

It was observed that the successive iterations of equation (23) were opposite in sign and therefore susceptible to dampening. In order to make the solution given by equation (23) useful, an alternate form of the solution was tried; namely,

$$\Delta E_{i+1} = \Delta M - \delta_o + e \sin [E_o + (\Delta E_{i-1} + \Delta E_i)/2] \tag{25}$$

For the same conditions used to test equation (23), equation (25) satisfied the iteration convergence criterion in 5 iterations. The dampened iteration proved to be highly efficient and accurate and represents the most useful solution to Kepler's equation attempted, which has included several partial derivative techniques.

If the above set of equations are to be used in formulations where high accuracy and precision are essential (such as computing the reference orbit or trajectory in the Encke's method of predicting), then using a fixed constant for K in equation (24) can limit accuracy and make computations susceptible to truncation error in computers. The variability of ΔE can be compensated by making K also variable in such a way as to retain proportionate accuracy of ΔE. In order to achieve more uniform and higher accuracies from the iteration, a K for the convergence test of equation (24) was computed as

$$K' = K\Delta M \text{ where } K = 0.0000001.$$

The convergence test given by equation (24) now becomes

$$\text{CONTINUE} = K' < \left|\Delta E_{i+1} - \Delta E_i\right| < K' = \text{EXIT}$$

TWO-BODY PARTIAL DERIVATIVES . . . . . Partial differential operators ∇, ∇', and D are defined with the following convention:

$$\nabla = \begin{bmatrix} \dfrac{\partial}{\partial X_o} \\ \dfrac{\partial}{\partial Y_o} \\ \dfrac{\partial}{\partial Z_o} \end{bmatrix}, \qquad \nabla' = \begin{bmatrix} \dfrac{\partial}{\partial \dot{X}_o} \\ \dfrac{\partial}{\partial \dot{Y}_o} \\ \dfrac{\partial}{\partial \dot{Z}_o} \end{bmatrix}, \qquad D = \begin{bmatrix} \nabla \\ \nabla' \end{bmatrix} \tag{26}$$

Given the equations

$$\bar{R} = \bar{R}_o f + \bar{V}_o g$$

$$\bar{V} = \bar{R}_o \dot{f} + \bar{V}_o \dot{g}$$

then the two-body partial derivatives of positon and velocity at time t with respect to injection vectors $\bar{R}_o$ $\bar{V}_o$ at time t_o become

$$\underset{(6 \times 6)}{P} = \left[\begin{array}{c|c} Q_1 & Q_2 \\ \hline Q_3 & Q_4 \end{array}\right] = \left[\begin{array}{ccc|ccc}
\dfrac{\partial X}{\partial X_o} & \dfrac{\partial X}{\partial Y_o} & \dfrac{\partial X}{\partial Z_o} & \dfrac{\partial X}{\partial \dot{X}_o} & \dfrac{\partial X}{\partial \dot{Y}_o} & \dfrac{\partial X}{\partial \dot{Z}_o} \\[2ex]
\dfrac{\partial Y}{\partial X_o} & \dfrac{\partial Y}{\partial Y_o} & \dfrac{\partial Y}{\partial Z_o} & \dfrac{\partial Y}{\partial \dot{X}_o} & \dfrac{\partial Y}{\partial \dot{Y}_o} & \dfrac{\partial Y}{\partial \dot{Z}_o} \\[2ex]
\dfrac{\partial Z}{\partial X_o} & \dfrac{\partial Z}{\partial Y_o} & \dfrac{\partial Z}{\partial Z_o} & \dfrac{\partial Z}{\partial \dot{X}_o} & \dfrac{\partial Z}{\partial \dot{Y}_o} & \dfrac{\partial Z}{\partial \dot{Z}_o} \\[2ex] \hline
\dfrac{\partial \dot{X}}{\partial X_o} & \dfrac{\partial \dot{X}}{\partial Y_o} & \dfrac{\partial \dot{X}}{\partial Z_o} & \dfrac{\partial \dot{X}}{\partial \dot{X}_o} & \dfrac{\partial \dot{X}}{\partial \dot{Y}_o} & \dfrac{\partial \dot{X}}{\partial \dot{Z}_o} \\[2ex]
\dfrac{\partial \dot{Y}}{\partial X_o} & \dfrac{\partial \dot{Y}}{\partial Y_o} & \dfrac{\partial \dot{Y}}{\partial Z_o} & \dfrac{\partial \dot{Y}}{\partial \dot{X}_o} & \dfrac{\partial \dot{Y}}{\partial \dot{Y}_o} & \dfrac{\partial \dot{Y}}{\partial \dot{Z}_o} \\[2ex]
\dfrac{\partial \dot{Z}}{\partial X_o} & \dfrac{\partial \dot{Z}}{\partial Y_o} & \dfrac{\partial \dot{Z}}{\partial Z_o} & \dfrac{\partial \dot{Z}}{\partial \dot{X}_o} & \dfrac{\partial \dot{Z}}{\partial \dot{Y}_o} & \dfrac{\partial \dot{Z}}{\partial \dot{Z}_o}
\end{array}\right] \tag{27}$$

CUBIC CORRECTION

$$
\begin{aligned}
&\qquad (3 \times 3) \\
Q_1 &= \bar{R}_o \nabla f^T + fI + \bar{V}_o \nabla g^T \\
Q_2 &= \bar{R}_o \nabla' f^T + gI + \bar{V}_o \nabla' g^T \\
Q_3 &= \bar{R}_o \nabla \dot{f}^T + \dot{f}I + \bar{V}_o \nabla \dot{g}^T \\
Q_4 &= \bar{R}_o \nabla' \dot{f}^T + \dot{g}I + \bar{V}_o \nabla' \dot{g}^T
\end{aligned}
\tag{28}
$$

$$
(3 \times 3) \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
&\quad\; 3 \times 6 \qquad\quad (3\times2) \qquad (2\times6) \qquad\quad 3 \times 6 \\
P_{\bar{R}} &= [Q_1 Q_2] = [\bar{R}_o \bar{V}_o]\,[Df\ Dg]^T + [fI\ gI] \\
P_{\bar{V}} &= [Q_3 Q_4] = [\bar{R}_o \bar{V}_o]\,[D\dot{f}\ D\dot{g}]^T + [\dot{f}I\ \dot{g}I]
\end{aligned}
\tag{29}
$$

where $P_{\bar{R}}$ and $P_{\bar{V}}$ are the two-body partials of position $\bar{R}$ and velocity $\bar{V}$ with respect to both $\bar{R}_o$ and $\bar{V}_o$, respectively.

Taking partials of f, g, $\dot{f}$, and $\dot{g}$ will give

$$
Df = \frac{1}{(1 - \beta_o)} [D\beta_o(f - 1) - \sin \Delta E \, D\Delta E] \tag{30}
$$

$$
Dg = \left(\frac{3}{2a} g\right) Da + \frac{1}{n}(\cos \Delta E \, D\Delta E - D\delta + D\delta_o) \tag{31}
$$

$$
D\dot{f} = \dot{f}\left(\frac{a}{R_o} D\beta_o + \frac{a}{R} D\beta - \frac{3}{2} aDa + \frac{\cos \Delta E}{\sin \Delta E} D\Delta E\right) \tag{32}
$$

$$
D\dot{g} = \frac{1}{(1 - \beta)} [D\beta(\dot{g} - 1) - \sin \Delta E \, D\Delta E] \tag{33}
$$

$$
D\delta = Dc + \beta D\Delta E \tag{34}
$$

$$
D\Delta E = \frac{a}{R}\left[Dc - D\delta_o - \frac{3}{2} \frac{n}{a}(t - t_o) Da\right] \tag{35}
$$

$$D\epsilon = \sin \Delta E \, D\beta_o + \cos \Delta E \, D\delta_o \tag{36}$$

$$D\beta = DD - \delta \, D\Delta E \tag{37}$$

$$DD = \cos \Delta E \, D\beta_o - \sin \Delta E \, D\delta_o \tag{38}$$

$$D\beta_o = \begin{bmatrix} \nabla \beta_o \\ \nabla' \beta_o \end{bmatrix} \tag{39}$$

$$\nabla \beta_o = \frac{V_o^{\,2}}{R_o} \bar{R}_o \tag{40}$$

$$\nabla' \beta_o = 2\bar{R}_o \bar{V}_o \tag{41}$$

$$D\delta_o = \begin{bmatrix} \nabla \delta_o \\ \nabla' \delta_o \end{bmatrix} \tag{42}$$

$$\nabla \delta_o = \frac{\bar{V}_o}{a^2} - \frac{a\delta_o \bar{R}_o}{R_o^{\,3}} \tag{43}$$

$$\nabla' \delta_o = \frac{\bar{R}_o}{a^2} - a\delta_o \bar{V}_o \tag{44}$$

$$Da = \begin{bmatrix} \nabla a \\ \nabla' a \end{bmatrix} \tag{45}$$

$$\nabla a = \frac{2a^2}{R_o^{\,3}} \bar{R}_o \tag{46}$$

$$\nabla' a = 2a^2 \bar{V}_o \tag{47}$$

INERTIAL TO EQUATORIAL ROTATIONS . . . . All two-body equations are computed in a space fixed or inertial coordinate system. For near earth orbits and trajectories, inertial coordinates can be assumed to be coincident with the equatorial coordinates at epoch time t_o. A rotation from the stationary inertial coordinates to the earth fixed equatorial coordinates is given by

$$M_{IE} = \begin{bmatrix} \cos\omega_e \Delta t & \sin\omega_e \Delta t & 0 \\ -\sin\omega_e \Delta t & \cos\omega_e \Delta t & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{inertial to equatorial rotation matrix} \tag{48}$$

where t_o = time of epoch or injection

t = time in trajectory

$\Delta t = t - t_o$

With matrix M_{IE}, the inertial position, velocity, and partials will be rotated to equatorial by

$$\bar{R}_{EQ} = M_{IE} \bar{R} \tag{49}$$

$$\bar{V}_{EQ} = M_{IE} \bar{V} - \bar{V}_E \tag{50}$$

$$\bar{V}_E = \omega_e \begin{bmatrix} -Y_{EQ} \\ X_{EQ} \\ 0 \end{bmatrix} \tag{51}$$

$$P_{EQ} = \left[\begin{array}{c|c} M_{IE} Q_1 M_{IE}^T & M_{IE} Q_2 M_{IE}^T \\ \hline M_{IE} Q_3 M_{IE}^T & M_{IE} Q_4 M_{IE}^T \end{array}\right] \tag{52}$$

where ω_e = earth's angular rate

T = refers to matrix transpose.

All equations given here for trajectory prediction are predicated on a spherical gravity field and the absence of all perturbations such as atmospheric drag, aerodynamic lift, thrusts, etc. The equations are very useful in evaluation considerations where simplicity and versatility are needed before solutions and problems can be economically simulated. Perturbations can be added to the above equations to strengthen their applicability to real problems. Principal attributes of these equations are their versatility, simplicity, and speed of computation on computers. The two-body partials can be used in orbit and trajectory fitting to observational data.

FCF:kkb

APPENDIX P

NON-GRAVITATIONAL PERTURBATIONS

Let the accelerations acting on a vehicle in freefall be expressed by

$$\bar{A} = \bar{G} + \Sigma \bar{G}_p + \Sigma \bar{P} \qquad (1)$$

where,

$$\bar{A} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \text{total acceleration components}$$

$$\bar{G} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \text{principal component of earth's gravity}$$

$$\Sigma \bar{G}_p = \begin{bmatrix} q'_x \\ q'_y \\ q'_z \end{bmatrix} = \text{perturbations of earth's gravity due to zonal harmonics}$$

$$\Sigma \bar{P} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \text{acceleration component due to non-gravitational perturbations}$$

Examples of non-gravitational perturbations would be accelerations due to lift, drag, and solar radiation. As a matter of convention, the directional components are defined as being directed along the X, Y, Z axes of a right-handed orthogonal coordinate system. The reference frame used here is an earth-centered, space-fixed inertial coordinate system.

When the vehicle is in close proximity of the earth's atmosphere and extremely long prediction intervals are not considered, then atmospheric

lift and drag become the significant perturbative sources. Drag and lift accelerations, a_d and a_ℓ respectively, are given by*

$$a_d = 1/2\ \rho\ V_\ell^2\ C_d\ S/m$$
$$a_\ell = 1/2\ \rho\ V_\ell^2\ C_\ell\ S/m \qquad (2)$$

where

m = mass of the vehicle

ρ = air density

V_ℓ = vehicle earth-related velocity

C_d = vehicle drag coefficient

C_ℓ = vehicle lift coefficient

S = vehicle effective cross-sectional area

A vehicle's drag coefficient is not constant, but rather a function of the vehicle's Mach speed and shape. Also, the value of the drag coefficient for different Mach speeds does not lend itself to analytical expression. Thus, a table of actual measured values of the vehicle drag coefficient versus Mach speed is required to compute acceleration due to drag. Such a table for an instrumentation pod is given in table I-1. Once a table is provided, it is only necessary to compute the vehicle's Mach speed in order to determine the value of the drag coefficient. Vehicle Mach speed is defined by

$$M = V_\ell/C \qquad (3)$$

where

C = acoustic velocity

$C = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6$

y = vehicle altitude above the earth's surface in thousands of feet.

* Puckett, Allen E., "Guided Missile Engineering," McGraw-Hill, New York, 1959.

TABLE P-1

DRAG COEFFICIENTS

MACH NUMBER	COEFFICIENT
.0	1.12
.63	1.27
.90	1.38
1.02	1.82
1.05	1.95
1.15	2.07
1.25	2.10
1.37	2.06
1.52	1.99
2.40	1.64
3.00	1.48
3.50	1.41
3.97	1.38
5.00	1.35
6.00	1.34
10.00	1.31
40.00	1.31

The values of the coefficients, a_i, for different values of y are given in table ?. A comparison of values of acoustical velocity, computed using polynomials with the coefficients a_i, with data from the ARDC model atmosphere of 1959 is shown in table P-3.

The lift coefficient, C_ℓ, may be approximated by 2α, where α is the angle of attack in radians, if the vehicle body is assumed to be basically blunt in shape. Air density is a function of altitude above the earth's surface and may be approximated by

$$\rho = 10^r \, e^s \tag{4}$$

where

$$s = b_o + b_1 y + b_2 y^3$$

r = scaling exponent

Values of r and b_i for different values of y are given in table P-4. comparison of air density, computed from equation 4, with data from the ARDC model atmosphere of 1959 is given in table P-5. The coefficients b_i and a_i are primarily a result of fitting third degree polynomials to data from the ARDC model atmosphere of 1959.

The accelerations due to non-gravitational perturbations may now be expressed by

$$\Sigma\overline{P} = - a_d \overline{D} + a_\ell \overline{L} \tag{5}$$

where

$\overline{D}$ = unit vector directed along the longitudinal axis of the vehicle.

L = unit vector normal to the longitudinal plane of the vehicle.

TABLE P-2

COEFFICIENTS FOR COMPUTING SPEED OF SOUND (FT. SEC.$^{-1}$)

i	1	2	3*	4	5	6	7
a_0	1.116243333×10^3	968.08	2.21401073×10^4	1105.7	1.581094244×10^3	846.5	1100.0
a_1	-3.778722222×10^0	-0-	-1.2858749×10^5	-0-	-3.233401688×10^0	-0-	-0-
a_2	$-1.01777778 \times 10^{-2}$	-0-	3.9502867×10^5	-0-	$5.65363161 \times 10^{-3}$	-0-	-0-
a_3	$+3.777777326 \times 10^{-5}$	-0-	2.6060088×10^5	-0-	$-1.523623994 \times 10^{-5}$	-0-	-0-
a_4	-0-	-0-	-3.5596453×10^6	-0-	-0-	-0-	-0-
a_5	-0-	-0-	6.6814040×10^6	-0-	-0-	-0-	-0-
a_6	-0-	-0-	-4.0833579×10^6	-0-	-0-	-0-	-0-

$a_i = 0$ for $i \geq 4$

where

$$i = \begin{cases} 1 & -2000 \leq H \leq 38000 \\ 2 & 38000 < H \leq 80000 \\ 3 & 80,000 < H \leq 155,000 \\ 4 & 155,000 < H \leq 175,000 \\ 5 & 175,000 < H \leq 260,000 \\ 6 & 260,000 < H \leq 300,000 \\ 7 & 300,000 < H \end{cases}$$

H = altitude above mean sea level in feet

* In this case $y = H/300,000$. Coefficients are from Gianopulos, G. N., "Generalized Powered Flight Trajectory Program for IBM 704 Computer," JPL Technical Report No. 32-38, Sept., 1960.

TABLE P-3

COMPARISON OF COMPUTED VS. ACTUAL* SPEED OF SOUND

ALTITUDE (FT)	ACCUSTICAL VELOCITY (FT SEC^{-1})	ACTUAL
5.0×10^3	1097.1	1097.1
1.0×10^4	1077.4	1077.4
1.5×10^4	1057.4	1057.4
2.0×10^4	1036.9	1036.9
2.5×10^4	1016.0	1016.1
3.0×10^4	999.7	994.8
3.5×10^4	973.1	973.1
4.0×10^4	968.0	968.0
5.0×10^4	968.0	968.0
6.0×10^4	968.0	968.0
7.0×10^4	968.0	968.0
8.0×10^4	968.0	968.0
8.5×10^4	975.4	973.4
9.0×10^4	983.9	983.4
9.5×10^4	993.3	993.3
1.0×10^5	1003.0	1003.2
1.5×10^5	1095.3	1096.3
2.0×10^5	1038.6	1038.7
2.5×10^5	888.0	888.1
3.0×10^5	846.5	846.5
3.5×10^5	1100.0	1100.0
4.0×10^5	1100.0	1100.0
4.5×10^5	1100.0	1100.0

* By "actual" is meant values given by "Handbook of Geophysics," ARDC, 1960.

TABLE P-4

COEFFICIENTS IN COMPUTING AIR DENSITY (CB · FT^{-3})

i	b_1	b_2	b_3	b_o
1	$-2.140228194 \times 10^{-2}$	$-4.112711777 \times 10^{-4}$	$1.948636863 \times 10^{-6}$	4.33696389×10^{0}
2	$-1.385607963 \times 10^{-1}$	$+5.754215325 \times 10^{-4}$	$-1.060362481 \times 10^{-6}$	$1.385541893 \times 10^{1}$
3	$+6.194417751 \times 10^{-1}$	$-2.284935788 \times 10^{-3}$	$2.543977892 \times 10^{-6}$	$-4.859279372 \times 10^{1}$
4	$-5.301440339 \times 10^{-1}$	$+1.052418458 \times 10^{-3}$	$-7.164439446 \times 10^{-7}$	$9.154608305 \times 10^{1}$
5	$-1.423384892 \times 10^{-2}$	$+6.971707417 \times 10^{-6}$	$-1.733376425 \times 10^{-9}$	$9.995812402 \times 10^{0}$
6	$-8.730600381 \times 10^{-3}$	$+1.995115962 \times 10^{-6}$	-2.192314[illegible]63×10^{-10}	$1.254637591 \times 10^{1}$

and

	1	2	3	4	5	6
r	-4	-6	-8	-11	-13	-15

where

$$
i = \begin{cases} 1 & 0 \leq H \leq 100{,}000 \\ 2 & 100{,}000 < H \leq 250{,}000 \\ 3 & 250{,}000 < H \leq 350{,}000 \\ 4 & 350{,}000 < H \leq 525{,}000 \\ 5 & 525{,}000 < H \leq 1{,}200{,}000 \\ 6 & 1{,}200{,}000 < H \end{cases}
$$

H = altitude above mean sea level in feet

TABLE P-5

COMPARISON OF COMPUTED VS. ACTUAL* AIR DENSITY

ALTITUDE (FT)	COMPUTED AIR DENSITY ($GB \cdot FT^{-3}$)	ACTUAL
2.5×10^4	3.570×10^{-2}	3.430×10^{-2}
5.0×10^4	1.196×10^{-2}	1.170×10^{-2}
7.5×10^4	3.457×10^{-3}	3.546×10^{-3}
1.0×10^5	1.033×10^{-3}	1.033×10^{-3}
1.25×10^5	3.166×10^{-4}	3.274×10^{-4}
1.50×10^5	1.146×10^4	1.146×10^{-4}
1.75×10^5	4.695×10^{-5}	4.544×10^{-5}
2.00×10^5	1.968×10^{-5}	1.968×10^{-5}
2.25×10^5	7.647×10^{-6}	- -
2.5×10^5	2.493×10^{-6}	2.493×10^{-6}
3.0×10^5	1.327×10^{-7}	1.327×10^{-7}
3.5×10^5	7.282×10^{-9}	7.282×10^{-9}
4.0×10^5	7.561×10^{-10}	7.561×10^{-10}
4.5×10^5	2.248×10^{-10}	2.248×10^{-10}
5.0×10^5	1.023×10^{-10}	9.789×10^{-11}
7.0×10^5	1.735×10^{-11}	1.689×10^{-11}
9.0×10^5	4.800×10^{-12}	4.809×10^{-12}
1.1×10^6	1.595×10^{-12}	1.592×10^{-12}
1.3×10^6	5.954×10^{-13}	5.946×10^{-13}
1.5×10^6	2.452×10^{-13}	2.450×10^{-13}
1.7×10^6	1.094×10^{-13}	1.095×10^{-13}
1.9×10^6	5.243×10^{-14}	5.243×10^{-14}
2.1×10^6	2.665×10^{-14}	2.662×10^{-14}

*By "actual" is meant values given by "Handbook of Geophysics," ARDC, 1960.

Here, for simplicity, $\overline{D}$ was taken to be directed coincidentally with the vehicle's earth-related velocity vector, $\overline{V}_\ell$. Thus,

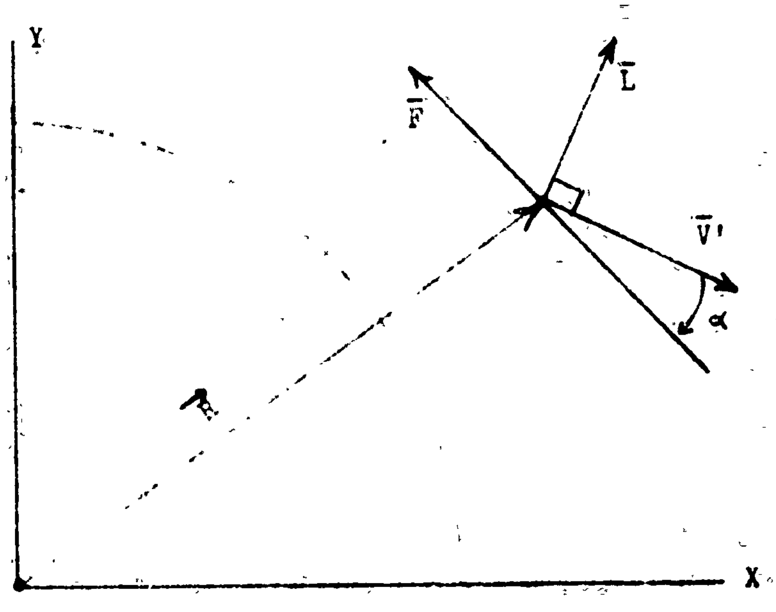
$$\overline{D} = \overline{V}' = \frac{\overline{V}_\ell}{v_\ell} = \begin{vmatrix} v'_x \\ v'_y \\ v'_z \end{vmatrix} \tag{6}$$

where

$$\overline{V}_\ell = \begin{vmatrix} v_x \\ v_y \\ v_z \end{vmatrix}$$

$$v_\ell = \left|\overline{V}_\ell\right|$$

Normally $\overline{L}$ is taken to be normal to some reference plane containing the longitudinal axis of the vehicle. For example, in the case of an aircraft, this plane is determined by the position of the wings. However, no assumptions of wings, nor their positioning, has been made. Therefore, this plane was taken to be the plane defined as being perpendicular to the equatorial plane and containing $\overline{V}'$. The flow of air $\overline{F}$, was taken to be perpendicular to the position vector, $\overline{R}$, of the vehicle and directed to oppose $\overline{V}'$.



Once the direction of $\bar{F}$ is determined, $\bar{L}$ becomes

$$\bar{L} = \begin{vmatrix} n_1 \\ n_2 \\ n_3 \end{vmatrix} = [\bar{V}' \cdot \cos \alpha + \bar{F}] \ (1/\sin \alpha) \qquad (7)$$

where

$$\alpha = \cos^{-1} \{-\bar{V}' \cdot \bar{F}\}$$

LBP:kkb

APPENDIX Q

EARTH'S GRAVITY FIELD

The earth's gravitational field may be derived from a scalar potential which obeys Poisson's equation; namely,

$$\nabla^2 \psi (r, \Phi, \lambda) = -\rho (r, \Phi, \lambda) \qquad (1)$$

where ψ = scalar potential

ρ = mass density

When the region of interest is above the earth's surface, $\rho = 0$ and equation (1) becomes

$$\nabla^2 \psi (r, \Phi, \lambda) = 0 \qquad (2)$$

A solution of equation (2) which is consistent with the physical earth is

$$\psi(r, \Phi, \lambda) = \frac{K}{a^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{a^{n+2}}{r^{n+1}} P_n^m (\sin \Phi) [C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda] \qquad (3)$$

where

Φ = geocentric latitude

λ = east longitude from Greenwich meridian

r = radial distance from earth's mass center

a = earth's equatorial radius (semimajor axis)

$P_n^m(\sin \Phi)$ = associated Legendre polynomials of the first kind

K = product of universal gravity constant and earth's mass

$C_{n,m}$, $S_{n,m}$ = empirical coefficients

The potential equation (3) is commonly referred to as an expansion in tesseral harmonics. The coefficients $C_{n,m}$ and $S_{n,m}$ have been evaluated

based on observations of satellite orbits and the corresponding changes in the associated basic orbital elements. Current values of the longitude dependent coefficients are not sufficiently well defined or significant to be used in general prediction representation. If it is assumed that the earth is longitudinally symmetric, then equation (3) becomes (after separation of the principal term)

$$\psi(r, \Phi) = \frac{K}{a^2}\left[\frac{1}{r} - \sum_{n=1}^{\infty} J_n \frac{a^{n+2}}{r^{n+1}} P_n (\sin \Phi)\right] \tag{5}$$

where

J_n = the zonal harmonics of the earth's gravity.

The principal term of equation (5), $\frac{1}{r}$, defines the potential of a spherical body with concentric distribution of mass. Perturbations due to the meridianal asymmetry of the earth are represented by the one to infinity summation terms.

Gravitational acceleration of the earth as a function of radial distance and geocentric latitude is given by the gradients of the potential function. (See figure Q-1.) Thus

$$\bar{g} = \bar{\nabla}\psi, \tag{6}$$

where $$\bar{\nabla} = \frac{\partial\psi}{\partial r}\bar{1}_r + \frac{1}{r}\frac{\partial\psi}{\partial\Phi}\bar{1}_\Phi \tag{7}$$

$$\frac{\partial\psi}{\partial r} = g_r = \text{radial component of gravity} \tag{8}$$

$$\frac{1}{r}\frac{\partial\psi}{\partial\Phi} = g_\Phi = \text{azimuthal component of gravity} \tag{9}$$

$\bar{1}_r, \bar{1}_\Phi$ = unit vectors of the radial and azimuthal gradients expressed in an earth centered, inertially fixed coordinate system.

Taking gradients of equation (5) yields

$$g_r = -\frac{K}{a^2}\left[\frac{1}{r^2} - \sum_{n=1}^{\infty}(n+1)\,J_n\left(\frac{a}{r}\right)^{n+2} P_n\,(\sin\phi)\right] \tag{10}$$

$$g_\phi = \frac{K}{a^2}\left[\sum_{n=1}^{\infty} J_n\left(\frac{a}{r}\right)^{n+2} P_n{}'\,(\sin\phi)\right] \tag{11}$$

where

$$P_n{}'\,(\sin\phi) = \frac{\partial}{\partial\phi}\left[P_n\,(\sin\phi)\right]$$

and the components of gravity will therefore be

$$\bar{g} = g_r\,\bar{1}_r + g_\phi\,\bar{1}_\phi \tag{12}$$

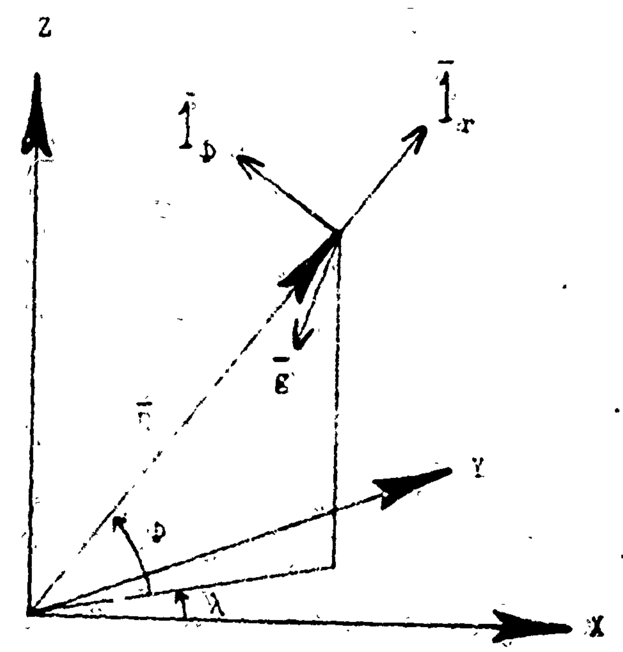


Figure Q-1

Components of Gravity

Legendre polynomials $P_n\ (\sin \phi)$ and $P_n{}'\ (\sin \phi)$ can be computed from the recurrsion relationship in the following manner:

$$P_o\ (\sin \phi) = 1$$

$$P_1\ (\sin \phi) = \sin \phi$$

$$P_2\ (\sin \phi) = (3 \sin^2 \phi - 1)/2$$

$$P_{n+1}(\sin \phi) = \left[\left(\frac{2n+1}{n+1}\right) \sin \phi\ P_n\ (\sin \phi) - \left(\frac{n}{n+1}\right) P_{n-1}\ (\sin \phi)\right] \tag{13}$$

$$P_o{}'\ (\sin \phi) = 0$$

$$P_1{}'\ (\sin \phi) = \cos \phi$$

$$P_2{}'\ (\sin \phi) = 3 \sin \phi \cos \phi$$

$$P'_{n+1}(\sin \phi) = \left[P'_{n-1}\ (\sin \phi) + (2n + 1)\ P_n\ (\sin \phi) \cos \phi\right]$$

If the total gravity is to be expressed in an orthogonal cartesian coordinate system such as the earth centered inertial system, then the unit vectors of equation (7) become

$$\bar{1}_r = \begin{bmatrix} \frac{X}{R} \\ \frac{Y}{R} \\ \frac{Z}{R} \end{bmatrix} = \begin{bmatrix} \cos \lambda \cos \phi \\ \sin \lambda \cos \phi \\ \sin \phi \end{bmatrix} \tag{14}$$

$$\bar{1}_\phi = \begin{bmatrix} -\cos \lambda \sin \phi \\ -\sin \lambda \sin \phi \\ \cos \phi \end{bmatrix} \tag{15}$$

CUBIC CORPORATION

The first nine zonal harmonics have been determined by Y. Kozai[1] from satellite orbital data. Kozai's values for the zonal harmonics are accepted as the most representative now available and are listed here for reference.

$$
\begin{aligned}
J_1 &= 0 \\
J_2 &= +\ 1082.48 \pm 0.04 \times 10^{-6} \\
J_3 &= -\ 2.562 \pm 0.007 \times 10^{-6} \\
J_4 &= -\ 1.84 \pm 0.09 \times 10^{-6} \\
J_5 &= -\ 0.064 \pm 0.007 \times 10^{-6} \\
J_6 &= +\ 0.39 \pm 0.009 \times 10^{-6} \\
J_7 &= -\ 0.470 \pm 0.010 \times 10^{-6} \\
J_8 &= -\ 0.02 \pm 0.07 \times 10^{-6} \\
J_9 &= +\ 0.117 \pm 0.011 \times 10^{-6}
\end{aligned}
\tag{16}
$$

Also given by Kozai are the constants K and a, which are:

$$
\begin{aligned}
K &= \gamma M = 3.986032 \times 10^{20}\ \text{cm}^3/\text{sec}^2 \\
a &= 6378165\ \text{meters} \\
a &= 20925696.335\ \text{feet} \\
\frac{K}{a^2} &= 32.14648177\ \text{ft/sec}^2
\end{aligned}
\tag{17}
$$

---

[1] Kozai, Yoshihide. "Numerical Results from Orbits." Smithsonian Institute Astrophysical Observatory Special Report No. 101.

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APPENDIX

GEODETIC SECOR LINE CROSSING

Introduction

The purpose of a line crossing operation is to estimate the distance between two points on the earth's surface. This distance is reduced to the shortest distance (i.e., the geodesic) along some reference spheroid (e.g., Clark 1866 or International).

The classical line crossing technique is illustrated by the SHIRAN and HIRAN systems. In these systems an aircraft flies across the baseline at a nearly constant height and nearly perpendicular to the baseline. During the flight ranges from the two base stations are simultaneously measured. (See figure R-1.) If the pairs of measured ranges are added to form a series of range sums (Rs), a curve similar to that shown in figure R-2 will result. The minimum of this curve corresponds to the time at which the aircraft was directly over the line. Using the two ranges and the altitude corresponding to the range sum minimum, an estimate of the geodesic may be found.

The use of the Geodetic SECOR system to perform line crossings is operationally similar but differs from the classical techniques in several ways. First, the height of the satellite will not be constant but will vary with time. Second, the satellite cannot be expected to cross the baseline near the midpoint nor cross perpendicular to the baseline. These two conditions, as will be shown later, cause the range sum minimum to occur at some time before or after the baseline is crossed. This prevents the use of the classical technique for the solution of the problem.

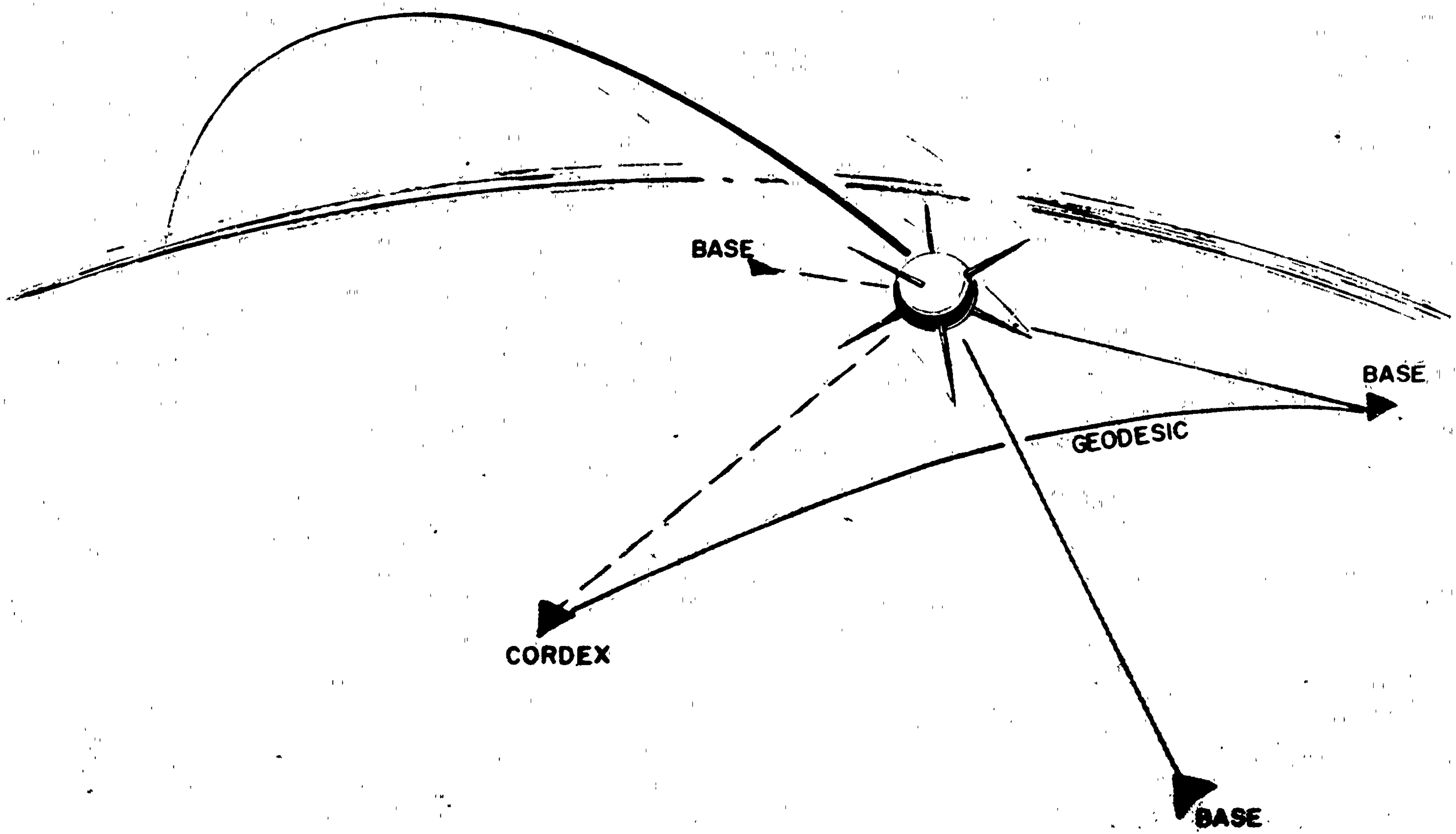
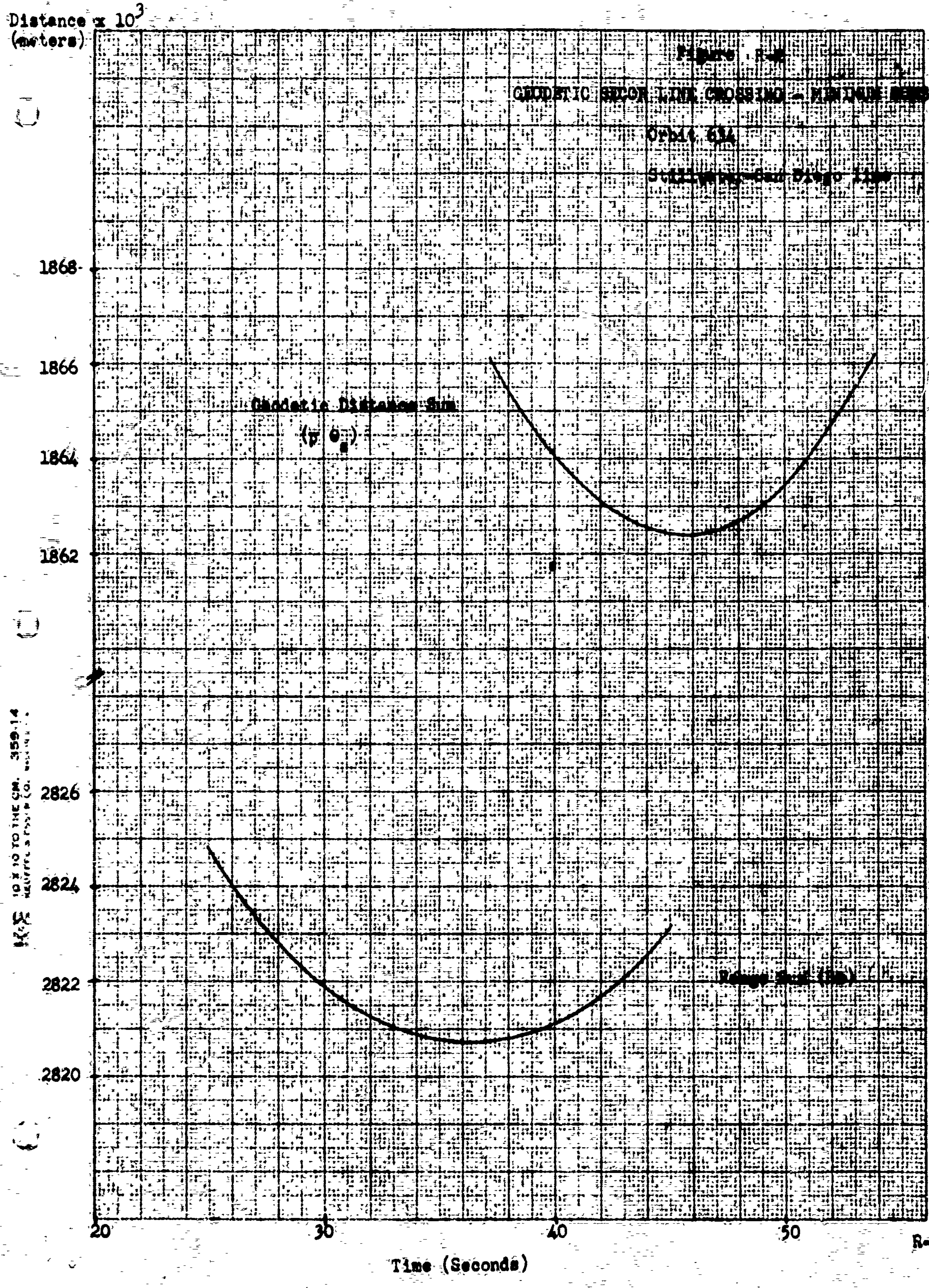


Figure R-1

LINE CROSSING MODE



Distance x 10^3 (meters)
1868
1866
1864
1862
2826
2824
2822
2820
Geodetic Distance Sum
20
30
40
50
Time (Seconds)

Method for Geodetic SECOR Line Crossing Computation

In the method for computing the line crossing from Geodetic SECOR satellite data the following are assumed to be known precisely:

R_E = distance from the center of the earth to the satellite.

$(\phi\lambda h)_1$ = latitude, longitude, height of base site number one.

h_2 = height of base site number two.

R_1, R_2 = simultaneous range observations from sites one and two to the satellite.

The latitude and longitude of the second site $(\phi\lambda)_2$ are assumed to be approximately known. A schematic representation of these quantities is shown in figure R-3.

The distance R_E is computed at each point from the satellite's equatorial coordinates $(X_E\ Y_E\ Z_E)$ by $R_E = [X_E^2 + Y_E^2 + Z_E^2]^{\frac{1}{2}}$. The satellite position in equatorial coordinates may be determined using Geodetic SECOR data from base site number one and two additional sites whose coordinates are known or from some other tracking or ephemeris data.

Since the range sum minimum cannot be used to determine the time of the line crossing, some other parameter must be found which will be minimum as the baseline is crossed. The parameter used is the central angle sum $\theta_s = \theta_1 + \theta_2$ (figure R-3) which takes on a minimum value when the vector $\vec{R}_E$ is coplanar with the vectors to the two base stations (i.e., $\vec{R}_{s1}$, $\vec{R}_{s2}$). The value of θ_s may be found at each point by applying the law of cosines to each triangle, shown in figure R-3. That is;

$$\cos\theta_1 = \frac{R_E^2 + R_{s1}^2 - R_1^2}{2R_E R_{s1}} \tag{1}$$

$$\cos\theta_2 = \frac{R_E^2 + R_{s2}^2 - R_2^2}{2R_E R_{s2}} \tag{2}$$

$$\theta_s = \theta_1 + \theta_2 \tag{3}$$

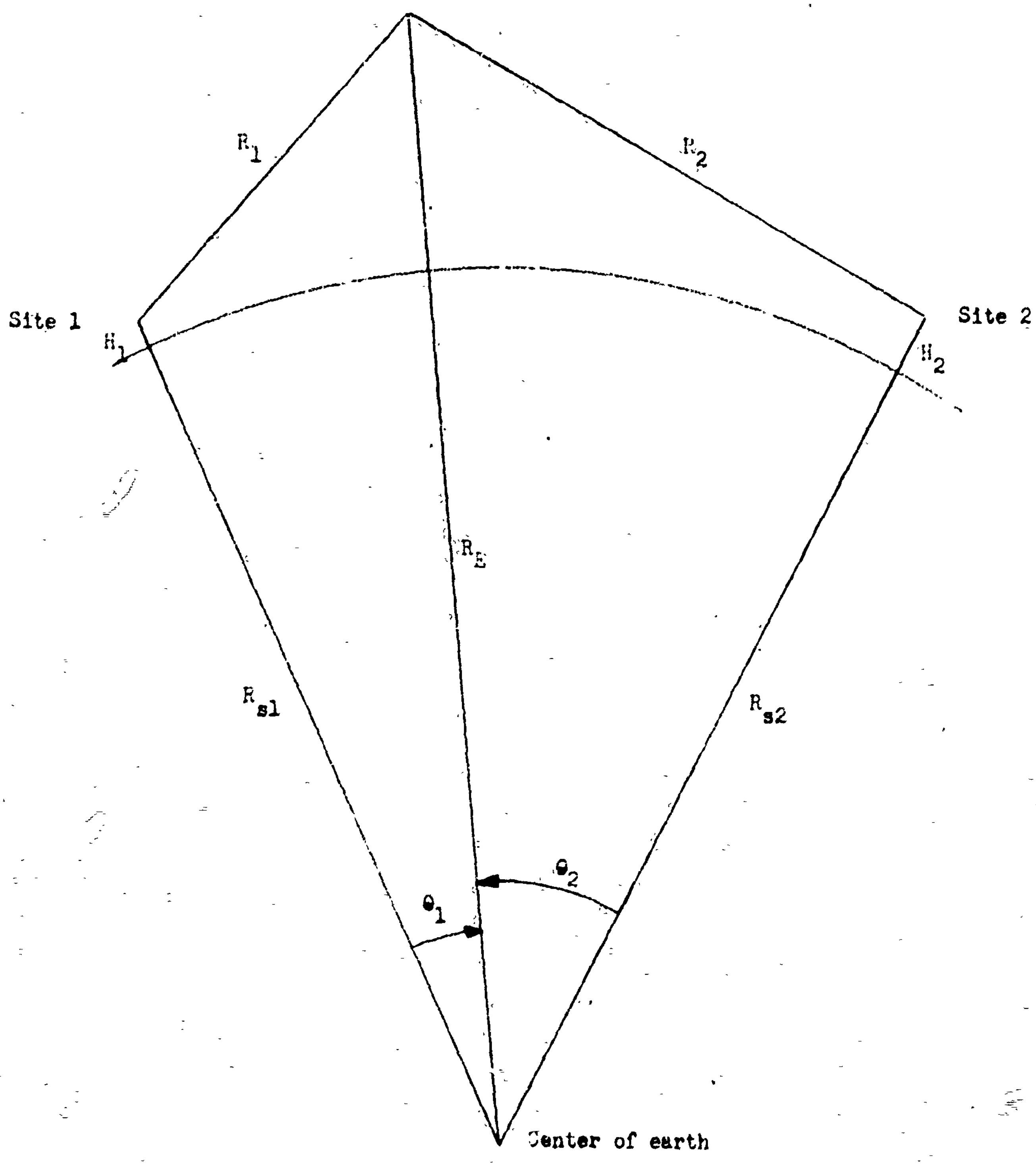


Figure R-3. Line Crossing Geometry

Minimum Sum Determination

The determination of a minimum central angle sum assumes that in a limited region about the minimum, the sum, $S(t)$, may be approximated by a second degree polynomial: $P(t) = a_0 + a_1 t + a_2 t^2$ if the dynamics of the vehicle are not too extreme. Having determined $P(t)$, the minimum may be obtained as follows:

$$\frac{d}{dt} P(t_M) = 0, \quad P_M = P(t_M) \tag{4}$$

where t_M is the time at which $P(t)$ is an extremum.

$$\frac{d}{dt} P(t_M) = a_1 + 2a_2 t_M \tag{5}$$

So,

$$t_M = -\frac{a_1}{2a_2} \tag{6}$$

Then:

$$P_M = a_0 - \frac{a_1^2}{2a_2} + \frac{a_1^2}{4a_2} = a_0 - \frac{a_1^2}{4a_2} \tag{7}$$

For a minimum:

$$\frac{d^2}{dt^2} P(t_M) > 0 \tag{8}$$

$$\frac{d^2}{dt^2} P(t) = 2a_2 > 0 \tag{9}$$

So $a_2 > 0$ for a minimum.

The polynomial $P(t)$ may be determined from the measured data (i.e., central angle sums) $S_1, S_2, \ldots, S_N$ by use of a least squares criterion. That is, $S = \sum_{i=1}^{N} [S_i - P(t_i)]^2 = \sum_{i=1}^{N} [S_i - a_0 - a_1 t_i - a_2 t_i^2]^2$ is a minimum.

CUBIC CORRELATION

For a minimum:

$$\frac{\partial s}{\partial a_o} = 0,\ \frac{\partial s}{\partial a_1} = 0,\ \frac{\partial s}{\partial a_2} = 0$$

$$\frac{\partial s}{\partial a_o} = -2 \sum_{i=1}^{N} [U_i - a_o - a_1 t_i - a_2 t_i^2] = 0$$

$$\frac{\partial s}{\partial a_1} = -2 \sum_{i=1}^{N} [U_i t_i - a_o t_i - a_1 t_i^2 - a_2 t_i^3] = 0$$

$$\frac{\partial s}{\partial a_2} = -2 \sum_{i=1}^{N} [U_i t_i^2 - a_o t_i^2 - a_1 t_i^3 - a_2 t_i^4] = 0$$

Rewriting in matrix form

$$\sum_{i=1}^{N} \begin{bmatrix} 1 & t_i & t_i^2 \\ t_i & t_i^2 & t_i^3 \\ t_i^2 & t_i^3 & t_i^4 \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} U_i \\ U_i t_i \\ U_i t_i^2 \end{bmatrix}$$

But,

$$\begin{bmatrix} 1 & t_i & t_i^2 \\ t_i & t_i^2 & t_i^3 \\ t_i^2 & t_i^3 & t_i^4 \end{bmatrix} = \begin{bmatrix} 1 \\ t_i \\ t_i^2 \end{bmatrix} \begin{bmatrix} 1 & t_i & t_i^2 \end{bmatrix}$$

Let,

$$\begin{bmatrix} T_i \end{bmatrix} = \begin{bmatrix} 1 \\ t_i \\ t_i^2 \end{bmatrix},\quad \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix}$$

$$\sum_{i=1}^{N} \underset{(3\times1)}{[T_i]} \underset{(1\times3)}{[T_i]^T} \underset{(3\times1)}{[A]} = \sum_{i=1}^{N} \underset{(3\times1)}{[T_i]} \underset{(1\times1)}{U_i}$$

Solving for $[A]$:

$$[A] = \left\{ \sum_{i=1}^{N} [T_i][T_i]^T \right\}^{-1} \left\{ \sum_{i=1}^{N} [T_i] U_i \right\} \tag{18}$$

Figure shows a sample curve fit to a central angle sum scaled by a mean earth's radius for a long-line satellite crossing. The residuals plotted represent $[U_i - F(t_i)]$ versus time.

Estimation of the Geodesic

The minimum distance between the two base stations (geodesic) is not a plane curve and thus may not be determined in closed form from the central angle minimum. An estimate may be made by assuming that the geodesic may be expressed in the form:

$$S_{GD} = \bar{\rho}\, \theta_{12} \tag{19}$$

$S_{GD} \longrightarrow$ geodesic

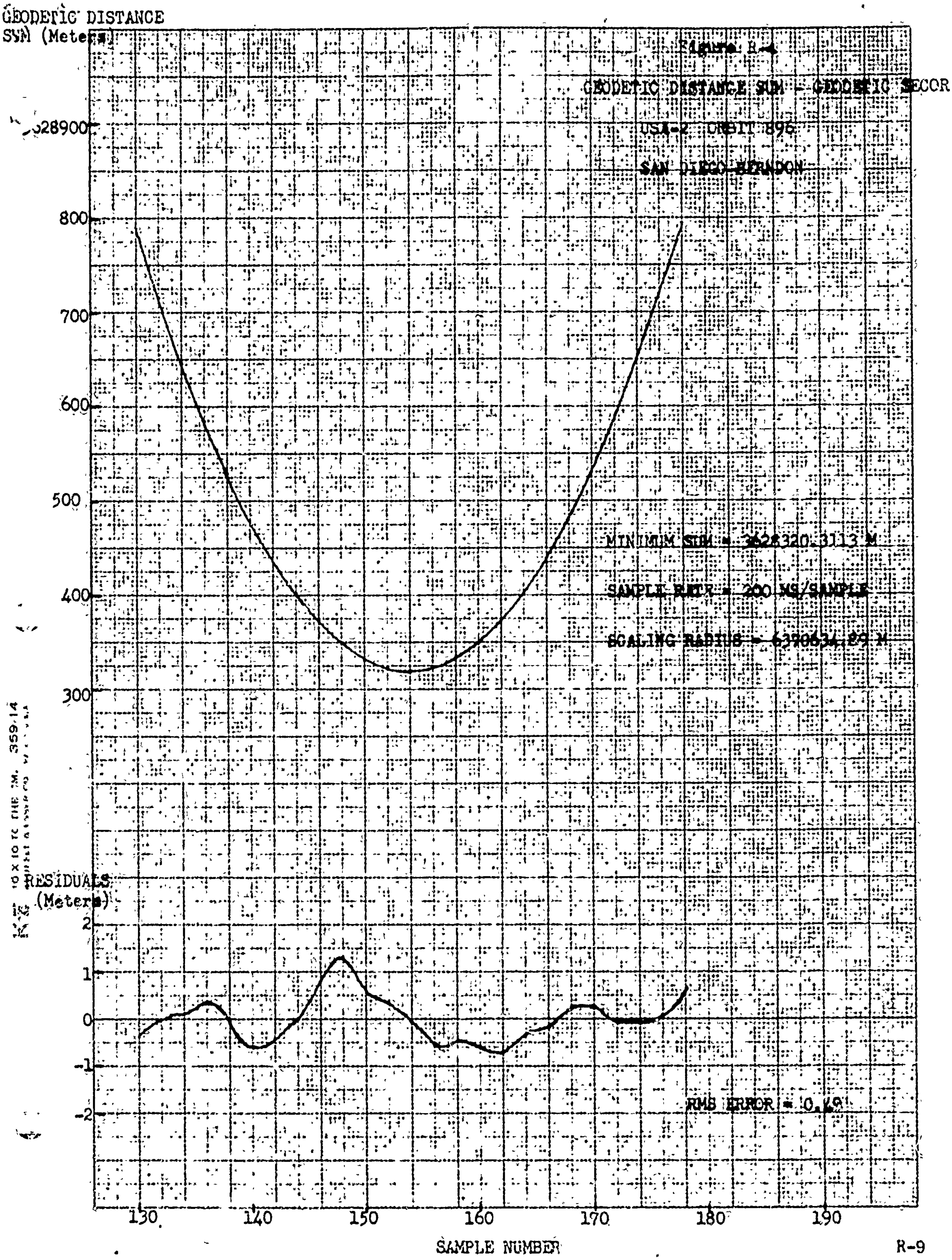
$\bar{\rho} \longrightarrow$ exact scaling radius

$\theta_{12} \longrightarrow$ central angle.

Now the estimate of θ_{12} is found directly from the minimum central angle sum (i.e., $\tilde{\theta}_{12} = (\theta_s)$ minimum). An estimate of the scaling radius $(\hat{\rho})$ is made using the survey data for base station 1 and the approximate survey data for base station 2 to compute a geodesic $(\hat{S}_{GD})$ and central angle $(\hat{\theta}_{12})$. The central angle is found from:

$$\cos \hat{\theta}_{12} = \frac{\vec{r}_{s1} \cdot \vec{r}_{s2}}{r_{s1}\, r_{s2}} \tag{20}$$

and $\hat{S}_{GD}$ is found using Sodano's method for the inverse computation.


GEODETIC DISTANCE
SUM (Meters)
GEODETIC DISTANCE SUM - GEODETIC SECOR
USA-2 ORBIT 896
SAN DIEGO-HERNDON
800
700
600
500
400
300
SAMPLE RATE = 200 MS/SAMPLE
RESIDUALS
(Meters)
2
1
0
-1
-2
RMS ERROR
130
140
150
160
170
180
190
SAMPLE NUMBER

The estimate for the scaling radius, ($\hat{\rho}$), is found from:

$$\hat{\rho} = \hat{S}_{GD}/\hat{\theta}_{12} \tag{21}$$

The final estimate for the geodesic is given by:

$$\tilde{S}_{GD} = \hat{\rho}\,\hat{\theta}_{12} \tag{22}$$

The relative error is approximated by:

$$\frac{\Delta \tilde{S}_{GD}}{S_{GD}} \doteq \frac{\Delta \hat{\rho}}{\hat{\rho}} + \frac{\Delta \tilde{\theta}_{12}}{\tilde{\theta}_{12}} \tag{23}$$

The first term of this expression reflects the uncertainty in the survey position of the second base site while the second term reflects **primarily** the errors arising from the determination of R_E and the ranging errors.

In order to determine more accurately the line length, a network of lines must be measured and a network adjustment procedure applied to improve the first estimates.

Effect of Vehicle Dynamics on the Range Sum and Central Angle Sums

As indicated above, the minimum range sum does not necessarily occur as the vehicle crosses the baseline. The relations derived below show, for the case of a circular orbit, the magnitude of the offset.

Figure F-5 shows the geometry of a line crossing configuration for a circular orbit. The coordinate system is chosen so that the orbit is centered at the origin of the coordinate system and lies in the y-z plane. The radius (ρ) is constant and the angular velocity ($\dot{\beta}$) is constant.

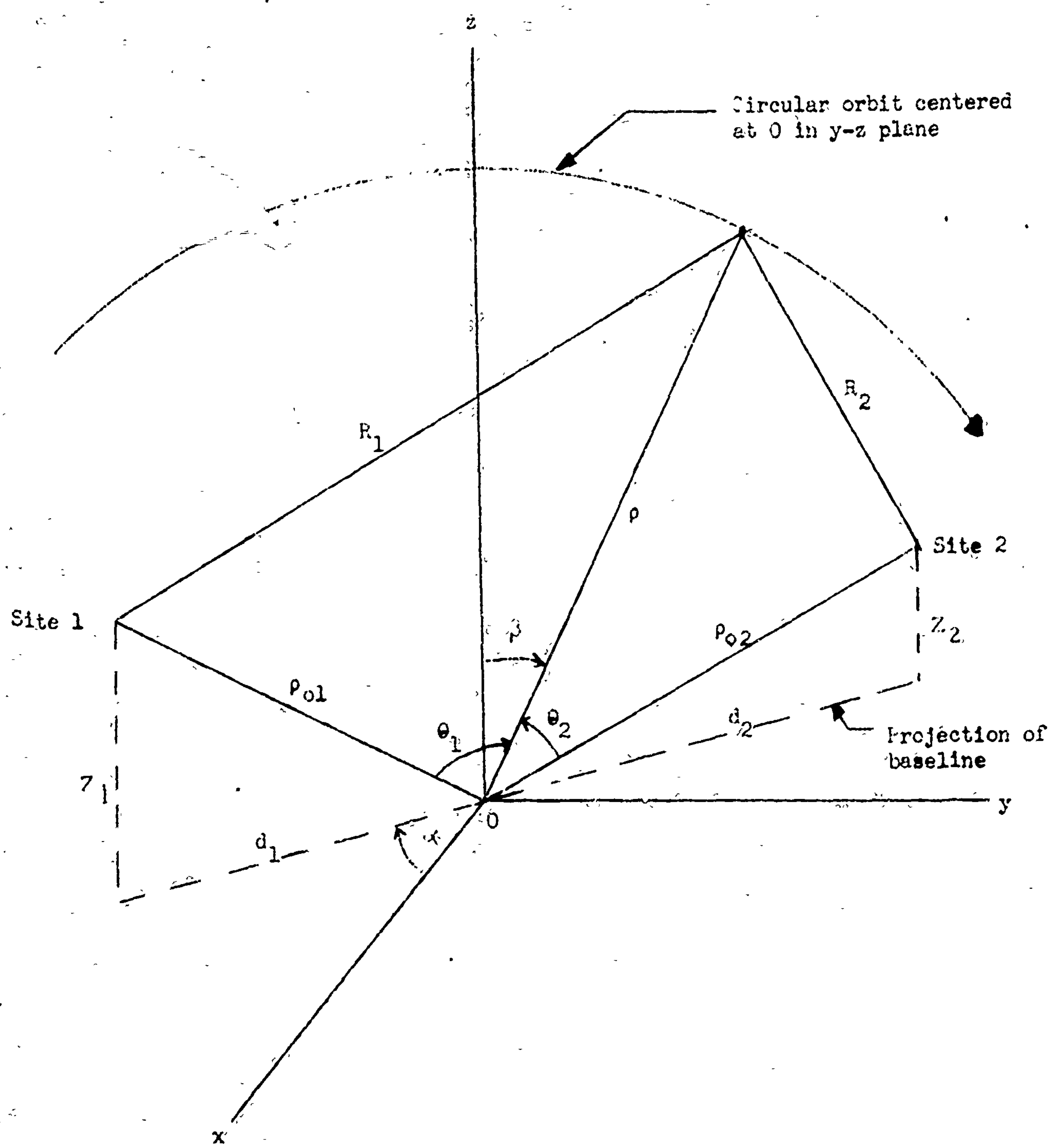


Figure R-5. Geometry for Circular Orbit

Using the geometry of figure R-6 the coordinates of the satellite (xyz) and of the two tracking sites $(XYZ)_1$ and $(XYZ)_2$ may be written:

$$\begin{aligned} x &= 0 & X_1 &= d_1 \cos\alpha & X_2 &= -d_2 \cos\alpha \\ y &= \rho \sin\beta & Y_1 &= -d_1 \sin\alpha & Y_2 &= d_2 \sin\alpha \\ z &= \rho \cos\beta & Z_1 &= Z_1 & Z_2 &= Z_2 \end{aligned} \tag{24}$$

The ranges and range rates from each tracking site are:

$$R_1 = \left[\rho^2 + d_1^2 + Z_1^2 + 2\rho(d_1 \sin\alpha \sin\beta - Z_1 \cos\beta)\right]^{1/2} \tag{25}$$

$$R_2 = \left[\rho^2 + d_2^2 + Z_2^2 - 2\rho(d_2 \sin\alpha \sin\beta + Z_2 \cos\beta)\right]^{1/2} \tag{26}$$

$$\dot{R}_1 = \frac{\rho\dot{\beta}}{R_1}(d_1 \sin\alpha \cos\beta + Z_1 \sin\beta) \tag{27}$$

$$\dot{R}_2 = \frac{\rho\dot{\beta}}{R_2}(-d_2 \sin\alpha \cos\beta + Z_2 \sin\beta) \tag{28}$$

Now the range sum (R_s) is given by $R_s = R_1 + R_2$, and the range sum rate $\dot{R}_s$ by:

$$\dot{R}_s = \dot{R}_1 + \dot{R}_2 = \rho\dot{\beta}\left\{\left[\frac{d_1}{R_1} - \frac{d_2}{R_2}\right] \sin\alpha \cos\beta + \left[\frac{Z_1}{R_1} + \frac{Z_2}{R_2}\right] \sin\beta\right\} \tag{29}$$

In order that $\dot{R}_s$ may be used, the range sum minimum must occur as the vehicle crosses the baseline. In the above situation this corresponds to $\beta = 0$ or

$$(\dot{R}_s)_{\beta=0} = \rho\dot{\beta}\left[\frac{d_1}{R_1} - \frac{d_2}{R_2}\right] \sin\alpha = 0 \tag{30}$$

The conditions under which $\dot{R}_s = 0$ when $\beta = 0$ are:

(1) $\rho\dot{\beta} = 0$ which is not of practical interest.

(2) $\alpha = 0$ which corresponds to perpendicular crossing of the baseline.

(3) $\frac{d_1}{R_1} = \frac{d_2}{R_2}$ which corresponds to bisecting the baseline.

In general, then, use of the range sum will introduce an error in the baseline length determination.

The central angle sum rate ($\dot{\theta}_s = \dot{\theta}_1 + \dot{\theta}_2$) may be obtained using the law of cosines to determine θ_1 and θ_2 as follows:

$$\theta_s = \left[\cos^{-1}\left(\frac{\rho^2 + \rho_{o1}^2 - R_1^2}{2\rho\rho_{o1}}\right) + \cos^{-1}\left(\frac{\rho^2 + \rho_{o2}^2 - R_2^2}{2\rho\rho_{o2}}\right)\right] \tag{31}$$

$$\dot{\theta}_s = \dot{\beta}\left[\frac{1}{\rho_{o1}\sin\theta_1}\left(d_1 \sin\alpha\cos\beta + Z_1\sin\beta\right) + \frac{1}{\rho_{o2}\sin\theta_2}\left(-d_2\sin\alpha\cos\beta + Z_2\sin\beta\right)\right] \tag{32}$$

In order that θ_s be a minimum as the satellite crosses the baseline, then $\dot{\theta}_s = 0$ at $\beta = 0$.

$$(\dot{\theta}_s)_{\beta=0} = \dot{\beta}\sin\alpha\left(\frac{d_1}{\rho_{o1}\sin\theta_1} - \frac{d_2}{\rho_{o2}\sin\theta_2}\right) \tag{33}$$

but at $\beta = 0$,

$$\frac{d_1}{\rho_{o1}} = \sin\theta_1 \text{ and } \frac{d_2}{\rho_{o2}} = \sin\theta_2 \tag{34}$$

so that $\dot{\theta}_s = 0$ at $\beta = 0$ and θ_s is a minimum. Thus for a circular orbit the central angle sum (θ_s) may be used in the line crossing computation while the range sum (R_s) may not.

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APPENDIX S

SAMPLE LISTINGS

This appendix provides a sample of each of the types of listing (printout) obtained during the data processing. A brief description of each heading is included unless the heading is self-explanatory.

S.1 Raw Listing. The raw listing was made directly from the copied raw tape (program EXAM1) after unpacking the tape format (subroutine FORMAT) and resolving the range (subroutine RESOLVE). In general, every fourth or fifth sample was listed.

The following quantities were output (sample listing 1):

T	Time mark which was recorded (indicated by a 1) every second. In the sample listing, these particular samples were missed
Q	Quality mark which was recorded as a 1 if one or more of the tracking servos were not locked
S	Station number
R	Run number
MO, DA	Month and day of track
HR, M, S, MS	Time (GMT) recorded at the tracking site in hours, minutes, seconds, milliseconds
RANGE	Resolved range in meters
DIF	First differences of the ranges
VF	Very fine channel in meters (1/2 and 1/4 meter bits not indicated)
FN	Fine channel in meters
CS	Coarse channel in meters
VC	Very coarse channel in meters
ER	Extended range in meters
DI - IC	Difference between the VF and VFIC channels (used to compute the ionospheric correction)

VF - F	Difference between the overlap bits of the very fine and and fine channels
F - C	Difference between the overlap bits of the fine and coarse channels
C - VC	Difference between the overlap bits of the coarse and very coarse channels

In earlier listings, the last three columns included the following quantities:

R - D2, R - D3, R - D4	Reference phase minus the phase three of the D channels. The numbers were scaled so that the least significant bit indicated one unit. The VF column is actually R - D1.

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GEODETIC SECOR USA-2 LARSON ORBIT 1

T	Q	S	R	MO	DA	HR	M	S	MS	RANGE	DIF	VF	FN	CS	VC
0	0	3	1	4	21	22	19	12	12	951267.00	-25.75	227	976	34560	421
0	0	3	1	4	21	22	19	12	262	951244.00	-23.00	204	944	34816	421
0	0	3	1	4	21	22	19	12	512	951223.25	-20.75	183	912	34048	421
0	0	3	1	4	21	22	19	12	762	951206.50	-16.75	166	912	34560	421
0	0	3	1	4	21	22	19	13	12	951192.25	-14.25	152	880	34048	421
0	0	3	1	4	21	22	19	13	262	951181.50	-10.75	141	896	34560	421
0	0	3	1	4	21	22	19	13	512	951173.75	-7.75	133	880	34304	421
0	0	3	1	4	21	22	19	13	762	951169.25	-4.50	129	864	34304	419
0	0	3	1	4	21	22	19	14	12	951167.25	-2.00	127	864	34304	419
0	0	3	1	4	21	22	19	14	262	951167.75	.50	127	880	34560	421
0	0	3	1	4	21	22	19	14	512	951171.75	4.00	131	880	34560	419
0	0	3	1	4	21	22	19	14	762	951178.75	7.00	138	896	34560	421
0	0	3	1	4	21	22	19	15	12	951188.50	9.75	148	896	34816	4198
0	0	3	1	4	21	22	19	15	262	951200.50	12.00	160	912	34304	4218
0	0	3	1	4	21	22	19	15	512	951215.75	15.25	175	912	34304	4218
0	0	3	1	4	21	22	19	15	762	951234.75	19.00	194	928	34304	4198
0	0	3	1	4	21	22	19	16	12	951256.25	21.50	216	944	34304	4198
0	0	3	1	4	21	22	19	16	262	951281.25	25.00	241	1008	34560	4259
0	0	3	1	4	21	22	19	16	512	951309.25	28.00	13	1008	34304	4198
0	0	3	1	4	21	22	19	16	762	951339.00	29.75	43	1024	34304	4198
0	0	3	1	4	21	22	19	17	12	951372.75	33.75	76	1088	34560	4239
0	0	3	1	4	21	22	19	17	262	951409.75	37.00	113	1136	34816	4218
0	0	3	1	4	21	22	19	17	512	951449.25	39.50	153	1152	34816	4198
0	0	3	1	4	21	22	19	17	762	951490.25	41.00	194	1168	34304	419
0	0	3	1	4	21	22	19	18	12	951536.50	46.25	240	1248	35072	4218
0	0	3	1	4	21	22	19	18	262	951584.75	48.25	32	1296	34816	4218
0	0	3	1	4	21	22	19	18	512	951635.50	50.75	83	1344	34816	4239
0	0	3	1	4	21	22	19	18	762	951690.75	55.25	138	1424	35328	4259
0	0	3	1	4	21	22	19	19	12	951747.25	56.50	195	1440	34816	4198
0	0	3	1	4	21	22	19	19	262	951807.50	60.25	255	1520	35072	4218
0	0	3	1	4	21	22	19	19	512	951870.25	62.75	62	1568	34816	4218
0	0	3	1	4	21	22	19	19	762	951936.75	66.50	128	1648	35328	4239
0	0	3	1	4	21	22	19	20	12	952005.50	68.75	197	1696	35328	4198
0	0	3	1	4	21	22	19	20	262	952077.50	72.00	13	1792	35328	4239
0	0	3	1	4	21	22	19	20	512	952152.25	74.75	88	1872	35072	4259
0	0	3	1	4	21	22	19	20	762	952230.75	78.50	166	1952	35840	4239
0	0	3	1	4	21	22	19	21	12	952311.00	80.25	247	2032	35584	4259
0	0	3	1	4	21	22	19	21	262	952394.75	83.75	74	2080	35584	4218
0	0	3	1	4	21	22	19	21	512	952481.75	87.00	161	2192	35840	4218
0	0	3	1	4	21	22	19	21	762	952571.25	89.50	251	2256	35584	4218
0	0	3	1	4	21	22	19	22	12	952663.50	92.25	87	2368	35840	4239
0	0	3	1	4	21	22	19	22	262	952759.50	96.00	183	2480	36096	4239
0	0	3	1	4	21	22	19	22	512	952857.75	98.25	25	2560	36096	4218
0	0	3	1	4	21	22	19	22	762	952959.50	101.75	127	2656	36352	4218
0	0	3	1	4	21	22	19	23	12	953063.50	104.00	231	2784	36352	4259
0	0	3	1	4	21	22	19	23	262	953171.25	107.75	83	2864	36352	4218
0	0	3	1	4	21	22	19	23	512	953281.25	110.00	193	2992	36608	4259
0	0	3	1	4	21	22	19	23	762	953394.75	113.50	50	3120	36608	4239
0	0	3	1	4	21	22	19	24	12	953511.25	116.50	167	3232	37120	4239
0	0	3	1	4	21	22	19	24	262	953630.00	118.75	30	3344	36864	4239
0	0	3	1	4	21	22	19	24	512	953752.50	122.50	152	3456	37376	4239
0	0	3	1	4	21	22	19	24	762	953877.50	125.00	21	3616	37376	4280
0	0	3	1	4	21	22	19	25	12	954005.00	127.50	149	3696	37376	4239
0	0	3	1	4	21	22	19	25	262	954135.75	130.75	23	3856	37376	4259
0	0	3	1	4	21	22	19	25	512	954270.25	134.50	158	3984	37888	4239
0	0	3	1	4	21	22	19	25	762	954406.75	136.50	38	48	37632	4280

B

LARSON ORBIT 1407

DIF	VF	FN	CS	VC	ER	DI-IC	VF-F	F-C,	C-VC
-25.75	227	976	34560	421888	1015808	16	1	-3	2
-23.00	204	944	34816	421888	1015808	17	1	-4	2
-20.75	183	912	34048	421888	1015808	18	2	-1	2
-16.75	166	912	34560	421888	1015808	18	1	-3	2
-14.25	152	880	34048	421888	1015808	18	2	-1	2
-10.75	141	896	34560	421888	1015808	18	0	-3	2
-7.75	133	880	34304	421888	1015808	17	1	-2	2
-4.50	129	864	34304	419840	1015808	17	2	-2	3
-2.00	127	864	34304	419840	1015808	17	1	-2	3
.50	127	880	34560	421888	1015808	17	0	-3	2
4.00	131	880	34560	419840	1015808	17	1	-3	3
7.00	138	896	34560	421888	1015808	17	0	-3	2
9.75	148	896	34816	419840	1048576	17	1	-4	3
12.00	160	912	34304	421888	1048576	18	1	-2	2
15.25	175	912	34304	421888	1015808	18	1	-2	2
19.00	194	928	34304	419840	1048576	17	2	-2	3
21.50	216	944	34304	419840	1048576	17	2	-2	3
25.00	241	1008	34560	425984	1048576	17	0	-3	0
28.00	13	1008	34304	419840	1048576	17	-14	-1	3
29.75	43	1024	34304	419840	1048576	17	2	-1	3
33.75	76	1088	34560	423936	1048576	17	0	-2	1
37.00	113	1136	34816	421888	1048576	16	0	-3	2
39.50	153	1152	34816	419840	1048576	17	1	-3	3
41.00	194	1168	34304	419840	1048576	18	3	-1	3
46.25	240	1248	35072	421888	1048576	16	1	-4	2
48.25	32	1296	34816	421888	1048576	17	1	-2	2
50.75	83	1344	34816	423936	1048576	17	1	-2	1
55.25	138	1424	35328	425984	1048576	17	-0	-4	0
56.50	195	1440	34816	419840	1048576	17	2	-2	3
60.25	255	1520	35072	421888	1048576	17	0	-3	2
62.75	62	1568	34816	421888	1048576	17	1	-1	2
66.50	128	1648	35328	423936	1048576	17	1	-3	1
68.75	197	1696	35328	419840	1081344	17	2	-3	3
72.00	13	1792	35328	423936	1048576	16	0	-2	1
74.75	88	1872	35072	425984	1048576	16	0	-1	0
78.50	166	1952	35840	423936	1081344	17	0	-4	1
80.25	247	2032	35584	425984	1048576	16	0	-3	0
83.75	74	2080	35584	421888	1048576	17	2	-2	3
87.00	161	2192	35840	421888	1081344	18	1	-3	3
89.50	251	2256	35584	421888	1048576	17	2	-2	3
92.25	87	2368	35840	423936	1048576	17	1	-2	2
96.00	183	2480	36096	423936	1048576	17	0	-3	2
98.25	25	2560	36096	421888	1048576	17	1	-2	3
101.75	127	2656	36352	421888	1048576	16	1	-3	3
104.00	231	2784	36352	425984	1081344	17	0	-3	1
107.75	83	2864	36352	421888	1048576	16	2	-2	3
110.00	193	2992	36608	425984	1048576	17	1	-3	1
113.50	50	3120	36608	423936	1081344	16	0	-2	2
116.50	167	3232	37120	423936	1048576	17	0	11	2
118.75	30	3344	36864	423936	1048576	17	0	13	2
122.50	152	3456	37376	423936	1048576	17	1	11	2
125.00	21	3616	37376	428032	1048576	16	-0	12	0
127.50	149	3696	37376	423936	1048576	17	2	12	2
130.75	23	3856	37376	425984	1048576	17	0	13	1
134.50	158	3984	37888	423936	1048576	17	0	11	2
136.50	38	48	37632	428032	1048576	17	-0	-2	1

S.2 Edited and Smoothed Data Listing. The edited and smoothed data listing was obtained during the editing and smoothing pass (program PASS2) on each raw tape. The following quantities were listed:

HR, M, S, MS	Time recorded on the raw tape in hours, minutes, seconds, and milliseconds
RAW RANGE	Raw range obtained from the range resolution (subroutine RESOLVE) prior to application of any calibration
ED. RANGE	Edited range obtained from the data editing portion of PASS2 (subroutine EDITSR) with calibration constants applied
SM. RANGE	Smoothed range obtained from the edited ranges by using least squares smoothing coefficients (subroutine SCR)
RESIDUAL	Difference between the edited and smoothed ranges
EDIT CORR	Edit correction applied to the raw range to obtain the edited range (minus calibration). With one exception, the edit correction is an integral multiple of the least significant ambiguity (256 meters). If a data sample is bad (cannot be reduced within the noise tolerance by using an integral number of ambiguities), the edit correction is indicated by a 9.0. In this case, the edited range is either an extrapolated range or the raw range, depending on the number of successive bad samples which have occurred.

ED. DIFF.	First differences of the edited ranges
SM. DIFF.	First differences of the smoothed ranges
RD	Range rate derived from the least squares smoothing coefficient (subroutine SCR) in units of meters per second
RDD	Range acceleration derived from the least squares smoothing coefficients (subroutine SCR) in units of meters per second per second
IC	Measured ionospheric correction derived from DI - IC and including the calibration constants
C	Program data quality indicator - = no correction necessary A = ambiguity correction B = bad sample

At the end of each data block, the number of bad samples (NUM BAD), the number of least significant ambiguities applied (NUM AMB), and the RMS of the smoothing residuals (RMS ERROR) are indicated.

A

GEODETIC SECOR USA 2 SAN DIEGO ORBIT

HR	M	S	MS	RAW RANGE	ED. RANGE	SM. RANGE	RESIDUAL	EDIT CORR.	ED.
0	25	19	628	1777600.00	1777577.00	1777577.13	-.13	0	
0	25	19	728	1777699.25	1777676.25	1777676.37	-.12	0	1
0	25	19	828	1777799.25	1777776.25	1777776.03	.22	0	1
0	25	19	928	1777899.50	1777876.50	1777876.06	.44	0	1
0	25	20	28	1777999.50	1777976.50	1777976.43	.07	0	1
0	25	20	128	1778100.25	1778077.25	1778077.15	.10	0	1
0	25	20	228	1778201.00	1778178.00	1778178.24	-.24	0	1
0	25	20	328	1778558.50	1778279.50	1778279.65	-.15	-256.00	1
0	25	20	428	1778405.00	1778382.00	1778381.42	.58	0	1
0	25	20	528	1778762.50	1778483.50	1778483.50	-.00	-256.00	1
0	25	20	628	1794993.00	1778586.00	1778585.97	.03	-16384.00	1
0	25	20	728	1778711.75	1778688.75	1778688.80	-.05	0	1
0	25	20	828	1778815.00	1778792.00	1778791.95	.05	0	1
0	25	20	928	1778918.25	1778895.25	1778895.50	-.25	0	1
0	25	21	28	1779022.00	1778999.00	1778999.36	-.36	0	1
0	25	21	128	1779126.50	1779103.50	1779103.57	-.07	0	1
0	25	21	228	1779231.25	1779208.25	1779208.11	.14	0	1
0	25	21	328	1779335.75	1779312.75	1779313.00	-.25	0	1
0	25	21	428	1779441.00	1779418.00	1779418.24	-.24	0	1
0	25	21	528	1779547.25	1779524.25	1779523.83	.42	0	1
0	25	21	628	1779652.75	1779629.75	1779629.77	-.02	0	1
0	25	21	728	1780015.75	1779736.75	1779736.13	.62	-256.00	1
0	25	21	828	1780122.00	1779843.00	1779842.75	.25	-256.00	1
0	25	21	928	1779972.75	1779949.75	1779949.71	.04	0	1
0	25	22	28	1780080.00	1780057.00	1780057.01	-.01	0	1
0	25	22	128	1780187.00	1780164.00	1780164.65	-.65	0	1
0	25	22	228	1780295.25	1780272.25	1780272.63	-.38	0	1
0	25	22	328	1780403.75	1780380.75	1780380.94	-.19	0	1
0	25	22	428	1780513.00	1780490.00	1780489.61	.39	0	1
0	25	22	528	1780622.00	1780599.00	1780598.61	.39	0	1
0	25	22	628	1780731.25	1780708.25	1780707.95	.30	0	1
0	25	22	728	1780840.75	1780817.75	1780817.60	.15	0	1
0	25	22	828	1780950.50	1780927.50	1780927.63	-.13	0	1
0	25	22	928	1781060.50	1781037.50	1781037.99	-.49	0	1
0	25	23	28	1781171.75	1781148.75	1781148.75	-.00	0	1
0	25	23	128	1781283.00	1781260.00	1781259.91	.09	0	1
0	25	23	228	1781394.50	1781371.50	1781371.40	.10	0	1
0	25	23	328	1781506.25	1781483.25	1781483.22	.03	0	1
0	25	23	428	1781618.25	1781595.25	1781595.38	-.13	0	1
0	25	23	528	1781730.75	1781707.75	1781707.90	-.15	0	1
0	25	23	628	1781843.50	1781820.50	1781820.79	-.29	0	1
0	25	23	728	1781957.00	1781934.00	1781934.05	-.05	0	1
0	25	23	828	1782070.25	1782047.25	1782047.63	-.38	0	1
0	25	23	928	1782184.50	1782161.50	1782161.54	-.04	0	1
0	25	24	28	1782299.25	1782276.25	1782275.81	.44	0	1
0	25	24	128	1782413.75	1782390.75	1782390.39	.36	0	1
0	25	24	228	1782529.00	1782506.00	1782505.30	.70	0	1
0	25	24	328	1782643.50	1782620.50	1782620.61	-.11	0	1
0	25	24	428	1782759.25	1782736.25	1782736.29	-.04	0	1
0	25	24	528	1782875.75	1782852.75	1782852.32	.43	0	1
0	25	24	628	1782991.50	1782968.50	1782968.68	-.18	0	1
0	25	24	728	1783108.00	1783085.00	1783085.37	-.37	0	1
0	25	24	828	1783224.75	1783201.75	1783202.41	-.66	0	1

NUM. BAD	NUM. AMB.	RMS ERROR
0	-50	.3232

SAN DIEGO ORBIT 1269

RESIDUAL	EDIT CORR.	ED. DIFF.	SM. DIFF.	RD	RDD	I.C.	C
-.13	0	99.25	99.25	991	36	22.00	-
-.12	0	100.00	99.66	994	36	21.33	-
.22	0	100.25	100.02	998	36	21.33	-
.44	0	100.00	100.37	1002	36	21.33	-
.07	0	100.75	100.72	1005	35	21.33	-
.10	0	100.75	101.09	1009	35	21.33	-
-.24	0	101.50	101.42	1012	35	21.33	-
-.15	-256.00	102.50	101.76	1016	35	22.00	A
.58	0	101.50	102.09	1019	35	21.33	-
-.00	-256.00	102.50	102.46	1023	35	22.00	A
.03	-16384.00	102.75	102.83	1026	35	22.00	A
-.05	0	103.25	103.16	1030	35	21.33	-
.05	0	103.25	103.55	1033	35	22.00	-
-.25	0	103.75	103.86	1037	35	22.00	-
-.36	0	104.50	104.21	1040	35	22.00	-
-.07	0	104.75	104.54	1044	35	21.33	-
.14	0	104.50	104.88	1047	35	21.33	-
-.25	0	105.25	105.25	1051	35	22.00	-
-.24	0	106.25	105.59	1054	35	21.33	-
.42	0	105.50	105.95	1058	35	21.33	-
-.02	0	107.00	106.35	1061	35	22.00	-
.62	-256.00	106.25	106.62	1064	34	21.33	A
.25	-256.00	106.75	106.96	1068	34	22.00	A
.04	0	107.25	107.30	1071	34	22.00	-
-.01	0	107.00	107.65	1075	34	22.00	-
-.65	0	108.25	107.98	1078	34	22.67	-
-.38	0	108.50	108.31	1082	34	22.67	-
-.19	0	109.25	108.67	1085	34	22.00	-
.39	0	109.00	109.00	1088	34	21.33	-
.39	0	109.25	109.34	1092	34	22.67	-
.30	0	109.50	109.66	1095	35	22.00	-
.15	0	109.75	110.02	1099	35	22.00	-
-.13	0	110.00	110.37	1102	35	22.67	-
-.49	0	111.25	110.76	1106	35	22.67	-
-.00	0	111.25	111.16	1109	35	22.67	-
.09	0	111.50	111.49	1113	35	22.00	-
.10	0	111.75	111.82	1116	35	22.67	-
.03	0	112.00	112.16	1120	35	22.00	-
-.13	0	112.50	112.52	1123	35	22.00	-
-.15	0	112.75	112.88	1127	35	22.67	-
-.29	0	113.50	113.26	1130	35	22.67	-
-.05	0	113.25	113.58	1134	35	22.00	-
-.38	0	114.25	113.91	1137	34	22.00	-
-.04	0	114.75	114.27	1141	34	22.00	-
.44	0	114.50	114.55	1144	34	21.33	-
.36	0	115.25	114.91	1148	35	22.00	-
.70	0	114.50	115.31	1151	35	22.00	-
-.11	0	115.75	115.68	1155	35	22.00	-
-.04	0	116.50	116.03	1158	35	22.00	-
.43	0	115.75	116.36	1162	34	22.00	-
-.18	0	116.50	116.69	1165	34	22.00	-
-.37	0	116.75	117.04	1168	34	22.00	-
-.66	0	116.75	117.34	1172	34	22.00	-

S.3 Satellite Position Data Listing. The satellite position-data listing was produced during the simultaneous mode satellite position calculation (program PASS3). In order to provide an easily read printout format, four sheets were used.

Sheet 1:

H, M, S, MS	Time recorded by station 1 which was used as the time reference in hours, minutes, seconds, and milliseconds
TRACKERS	The four numbers indicate which of the four tapes were time-synched (e.g. 1234 if all four tapes were synched; 1230 if tapes 1, 2, and 3 were synched)
RANGE 1, AZ 1, EL 1	Range, azimuth, and elevation as determined from the input survey data and the satellite position using the ranges from stations 1, 2, and 3. This information is included for each of the four stations.

Sheet 2:

H, M, S, MS	Time in hours, minutes, seconds, and milliseconds as on sheet 1
LATITUDE, LONGITUDE, HEIGHT	Latitude, west longitude, and height of the satellite as determined using stations 1, 2, and 3 in units of degrees and meters
EQ VELOCITY	Equatorial velocity determined from the ranges and range rates of stations 1, 2, and 3 in units of meters per second

Sheet 3:

The corrections determined for each of the four stations are listed on this sheet.

TROPO REFR CORR	The tropospheric correction computed using the analytic model (subroutine REF). The correction is printed in meters and must be subtracted from the smoothed range.
MEASURED IC	Ionospheric correction from the edited and smoothed data tapes. This correction is printed in meters and must be subtracted from the smoothed range (if used).
COMPUTED IC	Ionospheric correction computed using the analytic model (subroutine IONCR). This correction is in meters, and must be subtracted from the smoothed ranges.
TRANSIT TIME CORR	Transit time correction which makes the ranges correspond to the indicated time (computed in program PASS3). The correction is in meters and must be added to the smoothed ranges.

<u>Sheet 4:</u>

LSSQ OF PERMUTED SOLUTIONS	Average latitude, west longitude, and height of the satellite determined from the four permuted solutions.
VARIATION OF PERMUTED SOLUTIONS FROM LSSQ	Difference of the latitude, longitude, and height of each permuted solution and the LSSQ or average solution.
COMBINATION	The stations used in the four permuted solutions are: 123, 124, 134, 234.

A

GEODETIC SECUR USA 2 SATELLITE POSITION ORBIT

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	H	M	S	MS	TRACKERS	RANGE 1	AZ 1	EL 1	RANGE 2	AZ 2	EL 2	RANGE 3
1	14	9	28	348	1 2 3 4	2162018	307.3	16.8	2057921	241.7	18.7	1217663
2	14	9	28	548	1 2 3 4	2162112	307.4	16.8	2056852	241.7	18.7	1218540
3	14	9	28	748	1 2 3 4	2162206	307.4	16.8	2055783	241.7	18.8	1219417
4	14	9	28	949	1 2 3 4	2162302	307.4	16.8	2054714	241.8	18.8	1220295
5	14	9	29	149	1 2 3 4	2162398	307.5	16.8	2053646	241.8	18.8	1221174
6	14	9	29	348	1 2 3 4	2162495	307.5	16.8	2052578	241.8	18.8	1222054
7	14	9	29	548	1 2 3 4	2162593	307.6	16.8	2051510	241.8	18.8	1222934
8	14	9	29	748	1 2 3 4	2162691	307.6	16.8	2050442	241.8	18.9	1223816
9	14	9	29	949	1 2 3 4	2162791	307.6	16.8	2049375	241.8	18.9	1224698
10	14	9	30	149	1 2 3 4	2162891	307.7	16.8	2048308	241.9	18.9	1225581
11	14	9	30	348	1 2 3 4	2162992	307.7	16.8	2047242	241.9	18.9	1226465
12	14	9	30	548	1 2 3 4	2163094	307.8	16.8	2046175	241.9	19.0	1227350
13	14	9	30	748	1 2 3 4	2163196	307.8	16.8	2045110	241.9	19.0	1228235
14	14	9	30	949	1 2 3 4	2163300	307.8	16.8	2044044	241.9	19.0	1229122
15	14	9	31	149	1 2 3 4	2163403	307.9	16.8	2042979	242.0	19.0	1230009
16	14	9	31	348	1 2 3 4	2163508	307.9	16.8	2041914	242.0	19.0	1230897
17	14	9	31	548	1 2 3 4	2163614	308.0	16.8	2040849	242.0	19.1	1231786
18	14	9	31	748	1 2 3 4	2163720	308.0	16.8	2039784	242.0	19.1	1232675
19	14	9	31	949	1 2 3 4	2163827	308.0	16.8	2038720	242.0	19.1	1233565
20	14	9	32	149	1 2 3 4	2163935	308.1	16.8	2037657	242.1	19.1	1234457
21	14	9	32	348	1 2 3 4	2164044	308.1	16.8	2036593	242.1	19.1	1235349
22	14	9	32	548	1 2 3 4	2164153	308.2	16.8	2035530	242.1	19.2	1236242
23	14	9	32	748	1 2 3 4	2164264	308.2	16.8	2034467	242.1	19.2	1237136
24	14	9	32	949	1 2 3 4	2164375	308.2	16.8	2033404	242.1	19.2	1238031
25	14	9	33	149	1 2 3 4	2164486	308.3	16.8	2032341	242.2	19.2	1238926
26	14	9	33	348	1 2 3 4	2164598	308.3	16.8	2031280	242.2	19.2	1239822
27	14	9	33	548	1 2 3 4	2164712	308.4	16.8	2030218	242.2	19.3	1240719
28	14	9	33	748	1 2 3 4	2164826	308.4	16.8	2029156	242.2	19.3	1241617
29	14	9	33	949	1 2 3 4	2164941	308.4	16.8	2028095	242.2	19.3	1242516
30	14	9	34	149	1 2 3 4	2165057	308.5	16.8	2027034	242.3	19.3	1243415
31	14	9	34	348	1 2 3 4	2165174	308.5	16.8	2025974	242.3	19.3	1244315
32	14	9	34	548	1 2 3 4	2165291	308.6	16.8	2024913	242.3	19.4	1245217
33	14	9	34	748	1 2 3 4	2165410	308.6	16.8	2023853	242.3	19.4	1246119
34	14	9	34	949	1 2 3 4	2165529	308.6	16.8	2022793	242.3	19.4	1247022
35	14	9	35	149	1 2 3 4	2165648	308.7	16.8	2021734	242.3	19.4	1247925
36	14	9	35	348	1 2 3 4	2165768	308.7	16.8	2020674	242.4	19.4	1248830
37	14	9	35	548	1 2 3 4	2165889	308.8	16.8	2019615	242.4	19.5	1249735
38	14	9	35	748	1 2 3 4	2166011	308.8	16.8	2018557	242.4	19.5	1250641
39	14	9	35	949	1 2 3 4	2166133	308.8	16.8	2017498	242.4	19.5	1251548
40	14	9	36	149	1 2 3 4	2166257	308.9	16.8	2016440	242.4	19.5	1252455
41	14	9	36	348	1 2 3 4	2166381	308.9	16.8	2015383	242.5	19.6	1253363
42	14	9	36	548	1 2 3 4	2166506	309.0	16.8	2014326	242.5	19.6	1254272
43	14	9	36	748	1 2 3 4	2166632	309.0	16.8	2013269	242.5	19.6	1255182
44	14	9	36	949	1 2 3 4	2166759	309.0	16.8	2012212	242.5	19.6	1256092
45	14	9	37	149	1 2 3 4	2166886	309.1	16.7	2011156	242.5	19.6	1257003
46	14	9	37	348	1 2 3 4	2167014	309.1	16.7	2010100	242.6	19.7	1257916
47	14	9	37	550	1 2 3 4	2167143	309.2	16.7	2009044	242.6	19.7	1258829
48	14	9	37	750	1 2 3 4	2167272	309.2	16.7	2007988	242.6	19.7	1259743
49	14	9	37	949	1 2 3 4	2167402	309.2	16.7	2006933	242.6	19.7	1260657
50	14	9	38	149	1 2 3 4	2167534	309.3	16.7	2005878	242.6	19.7	1261573
51	14	9	38	350	1 2 3 4	2167666	309.3	16.7	2004823	242.7	19.8	1262489
52	14	9	38	550	1 2 3 4	2167799	309.3	16.7	2003768	242.7	19.8	1263406
53	14	9	38	750	1 2 3 4	2167933	309.4	16.7	2002714	242.7	19.8	1264324
54	14	9	38	949	1 2 3 4	2168067	309.4	16.7	2001660	242.7	19.8	1265242

B

SATELLITE POSITION ORBIT 1319

PAGE 4

GRAND FORLS			I SAN DIEGO			I LARSON AFB		
RANGE 2	AZ 2	EL 2	RANGE 3	AZ 3	EL 3	RANGE 4	AZ 4	EL 4
2057921	241.7	18.7	1217663	17.0	46.0	1388796	155.1	37.3
2056852	241.7	18.7	1218540	17.0	46.0	1388091	155.0	37.3
2055783	241.7	18.8	1219417	17.0	45.9	1387386	155.0	37.3
2054714	241.8	18.8	1220295	17.0	45.9	1386682	154.9	37.4
2053646	241.8	18.8	1221174	17.1	45.8	1385979	154.9	37.4
2052578	241.8	18.8	1222054	17.1	45.8	1385277	154.8	37.4
2051510	241.6	18.8	1222934	17.1	45.7	1384575	154.8	37.5
2050442	241.8	18.9	1223816	17.1	45.6	1383875	154.7	37.5
2049375	241.8	18.9	1224698	17.1	45.6	1383176	154.7	37.5
2048308	241.9	18.9	1225581	17.1	45.5	1382477	154.6	37.5
2047242	241.9	18.9	1226465	17.1	45.5	1381780	154.6	37.6
2046175	241.9	19.0	1227350	17.1	45.4	1381083	154.5	37.6
2045110	241.9	19.0	1228235	17.1	45.4	1380388	154.5	37.6
2044044	241.9	19.0	1229122	17.1	45.3	1379693	154.4	37.7
2042979	242.0	19.0	1230009	17.1	45.3	1379000	154.3	37.7
2041914	242.0	19.0	1230897	17.1	45.2	1378308	154.3	37.7
2040849	242.0	19.1	1231786	17.1	45.2	1377616	154.2	37.8
2039784	242.0	19.1	1232675	17.2	45.1	1376925	154.2	37.8
2038720	242.0	19.1	1233565	17.2	45.1	1376236	154.1	37.8
2037657	242.1	19.1	1234457	17.2	45.0	1375547	154.1	37.8
2036593	242.1	19.1	1235349	17.2	45.0	1374860	154.0	37.9
2035530	242.1	19.2	1236242	17.2	44.9	1374173	154.0	37.9
2034467	242.1	19.2	1237136	17.2	44.9	1373487	153.9	37.9
2033404	242.1	19.2	1238031	17.2	44.8	1372803	153.9	38.0
2032341	242.2	19.2	1238926	17.2	44.7	1372119	153.8	38.0
2031280	242.2	19.2	1239822	17.2	44.7	1371437	153.7	38.0
2030218	242.2	19.3	1240719	17.2	44.6	1370755	153.7	38.1
2029156	242.2	19.3	1241617	17.2	44.6	1370074	153.6	38.1
2028095	242.2	19.3	1242516	17.2	44.5	1369394	153.6	38.1
2027034	242.3	19.3	1243415	17.2	44.5	1368716	153.5	38.2
2025974	242.3	19.3	1244315	17.3	44.4	1368038	153.5	36.2
2024913	242.3	19.4	1245217	17.3	44.4	1367360	153.4	38.2
2023853	242.3	19.4	1246119	17.3	44.3	1366684	153.4	38.2
2022793	242.3	19.4	1247022	17.3	44.3	1366009	153.3	38.3
2021734	242.3	19.4	1247925	17.3	44.2	1365336	153.2	38.3
2020674	242.4	19.4	1248830	17.3	44.2	1364663	153.2	38.3
2019615	242.4	19.5	1249735	17.3	44.1	1363992	153.1	38.4
2018557	242.4	19.5	1250641	17.3	44.1	1363321	153.1	38.4
2017498	242.4	19.5	1251548	17.3	44.0	1362652	153.0	38.4
2016440	242.4	19.5	1252455	17.3	44.0	1361983	153.0	38.5
2015383	242.5	19.6	1253363	17.3	43.9	1361315	152.9	38.5
2014326	242.5	19.6	1254272	17.3	43.9	1360648	152.8	38.5
2013269	242.5	19.6	1255182	17.3	43.8	1359982	152.8	38.5
2012212	242.5	19.6	1256092	17.3	43.8	1359318	152.7	38.6
2011156	242.5	19.6	1257003	17.4	43.7	1358654	152.7	38.6
2010100	242.6	19.7	1257916	17.4	43.7	1357992	152.6	38.6
2009044	242.6	19.7	1258829	17.4	43.6	1357330	152.6	38.7
2007988	242.6	19.7	1259743	17.4	43.6	1356669	152.5	38.7
2006933	242.6	19.7	1260657	17.4	43.5	1356010	152.4	38.7
2005878	242.6	19.7	1261573	17.4	43.5	1355351	152.4	38.8
2004823	242.7	19.8	1262489	17.4	43.4	1354693	152.3	38.8
2003768	242.7	19.8	1263406	17.4	43.4	1354036	152.3	38.8
2002714	242.7	19.8	1264324	17.4	43.3	1353380	152.2	38.8
2001660	242.7	19.8	1265242	17.4	43.3	1352725	152.2	38.9

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GEODETIC SECOR USA 2 SATELLITE POSITION ORBIT 13

	H	M	S	MS	LATITUDE	LONGITUDE	HEIGHT	XE	YE	
1	14	9	28	348	39.1712	114.6279	925569	-2362458.55	-5153421.48	4591
2	14	9	28	548	39.1817	114.6221	925574	-2361591.78	-5152901.79	4592
3	14	9	28	748	39.1921	114.6164	925579	-2360724.59	-5152381.43	4593
4	14	9	28	949	39.2025	114.6106	925583	-2359857.29	-5151860.62	4594
5	14	9	29	149	39.2129	114.6048	925588	-2358990.14	-5151339.90	4595
6	14	9	29	348	39.2234	114.5990	925592	-2358122.65	-5150818.50	4596
7	14	9	29	548	39.2338	114.5932	925597	-2357255.07	-5150296.99	4597
8	14	9	29	748	39.2442	114.5875	925601	-2356387.48	-5149775.42	4598
9	14	9	29	949	39.2546	114.5817	925606	-2355519.79	-5149253.82	4600
10	14	9	30	149	39.2651	114.5759	925610	-2354652.22	-5148731.72	4601
11	14	9	30	348	39.2755	114.5701	925615	-2353784.45	-5148209.62	4602
12	14	9	30	548	39.2859	114.5643	925620	-2352916.50	-5147687.15	4603
13	14	9	30	748	39.2963	114.5585	925624	-2352048.46	-5147164.85	4604
14	14	9	30	949	39.3068	114.5527	925629	-2351180.26	-5146642.36	4605
15	14	9	31	149	39.3172	114.5469	925633	-2350311.88	-5146119.92	4606
16	14	9	31	348	39.3276	114.5411	925638	-2349443.51	-5145597.12	4607
17	14	9	31	548	39.3380	114.5353	925642	-2348575.07	-5145073.66	4608
18	14	9	31	748	39.3484	114.5295	925647	-2347706.46	-5144550.37	4609
19	14	9	31	949	39.3589	114.5237	925651	-2346837.85	-5144026.95	4610
20	14	9	32	149	39.3693	114.5179	925656	-2345969.05	-5143503.54	4611
21	14	9	32	348	39.3797	114.5121	925660	-2345100.14	-5142979.68	4612
22	14	9	32	548	39.3901	114.5063	925665	-2344231.16	-5142455.51	4613
23	14	9	32	748	39.4005	114.5005	925670	-2343361.93	-5141931.20	4614
24	14	9	32	949	39.4110	114.4946	925674	-2342492.48	-5141406.81	4615
25	14	9	33	149	39.4214	114.4888	925679	-2341622.98	-5140882.54	4616
26	14	9	33	348	39.4318	114.4830	925683	-2340753.47	-5140358.16	4617
27	14	9	33	548	39.4422	114.4772	925688	-2339883.88	-5139833.18	4618
28	14	9	33	748	39.4526	114.4713	925692	-2339014.17	-5139307.72	4619
29	14	9	33	949	39.4630	114.4655	925697	-2338144.45	-5138782.54	4620
30	14	9	34	149	39.4735	114.4597	925702	-2337274.66	-5138257.16	4621
31	14	9	34	348	39.4839	114.4539	925706	-2336404.74	-5137731.04	4622
32	14	9	34	548	39.4943	114.4480	925710	-2335534.51	-5137204.52	4623
33	14	9	34	748	39.5047	114.4422	925715	-2334664.34	-5136678.30	4624
34	14	9	34	949	39.5151	114.4364	925720	-2333793.89	-5136151.84	4625
35	14	9	35	149	39.5256	114.4305	925724	-2332923.06	-5135625.59	4626
36	14	9	35	348	39.5360	114.4247	925729	-2332051.98	-5135099.20	4627
37	14	9	35	548	39.5464	114.4188	925734	-2331180.88	-5134572.87	4628
38	14	9	35	748	39.5568	114.4130	925738	-2330309.65	-5134046.21	4629
39	14	9	35	949	39.5672	114.4071	925743	-2329438.39	-5133519.16	4630
40	14	9	36	149	39.5776	114.4013	925748	-2328567.20	-5132992.11	4631
41	14	9	36	348	39.5880	114.3954	925752	-2327696.14	-5132464.47	4632
42	14	9	36	548	39.5984	114.3896	925757	-2326825.13	-5131936.43	4633
43	14	9	36	748	39.6089	114.3837	925761	-2325953.84	-5131408.13	4634
44	14	9	36	949	39.6193	114.3779	925766	-2325082.50	-5130880.18	4635
45	14	9	37	149	39.6297	114.3720	925771	-2324210.98	-5130352.37	4636
46	14	9	37	348	39.6401	114.3662	925775	-2323339.07	-5129824.05	4637
47	14	9	37	550	39.6505	114.3603	925780	-2322466.84	-5129295.49	4638
48	14	9	37	750	39.6609	114.3544	925784	-2321594.36	-5128766.70	4640
49	14	9	37	949	39.6713	114.3486	925789	-2320721.72	-5128238.17	4641
50	14	9	38	149	39.6817	114.3427	925794	-2319849.12	-5127708.68	4642
51	14	9	38	350	39.6921	114.3368	925798	-2318976.55	-5127178.87	4643
52	14	9	38	550	39.7026	114.3309	925802	-2318103.93	-5126648.43	4644
53	14	9	38	750	39.7130	114.3251	925807	-2317231.34	-5126118.19	4645
54	14	9	38	949	39.7234	114.3192	925811	-2316358.70	-5125588.09	4646

B

SATELLITE POSITION   ORHIT 1319

XE	YE	ZE		EO.	VELOCITY
2458.55	-5153421.48	4591765.12	4335	2600	5155
1591.78	-5152901.79	4592795.91	4335	2601	5154
0724.59	-5152381.43	4593826.59	4336	2603	5153
9857.29	-5151860.62	4594857.14	4336	2604	5152
8990.14	-5151339.90	4595887.54	4337	2605	5152
8122.65	-5150818.50	4596917.93	4337	2607	5151
7255.07	-5150296.99	4597948.04	4338	2607	5150
6387.48	-5149775.42	4598977.96	4338	2608	5149
5519.79	-5149253.82	4600007.62	4339	2609	5148
4652.22	-5148731.72	4601037.19	4339	2610	5147
3784.45	-5148209.62	4602066.59	4340	2611	5146
2916.50	-5147687.15	4603095.68	4340	2612	5145
2048.46	-5147164.85	4604124.53	4341	2612	5143
1180.26	-5146642.36	4605153.07	4341	2613	5142
0311.88	-5146119.92	4606181.26	4342	2614	5141
9443.51	-5145597.12	4607209.31	4342	2615	5140
8575.07	-5145073.66	4608237.33	4343	2616	5139
7706.46	-5144550.37	4609265.11	4343	2617	5138
6837.85	-5144026.95	4610292.57	4344	2618	5138
5969.05	-5143503.54	4611320.03	4344	2619	5137
5100.14	-5142979.68	4612347.44	4345	2620	5136
4231.16	-5142455.51	4613374.71	4346	2620	5135
3361.93	-5141931.20	4614401.71	4346	2621	5134
2492.48	-5141406.81	4615428.32	4347	2622	5133
1622.98	-5140882.54	4616454.68	4347	2623	5132
0753.47	-5140358.16	4617480.75	4348	2624	5131
9883.88	-5139833.18	4618506.94	4348	2625	5130
9014.17	-5139307.72	4619533.07	4349	2626	5130
8144.45	-5138782.54	4620558.82	4349	2628	5129
7274.66	-5138257.16	4621584.47	4350	2629	5128
6404.74	-5137731.04	4622610.15	4351	2630	5127
5534.51	-5137204.52	4623635.84	4351	2630	5127
4664.34	-5136678.30	4624661.17	4352	2631	5126
3793.89	-5136151.84	4625686.30	4353	2632	5125
2923.06	-5135625.59	4626711.05	4354	2632	5123
2051.98	-5135099.20	4627735.56	4355	2633	5122
1180.88	-5134572.87	4628759.84	4355	2633	5121
0309.65	-5134046.21	4629784.03	4356	2635	5120
9438.39	-5133519.16	4630808.05	4356	2636	5119
8567.20	-5132992.11	4631831.73	4356	2637	5118
7696.14	-5132464.47	4632855.23	4356	2638	5117
6825.13	-5131936.43	4633878.53	4357	2639	5116
5953.84	-5131408.13	4634901.64	4358	2640	5115
5082.50	-5130880.18	4635924.49	4358	2641	5114
4210.98	-5130352.37	4636947.20	4359	2641	5114
3339.07	-5129824.05	4637969.84	4360	2643	5113
2466.84	-5129295.49	4638992.38	4361	2644	5112
1594.36	-5128766.70	4640014.77	4362	2645	5112
0721.72	-5128238.17	4641037.09	4362	2646	5111
9849.12	-5127708.68	4642059.34	4363	2647	5110
8976.55	-5127178.87	4643081.35	4363	2649	5109
8103.93	-5126648.43	4644103.22	4363	2650	5108
7231.34	-5126116.19	4645124.70	4363	2651	5107
6358.76	-5125588.09	4646145.99	4363	2652	5106

A

GEODETIC SECOR USA 2 SATELLITE POSITION ORBIT

		TROPO.	REFR.	CORR.			MEASURED	IC			COMPUTED	I
1	14.3	4.7	7.9	3.1	25.5	25.2	36.2	16.0	64.6	23.4	37.7	15.
2	14.3	4.7	7.9	3.1	26.2	24.5	35.5	15.3	64.5	23.4	37.7	15.
3	14.3	4.7	7.9	3.1	118.8	24.5	35.5	15.3	64.4	23.4	37.7	15.
4	14.3	4.7	7.9	3.1	75.5	25.2	36.8	16.0	64.3	23.4	37.6	15.
5	14.3	4.7	7.9	3.1	36.2	23.8	36.8	15.3	64.2	23.4	37.6	15.
6	14.2	4.7	7.9	3.1	26.2	25.2	35.5	14.0	64.2	23.4	37.6	15.
7	14.2	4.7	7.8	3.1	99.5	25.2	36.2	14.0	64.1	23.4	37.6	15.
8	14.2	4.7	7.8	3.1	30.2	24.5	36.8	14.7	64.0	23.4	37.5	15.
9	14.2	4.7	7.8	3.1	27.5	25.2	35.5	14.7	63.9	23.4	37.5	15.
10	14.2	4.7	7.8	3.1	125.5	23.8	37.5	14.7	63.8	23.3	37.5	15.
11	14.1	4.7	7.8	3.1	86.2	24.5	35.5	15.3	63.8	23.3	37.5	15.
12	14.1	4.7	7.8	3.1	64.8	24.5	35.2	15.3	63.7	23.3	37.4	15.
13	14.1	4.7	7.8	3.1	39.5	23.8	36.2	14.7	63.6	23.3	37.4	15.
14	14.1	4.7	7.8	3.1	34.8	24.5	36.8	14.7	63.5	23.3	37.4	15.
15	14.1	4.7	7.8	3.1	27.5	23.8	36.2	15.3	63.4	23.3	37.3	15.
16	14.0	4.7	7.8	3.1	22.2	24.5	36.2	16.7	63.4	23.3	37.3	15.
17	14.0	4.7	7.8	3.1	20.8	23.8	36.2	16.0	63.3	23.3	37.3	15.
18	14.0	4.7	7.8	3.1	8.2	24.5	35.5	16.0	63.2	23.3	37.3	15.
19	14.0	4.7	7.8	3.1	22.2	23.8	35.5	16.7	63.1	23.3	37.2	15.
20	14.0	4.7	7.8	3.1	51.5	25.2	36.8	16.7	63.0	23.3	37.2	15.
21	13.9	4.7	7.8	3.1	76.8	23.8	36.2	16.0	63.0	23.2	37.2	15.
22	13.9	4.7	7.8	3.1	97.5	25.2	36.2	16.0	62.9	23.2	37.2	15.
23	13.9	4.7	7.7	3.1	36.2	23.8	36.2	15.3	62.8	23.2	37.1	15.
24	13.9	4.7	7.7	3.1	12.8	25.2	31.5	15.3	62.7	23.2	37.1	15.
25	13.9	4.7	7.7	3.1	52.8	23.8	36.2	14.0	62.7	23.2	37.1	15.
26	13.8	4.7	7.7	3.1	94.8	25.2	36.2	16.0	62.6	23.2	37.1	15.
27	13.8	4.7	7.7	3.1	36.2	24.5	36.8	16.0	62.5	23.2	37.0	15.
28	13.8	4.7	7.7	3.1	18.2	25.2	35.5	16.7	62.4	23.2	37.0	15.
29	13.8	4.7	7.7	3.1	66.8	25.2	34.8	16.0	62.3	23.2	37.0	15.
30	13.8	4.7	7.7	3.1	109.5	24.5	35.5	15.3	62.3	23.2	36.9	15.
31	13.7	4.7	7.7	3.1	130.8	25.2	35.5	14.0	62.2	23.2	36.9	15.
32	13.7	4.7	7.7	3.1	18.8	24.5	35.5	14.0	62.1	23.2	36.9	15.
33	13.7	4.7	7.7	3.1	28.8	23.8	36.2	15.3	62.0	23.1	36.9	15.
34	13.7	4.7	7.7	3.1	68.2	23.8	35.5	15.3	62.0	23.1	36.8	15.
35	13.7	4.7	7.7	3.1	95.5	23.8	35.5	14.7	61.9	23.1	36.8	15.
36	13.6	4.7	7.7	3.1	122.8	24.5	34.8	15.3	61.8	23.1	36.8	15.
37	13.6	4.7	7.7	3.1	21.5	23.8	36.2	16.0	61.7	23.1	36.8	15.
38	13.6	4.7	7.7	3.1	6.5	24.5	36.2	16.0	61.7	23.1	36.7	15.6
39	13.6	4.7	7.6	3.1	14.2	23.8	34.8	16.7	61.6	23.1	36.7	15.6
40	13.6	4.7	7.6	3.1	12.2	23.8	36.8	15.3	61.5	23.1	36.7	15.6
41	13.5	4.7	7.6	3.1	18.8	24.5	36.2	16.0	61.4	23.1	36.7	15.6
42	13.5	4.7	7.6	3.1	18.8	24.5	36.2	16.7	61.4	23.1	36.6	15.6
43	13.5	4.7	7.6	3.1	22.2	23.8	35.5	15.3	61.3	23.1	36.6	15.6
44	13.5	4.7	7.6	3.1	12.5	24.5	36.2	15.3	61.2	23.1	36.6	15.6
45	13.5	4.7	7.6	3.1	15.5	24.5	36.2	16.7	61.1	23.0	36.5	15.6
46	13.4	4.6	7.6	3.1	7.5	24.5	36.8	15.3	61.1	23.0	36.5	15.6
47	13.4	4.6	7.6	3.1	21.5	24.5	35.5	15.3	61.0	23.0	36.5	15.6
48	13.4	4.6	7.6	3.1	41.5	24.5	36.8	15.3	60.9	23.0	36.5	15.6
49	13.4	4.6	7.6	3.1	122.2	24.5	36.8	16.0	60.8	23.0	36.4	15.6
50	13.4	4.6	7.6	3.1	112.8	23.8	35.5	16.0	60.8	23.0	36.4	15.6
51	13.4	4.6	7.6	3.1	99.5	23.8	36.2	16.7	60.7	23.0	36.4	15.6
52	13.3	4.6	7.6	3.1	70.8	24.5	36.2	16.0	60.6	23.0	36.4	15.6
53	13.3	4.6	7.6	3.1	32.2	23.8	35.5	16.7	60.5	23.0	36.3	15.7
54	13.3	4.6	7.6	3.1	16.2	23.2	36.2	15.3	60.5	23.0	36.3	15.7

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2 SATELLITE POSITION ORBIT 1255

PAGE 2

SURED IC		COMPUTED IC				TRANSIT TIME			CORR.
.2	16.0	64.6	23.4	37.7	15.5	-55.7	-21.2	-39.9	7.6
.5	15.3	64.5	23.4	37.7	15.5	-55.7	-21.2	-39.9	7.7
.5	15.3	64.4	23.4	37.7	15.5	-55.6	-21.1	-39.9	7.8
.8	16.0	64.3	23.4	37.6	15.5	-55.6	-21.1	-39.8	7.8
.8	15.3	64.2	23.4	37.6	15.5	-55.6	-21.0	-39.8	7.9
.5	14.0	64.2	23.4	37.6	15.5	-55.5	-21.0	-39.7	8.0
.2	14.0	64.1	23.4	37.6	15.5	-55.5	-20.9	-39.7	8.0
.8	14.7	64.0	23.4	37.5	15.5	-55.5	-20.9	-39.7	8.1
.5	14.7	63.9	23.4	37.5	15.5	-55.4	-20.8	-39.6	8.2
.5	14.7	63.8	23.3	37.5	15.5	-55.4	-20.7	-39.6	8.3
.5	15.3	63.8	23.3	37.5	15.5	-55.4	-20.7	-39.5	8.3
.2	13.3	63.7	23.3	37.4	15.5	-55.4	-20.6	-39.5	8.4
.2	14.7	63.6	23.3	37.4	15.5	-55.3	-20.6	-39.5	8.5
.8	14.7	63.5	23.3	37.4	15.5	-55.3	-20.5	-39.4	8.5
.2	15.3	63.4	23.3	37.3	15.5	-55.3	-20.5	-39.4	8.6
.2	16.7	63.4	23.3	37.3	15.5	-55.2	-20.4	-39.4	8.7
.2	16.0	63.3	23.3	37.3	15.5	-55.2	-20.3	-39.3	8.7
.5	16.0	63.2	23.3	37.3	15.5	-55.2	-20.3	-39.3	8.8
.5	16.7	63.1	23.3	37.2	15.5	-55.2	-20.2	-39.2	8.9
.8	16.7	63.0	23.3	37.2	15.5	-55.1	-20.2	-39.2	8.9
.2	16.0	63.0	23.2	37.2	15.5	-55.1	-20.1	-39.2	9.0
.2	16.0	62.9	23.2	37.2	15.5	-55.1	-20.1	-39.1	9.1
.2	15.3	62.8	23.2	37.1	15.5	-55.0	-20.0	-39.1	9.1
.5	15.3	62.7	23.2	37.1	15.5	-55.0	-20.0	-39.0	9.2
.2	14.0	62.7	23.2	37.1	15.6	-55.0	-19.9	-39.0	9.3
.2	16.0	62.6	23.2	37.1	15.6	-55.0	-19.8	-39.0	9.3
.8	16.0	62.5	23.2	37.0	15.6	-54.9	-19.8	-38.9	9.4
.5	16.7	62.4	23.2	37.0	15.6	-54.9	-19.7	-38.9	9.5
.8	16.0	62.3	23.2	37.0	15.6	-54.9	-19.7	-38.9	9.5
.5	15.3	62.3	23.2	36.9	15.6	-54.8	-19.6	-38.8	9.6
.5	14.0	62.2	23.2	36.9	15.6	-54.8	-19.6	-38.8	9.7
.5	14.0	62.1	23.2	36.9	15.6	-54.8	-19.5	-38.8	9.7
.2	15.3	62.0	23.1	36.9	15.6	-54.8	-19.5	-38.7	9.8
.5	15.3	62.0	23.1	36.8	15.6	-54.7	-19.4	-38.7	9.9
.5	14.7	61.9	23.1	36.8	15.6	-54.7	-19.3	-38.6	9.9
.8	15.3	61.8	23.1	36.8	15.6	-54.7	-19.3	-38.6	10.0
.2	16.0	61.7	23.1	36.8	15.6	-54.6	-19.2	-38.6	10.1
.2	16.0	61.7	23.1	36.7	15.6	-54.6	-19.2	-38.5	10.1
.8	16.7	61.6	23.1	36.7	15.6	-54.6	-19.1	-38.5	10.2
.8	15.3	61.5	23.1	36.7	15.6	-54.6	-19.1	-38.4	10.3
2	16.0	61.4	23.1	36.7	15.6	-54.6	-19.0	-38.4	10.3
2	16.7	61.4	23.1	36.6	15.6	-54.5	-18.9	-38.4	10.4
5	15.3	61.3	23.1	36.6	15.6	-54.5	-18.9	-38.3	10.5
2	15.3	61.2	23.1	36.6	15.6	-54.5	-18.8	-38.3	10.5
2	16.7	61.1	23.0	36.5	15.6	-54.4	-18.8	-38.3	10.6
8	15.3	61.1	23.0	36.5	15.6	-54.4	-18.7	-38.2	10.6
5	15.3	61.0	23.0	36.5	15.6	-54.4	-18.7	-38.2	10.7
8	15.3	60.9	23.0	36.5	15.6	-54.4	-18.6	-38.1	10.8
8	16.0	60.8	23.0	36.4	15.6	-54.3	-18.6	-38.1	10.8
5	16.0	60.8	23.0	36.4	15.6	-54.3	-18.5	-38.1	10.9
2	16.7	60.7	23.0	36.4	15.6	-54.3	-18.4	-38.0	11.0
2	15.0	60.6	23.0	36.4	15.6	-54.2	-18.4	-38.0	11.0
5	16.7	60.5	23.0	36.3	15.7	-54.2	-18.3	-38.0	11.1
2	15.3	60.5	23.0	36.3	15.7	-54.2	-18.3	-37.9	11.2

A

GEODETIC SECOR USA 2 SATELLITE POSITION ORBI

	LSSQ OF PERMUTED SOLUTIONS			VARIATION OF PERMUTED SOLUTIONS FROM LSSQ COM							
1	39.1712	114.6279	925570	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
2	39.1817	114.6221	925575	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
3	39.1921	114.6164	925579	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
4	39.2025	114.6106	925584	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
5	39.2129	114.6048	925588	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
6	39.2234	114.5990	925593	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
7	39.2338	114.5932	925597	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
8	39.2442	114.5874	925602	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
9	39.2546	114.5817	925606	.0000	.0000	-0	.0000	-.0000	1	-.0000	.
10	39.2651	114.5759	925611	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
11	39.2755	114.5701	925616	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
12	39.2859	114.5643	925620	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
13	39.2963	114.5585	925625	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
14	39.3068	114.5527	925629	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
15	39.3172	114.5469	925634	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
16	39.3276	114.5411	925638	.0000	.0000	-0	.0000	-.0000	1	-.0000	.
17	39.3380	114.5353	925643	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
18	39.3484	114.5295	925647	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
19	39.3589	114.5237	925652	.0000	.0000	-0	.0000	-.0000	1	-.0000	.
20	39.3693	114.5179	925656	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
21	39.3797	114.5121	925661	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
22	39.3901	114.5063	925665	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
23	39.4005	114.5004	925670	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
24	39.4110	114.4946	925674	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
25	39.4214	114.4888	925679	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
26	39.4318	114.4830	925683	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
27	39.4422	114.4772	925688	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
28	39.4526	114.4713	925692	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
29	39.4630	114.4655	925697	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
30	39.4735	114.4597	925701	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
31	39.4839	114.4539	925706	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
32	39.4943	114.4480	925711	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
33	39.5047	114.4422	925716	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
34	39.5151	114.4364	925720	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
35	39.5256	114.4305	925725	.0000	.0000	-0	.0000	-.0000	1	-.0000	.
36	39.5360	114.4247	925729	.0000	.0000	-0	.0000	-.0000	3	-.0000	.
37	39.5464	114.4188	925734	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
38	39.5568	114.4130	925738	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
39	39.5672	114.4071	925743	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
40	39.5776	114.4013	925748	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
41	39.5880	114.3954	925752	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
42	39.5984	114.3896	925757	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
43	39.6089	114.3837	925761	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
44	39.6193	114.3779	925766	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
45	39.6297	114.3720	925770	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
46	39.6401	114.3662	925775	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
47	39.6505	114.3603	925780	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
48	39.6609	114.3544	925784	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
49	39.6713	114.3486	925789	-.0000	-.0000	1	-.0000	.0000	-1	.0000	-.
50	39.6817	114.3427	925793	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
51	39.6921	114.3368	925798	-.0000	-.0000	0	-.0000	.0000	-0	.0000	-.
52	39.7026	114.3309	925803	.0000	.0000	-0	.0000	-.0000	0	-.0000	.
53	39.7130	114.3251	925807	.0000	.0000	-1	.0000	-.0000	1	-.0000	.
54	39.7234	114.3192	925812	.0000	.0000	-1	.0000	-.0000	1	-.0000	.

B

2 SATELLITE POSITION ORBIT 1319

PAGE 4

PERMUTED SOLUTIONS FROM LSSO COMBINATION

-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
1	-.0000	.0000	-1	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
0	-.0000	.0000	-0	.0000	-.0000	-0	.0000	.0000	0
-0	.0000	-.0000	0	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0
-1	.0000	-.0000	1	-.0000	.0000	0	-.0000	-.0000	-0

S.4 <u>3-3 CORDEX Solution</u>. The 3-3 CORDEX solution (program PASS4) lists a summary of results followed by a listing of each discrete solution.

TM1, TM2, TM3	These three rows indicate for each time span used: 1. the first time in hours, minutes, seconds, and milliseconds 2. the last time in hours, minutes, seconds, and milliseconds 3. the time between samples in hours, minutes, seconds, and milliseconds 4. the logical tape unit on which the satellite position tape was mounted.
FST INPUT	Latitude, west longitude, and height of the CORDEX station input (survey values) in units of degrees and meters
FST AVER	Latitude, west longitude, and height of the CORDEX station determined by averaging all the discrete solutions in units of degrees and meters.
FST BIAS	Difference between the survey and the average coordinates of latitude, west longitude, and height in units of degrees and meters
RMS ERROR	The rms of the deviations of each solution from the average solution. Units are degrees and meters.

The following quantities are listed for each discrete solution.

SAMP	Sample number

LATITUDE LONGITUDE, HEIGHT	Latitude, west longitude, and height of the CORDEX station in degrees and meters
DEVIATION FROM INPUT POSITION	Deviation of each solution from the input survey position in latitude, west longitude, and height
DEVIATION FROM AVERAGE POSITION	Deviation of each solution from the average solution in latitude, west longitude, and height

A

GEODETIC SECOR UNKNOWN STATION LOCATION LARGE QUAD

TM1	13	52	21	895	13	52	38	92	0	0	0 200	1	FST INPUT
TM2	16	3	52	86	16	4	5	86	0	0	0 200	2	FST AVER
TM3	14	8	55	948	14	9	5	848	0	0	0 200	3	FST BIAS

NUMBER OF SAMPLES= 50 RMS ERROR

SAMP	LATITUDE	LONGITUDE	HEIGHT	DEVIATION FROM INPUT POSITION		
1	47.18498	119.33635	336.7	-.00015	-.00013	-33.7
2	47.18500	119.33636	340.7	-.00014	-.00012	-29.7
3	47.18502	119.33637	344.6	-.00012	-.00010	-25.8
4	47.18505	119.33638	350.1	-.00009	-.00009	-20.3
5	47.18509	119.33639	355.6	-.00005	-.00009	-14.8
6	47.18513	119.33639	361.8	-.00001	-.00008	-8.7
7	47.18516	119.33639	365.1	.00002	-.00009	-5.3
8	47.18515	119.33639	364.2	.00001	-.00009	-6.3
9	47.18515	119.33639	364.2	.00001	-.00009	-6.2
10	47.18514	119.33639	363.6	.00001	-.00009	-6.8
11	47.18513	119.33638	362.1	-.00000	-.00009	-8.4
12	47.18513	119.33638	360.6	-.00001	-.00010	-9.8
13	47.18512	119.33638	359.3	-.00002	-.00010	-11.1
14	47.18510	119.33638	356.3	-.00004	-.00010	-14.2
15	47.18508	119.33638	354.4	-.00006	-.00010	-16.0
16	47.18508	119.33638	352.8	-.00006	-.00009	-17.6
17	47.18507	119.33638	351.0	-.00007	-.00010	-19.4
18	47.18507	119.33637	350.0	-.00007	-.00010	-20.4
19	47.18507	119.33637	350.3	-.00007	-.00010	-20.1
20	47.18507	119.33639	352.3	-.00006	-.00009	-18.1
21	47.18508	119.33639	353.5	-.00006	-.00009	-16.9
22	47.18509	119.33639	355.6	-.00005	-.00009	-14.8
23	47.18510	119.33640	358.0	-.00004	-.00008	-12.4
24	47.18510	119.33640	359.7	-.00003	-.00008	-10.7
25	47.18510	119.33638	358.1	-.00004	-.00010	-12.3
26	47.18510	119.33636	357.0	-.00004	-.00011	-13.4
27	47.18511	119.33635	356.4	-.00003	-.00013	-14.0
28	47.18510	119.33633	352.8	-.00004	-.00015	-17.6
29	47.18510	119.33632	351.6	-.00004	-.00016	-18.8
30	47.18510	119.33631	351.4	-.00004	-.00017	-19.0
31	47.18510	119.33632	352.5	-.00004	-.00016	-17.9
32	47.18509	119.33632	352.1	-.00005	-.00015	-18.3
33	47.18509	119.33633	353.0	-.00005	-.00014	-17.4
34	47.18508	119.33635	353.7	-.00006	-.00013	-16.7
35	47.18508	119.33636	354.2	-.00006	-.00012	-16.2
36	47.18508	119.33636	354.1	-.00006	-.00012	-16.3
37	47.18508	119.33635	352.2	-.00006	-.00013	-18.2
38	47.18508	119.33633	350.4	-.00006	-.00015	-20.0
39	47.18508	119.33632	348.6	-.00006	-.00016	-21.9
40	47.18507	119.33631	346.1	-.00007	-.00016	-24.3
41	47.18506	119.33631	345.4	-.00008	-.00016	-25.0
42	47.18507	119.33632	346.0	-.00007	-.00016	-24.4
43	47.18507	119.33632	346.3	-.00007	-.00016	-24.1
44	47.18508	119.33633	349.1	-.00006	-.00014	-21.3
45	47.18509	119.33635	352.0	-.00005	-.00013	-18.4
46	47.18511	119.33637	356.7	-.00003	-.00011	-13.7
47	47.18514	119.33639	362.1	.00000	-.00009	-8.3
48	47.18513	119.33639	361.3	-.00001	-.00009	-9.1
49	47.18512	119.33638	359.3	-.00002	-.00010	-11.1
50	47.18511	119.33638	358.3	-.00003	-.00010	-12.1

B 5/6/64

ATION LOCATION LARGE QUAD 1291L-1167-1319

0 200	1	FST INPUT	47.18514	119.33648	370.4
0 200	2	FST AVER	47.18509	119.33636	354.1
0 200	3	FST BIAS	-.00005	-.00012	-16.4
		RMS ERROR	.00003	.00003	6.1

PAGE 1

DEVIATION FROM INPUT POSITION			DEVIATION FROM AVERAGE POSITION		
-.00016	-.00013	-33.7	-.00011	-.00001	-17.4
-.00014	-.00012	-29.7	-.00009	-.00000	-13.4
-.00012	-.00010	-25.8	-.00008	.00001	-9.5
-.00009	-.00009	-20.3	-.00004	.00002	-3.8
-.00005	-.00009	-14.8	-.00000	.00003	1.5
-.00001	-.00008	-8.7	.00004	.00003	7.7
.00002	-.00009	-5.3	.00007	.00003	11.1
.00001	-.00009	-6.3	.00006	.00003	10.1
.00001	-.00009	-6.2	.00006	.00003	10.1
.00001	-.00009	-6.8	.00005	.00003	9.5
-.00000	-.00009	-8.4	.00004	.00002	8.0
-.00001	-.00010	-9.8	.00003	.00002	6.5
-.00002	-.00010	-11.1	.00003	.00002	5.2
-.00004	-.00010	-14.2	.00001	.00002	2.2
-.00006	-.00010	-16.0	-.00001	.00002	.4
-.00006	-.00009	-17.6	-.00002	.00002	-1.3
-.00007	-.00010	-19.4	-.00002	.00001	-3.0
-.00007	-.00010	-20.4	-.00002	.00001	-4.0
-.00007	-.00010	-20.1	-.00002	.00001	-3.8
-.00006	-.00009	-18.1	-.00002	.00002	-1.8
-.00006	-.00009	-16.9	-.00001	.00003	-.6
-.00005	-.00009	-14.8	-.00001	.00003	1.6
-.00004	-.00008	-12.4	.00000	.00004	3.9
-.00003	-.00008	-10.7	.00001	.00004	5.6
-.00004	-.00010	-12.3	.00001	.00002	4.1
-.00004	-.00011	-13.4	.00001	.00000	2.9
-.00003	-.00013	-14.0	.00002	-.00001	2.3
-.00004	-.00015	-17.6	.00001	-.00004	-1.2
-.00004	-.00016	-18.8	.00000	-.00005	-2.4
-.00004	-.00017	-19.0	.00001	-.00005	-2.7
-.00004	-.00016	-17.9	.00001	-.00004	-1.6
-.00005	-.00015	-18.3	-.00000	-.00004	-2.0
-.00005	-.00014	-17.4	-.00000	-.00003	-1.1
-.00006	-.00013	-16.7	-.00001	-.00001	-.4
-.00006	-.00012	-16.2	-.00001	-.00001	.1
-.00006	-.00012	-16.3	-.00001	-.00001	.1
-.00006	-.00013	-18.2	-.00001	-.00002	-1.8
-.00006	-.00015	-20.0	-.00001	-.00003	-3.7
-.00006	-.00016	-21.9	-.00001	-.00004	-5.5
-.00007	-.00016	-24.3	-.00003	-.00005	-8.0
-.00008	-.00016	-25.0	-.00003	-.00005	-8.7
-.00007	-.00016	-24.4	-.00002	-.00005	-8.0
-.00007	-.00016	-24.1	-.00002	-.00004	-7.7
-.00006	-.00014	-21.3	-.00001	-.00003	-5.0
-.00005	-.00013	-18.4	.00000	-.00002	-2.1
-.00003	-.00011	-13.7	.00002	.00001	2.7
.00000	-.00009	-8.3	.00005	.00002	8.0
-.00001	-.00009	-9.1	.00004	.00003	7.2
-.00002	-.00010	-11.1	.00003	.00002	5.2
-.00003	-.00010	-12.1	.00002	.00002	4.3

S.5 <u>3-2 CORDEX Solution</u>. The 3-2 CORDEX solution (program PASS432) listing is identical with the 3-3 CORDEX solution listing (paragraph S.4) except:

1. only two spans of data are used, and

2. the height of the CORDEX station is input, and not calculated, so that no error is indicated in the height.

A

GEODETIC SECOR   UNKNOWN STATION 3-2   ORBIT

T1	21	56	12 157	21	53	54 986	0	0	9 200	1	F
T2	5	1	55 638	5	12	6 137	0	0	0 200	2	F
											F

NUMBER OF SAMPLES= 53   R

SAMP	LATITUDE	LONGITUDE	HEIGHT	DEVIATION FROM	INP
1	38.90177	282.67447	121.0	-.00041	.00
2	38.90173	282.67447	121.0	-.00045	.00
3	38.90176	282.67447	121.0	-.00045	.00
4	38.90176	282.67446	121.0	-.00042	.00
5	38.90181	282.67446	121.0	-.00038	.00
6	38.90182	282.67446	121.0	-.00036	.00
7	38.90183	282.67446	121.0	-.00037	.00
8	38.90168	282.67448	121.0	-.00050	.00
9	38.90171	282.67448	121.0	-.00048	.00
10	38.90176	282.67447	121.0	-.00040	.00
11	38.90176	282.67447	121.0	-.00042	.00
12	38.90179	282.67446	121.0	-.00039	.00
13	38.90174	282.67446	121.0	-.00044	.00
14	38.90173	282.67446	121.0	-.00045	.00
15	38.90173	282.67445	121.0	-.00045	.00
16	38.90178	282.67444	121.0	-.00040	.00
17	38.90183	282.67445	121.0	-.00037	.00
18	38.90181	282.67445	121.0	-.00038	.00
19	38.90178	282.67446	121.0	-.00040	.00
20	38.90177	282.67446	121.0	-.00041	.00
21	38.90178	282.67446	121.0	-.00040	.00
22	38.90175	282.67447	121.0	-.00043	.00
23	38.90175	282.67447	121.0	-.00044	.00
24	38.90174	282.67446	121.0	-.00044	.00
25	38.90182	282.67444	121.0	-.00036	.00
26	38.90174	282.67445	121.0	-.00044	.00
27	38.90172	282.67445	121.0	-.00046	.00
28	38.90176	282.67444	121.0	-.00042	.00
29	38.90173	282.67444	121.0	-.00045	.00
30	38.90173	282.67444	121.0	-.00045	.00
31	38.90171	282.67445	121.0	-.00049	.00
32	38.90173	282.67444	121.0	-.00045	.00
33	38.90171	282.67444	121.0	-.00048	.00
34	38.90167	282.67443	121.0	-.00051	.00
35	38.90176	282.67442	121.0	-.00046	.00
36	38.90175	282.67442	121.0	-.00043	.00
37	38.90174	282.67442	121.0	-.00044	.00
38	38.90176	282.67443	121.0	-.00042	.00
39	38.90172	282.67443	121.0	-.00046	.00
40	38.90179	282.67442	121.0	-.00039	.00
41	38.90176	282.67443	121.0	-.00042	.00
42	38.90179	282.67442	121.0	-.00039	.00
43	38.90174	282.67443	121.0	-.00044	.00
44	38.90176	282.67443	121.0	-.00042	.00
45	38.90171	282.67444	121.0	-.00048	.00
46	38.90177	282.67443	121.0	-.00041	.00
47	38.90173	282.67443	121.0	-.00045	.00
48	38.90172	282.67442	121.0	-.00046	.00
49	38.90177	282.67441	121.0	-.00041	.00
50	38.90169	282.67442	121.0	-.00049	.00

B

STATION 3-2 ORBIT 1345--896 HERNDON

9 200	1	FST INPUT	38.99218	282.67312	121.0
0 200	2	FST AVER	38.99175	282.67444	121.0
		FST BIAS	-.00043	.00132	-.0
		RMS ERROR	.00004	.00002	.0
				PAGE 1	

DEVIATION FROM INPUT POSITION			DEVIATION FROM AVERAGE POSITION		
-.00041	.00135	0	.00002	.00002	.0
-.00045	.00135	0	-.00002	.00002	.0
-.00045	.00135	0	-.00001	.00002	.0
-.00042	.00134	0	.00002	.00002	.0
-.00038	.00134	0	.00005	.00001	.0
-.00036	.00134	0	.00008	.00001	.0
-.00037	.00134	0	.00007	.00002	.0
-.00050	.00136	0	-.00006	.00004	.0
-.00048	.00136	0	-.00004	.00004	.0
-.00040	.00135	0	.00004	.00003	.0
-.00042	.00135	0	.00001	.00002	.0
-.00039	.00134	0	.00004	.00002	.0
-.00044	.00134	0	-.00001	.00002	.0
-.00045	.00134	0	-.00001	.00001	.0
-.00045	.00133	0	-.00002	.00001	.0
-.00040	.00132	0	.00004	-.00000	.0
-.00037	.00133	0	.00006	.00000	.0
-.00038	.00133	0	.00005	.00000	.0
-.00040	.00134	0	.00004	.00001	.0
-.00041	.00134	0	.00002	.00002	.0
-.00040	.00134	0	.00003	.00002	.0
-.00043	.00135	0	.00000	.00002	.0
-.00044	.00135	0	-.00000	.00002	.0
-.00044	.00134	0	-.00001	.00001	.0
-.00036	.00132	0	.00008	-.00000	.0
-.00044	.00133	0	-.00000	.00000	.0
-.00046	.00133	0	-.00003	.00001	.0
-.00042	.00132	0	.00002	-.00001	.0
-.00045	.00132	0	-.00002	-.00000	.0
-.00045	.00132	0	-.00001	-.00001	.0
-.00048	.00133	0	-.00005	.00000	.0
-.00045	.00132	0	-.00002	-.00001	.0
-.00048	.00132	0	-.00005	-.00001	.0
-.00051	.00131	0	-.00007	-.00001	.0
-.00040	.00130	0	.00003	-.00003	.0
-.00043	.00130	0	.00001	-.00002	.0
-.00044	.00130	0	-.00001	-.00002	.0
-.00042	.00131	0	.00001	-.00002	.0
-.00046	.00131	0	-.00002	-.00001	.0
-.00039	.00130	0	.00004	-.00002	.0
-.00042	.00131	0	.00001	-.00002	.0
-.00039	.00130	0	.00005	-.00002	.0
-.00044	.00131	0	-.00001	-.00001	.0
-.00042	.00131	0	.00002	-.00002	.0
-.00048	.00132	0	-.00004	-.00001	.0
-.00041	.00131	0	.00002	-.00002	.0
-.00045	.00131	0	-.00002	-.00002	.0
-.00046	.00130	0	-.00002	-.00002	.0
-.00041	.00129	0	.00002	-.00004	.0
-.00049	.00130	0	-.00006	-.00003	.0

S. 6 Line Crossing Listing. The span of data used in the determination of the minimum angle sum is listed with the following quantities included:

TIME	Time in hours, minutes, seconds, and milliseconds
RANGE 1, RANGE 2	Ranges in meters from the two stations defining the baseline
HEIGHT	Satellite height in meters
GEOD SUM	Geodetic sum determined by multiplying the central angle sum by the scaling radius
RANGE SUM	Sum of RANGE 1 and RANGE 2
ANGLE SUM	Sum of the central angle in radians
E1, E2	Elevation angles in degrees observed at the ends of the baseline
RESIDUAL	Difference between the geodetic distance sum and the polynomial fit in meters
LAT, LONG	Latitude and west longitude of the satellite in degrees

Following the above listing, the results of the line computation are printed as follows:

MEAS MIN SUM	Measured minimum geodetic distance sum determined from the polynomial fit (meters)
COMP GEODESIC	Geodetic distance (geodesic) determined from the input survey data
RMS	The rms of the polynomial fit residuals
CENTRAL ANGLE	Central angle determined from the input survey data

SCALING RADIUS	Scaling radius determined from the computed geodesic and the central angle (meters)
MIN CENTRAL ANGLE	Minimum central angle determined from the polynomial fit in radians

A

GEODETIC SECOR -- LINE CROSSING SW-SD INT S

	TIME	RANGE1	RANGE2	HEIGHT	GEOD. SUM	RAN
282	2 26 39 476	1476325.03	1268085.27	936610.45	1866450.82	2744
283	2 26 39 877	1475879.63	1268249.27	936607.01	1866130.52	2744
284	2 26 40 276	1475438.97	1268420.17	936604.50	1865823.61	2743
285	2 26 40 677	1475006.58	1268596.18	936601.99	1865530.01	2743
286	2 26 41 76	1474573.48	1268777.68	936599.80	1865249.68	2743
287	2 26 41 476	1474146.43	1268961.39	936595.39	1864982.65	2743
288	2 26 41 877	1473727.69	1269150.64	936590.63	1864729.07	2742
289	2 26 42 276	1473310.60	1269345.91	936585.35	1864489.11	2742
290	2 26 42 677	1472896.61	1269548.46	936581.54	1864262.45	2742
291	2 26 43 78	1472491.89	1269757.07	936578.42	1864049.10	2742
292	2 26 43 477	1472091.15	1269970.08	936575.26	1863849.03	2742
293	2 26 43 878	1471695.37	1270189.73	936572.97	1863662.36	2741
294	2 26 44 278	1471304.05	1270415.30	936570.66	1863489.18	2741
295	2 26 44 677	1470916.65	1270646.43	936567.72	1863329.53	2741
296	2 26 45 78	1470533.90	1270882.89	936564.49	1863183.32	2741
297	2 26 45 477	1470155.78	1271124.71	936560.98	1863050.54	2741
298	2 26 45 878	1469782.51	1271373.45	936558.47	1862931.17	2741
299	2 26 46 278	1469413.76	1271625.25	936553.92	1862825.26	2741
300	2 26 46 677	1469050.08	1271878.10	936546.30	1862732.73	2740
301	2 26 47 76	1468692.12	1272142.80	936543.82	1862653.50	2740
302	2 26 47 476	1468339.41	1272414.70	936542.80	1862587.61	2740
303	2 26 47 877	1467991.81	1272691.95	936541.83	1862535.09	2740
304	2 26 48 276	1467648.71	1272978.75	936543.51	1862495.97	2740
305	2 26 48 677	1467309.86	1273261.51	936537.20	1862470.25	2740
306	2 26 49 76	1466975.95	1273551.78	936533.82	1862457.89	2740
307	2 26 49 476	1466646.74	1273851.73	936532.60	1862458.89	2740
308	2 26 49 875	1466327.48	1274156.75	936526.81	1862473.24	2740
309	2 26 50 276	1466003.07	1274461.85	936525.73	1862500.95	2740
310	2 26 50 676	1465688.39	1274771.77	936519.75	1862542.01	2740
311	2 26 51 75	1465378.87	1275089.94	936515.92	1862596.41	2740
312	2 26 51 476	1465074.46	1275418.09	936515.45	1862664.19	2740
313	2 26 51 875	1464774.78	1275746.28	936510.96	1862745.30	2740
314	2 26 52 276	1464479.86	1276083.85	936509.18	1862839.75	2740
315	2 26 52 676	1464189.43	1276425.10	936505.87	1862947.49	2740
316	2 26 53 75	1463903.97	1276769.85	936501.20	1863068.52	2740
317	2 26 53 476	1463623.84	1277126.51	936499.26	1863202.88	2740
318	2 26 53 875	1463348.48	1277480.32	936495.58	1863350.52	2740
319	2 26 54 276	1463077.81	1277842.65	936491.82	1863511.42	2740
320	2 26 54 676	1462812.42	1278210.47	936488.35	1863685.58	2741
321	2 26 55 75	1462551.82	1278583.36	936484.55	1863872.97	2741
322	2 26 55 474	1462296.32	1278961.56	936480.76	1864073.62	2741
323	2 26 55 875	1462045.80	1279345.81	936477.44	1864287.56	2741
324	2 26 56 275	1461800.08	1279736.44	936474.69	1864514.79	2741
325	2 26 56 674	1461559.14	1280132.24	936471.65	1864755.30	2741
326	2 26 57 75	1461322.87	1280533.12	936468.24	1865008.97	2741
327	2 26 57 474	1461091.42	1280938.24	936463.92	1865275.84	2742
328	2 26 57 875	1460865.01	1281349.02	936459.84	1865555.96	2742
329	2 26 58 275	1460643.75	1281765.93	936456.33	1865849.42	2742
330	2 26 58 674	1460427.49	1282189.33	936453.55	1866156.23	2742
331	2 26 59 75	1460210.06	1282618.09	936450.70	1866476.16	2742

MEAS. MIN. SUM	COMP. GEODESIC	RMS	CENTRAL A
1962457.0111	1862456.6784	2.0050	.2923

B

NG SW-SD INT SPH ORBIT 1131

HT	GEOD. SUM	RANGE SUM	ANGLE SUM	E1	E2	RESIDUAL	LAT.	LONG.
.45	1866450.82	2744410.30	.29293967	34	44	3.43	35.1	108.9
.01	1866130.52	2744126.91	.29288940	34	44	2.89	35.1	108.9
.50	1865823.61	2743859.14	.29284123	34	44	2.39	35.1	108.9
.99	1865530.01	2743599.76	.29279515	34	44	1.84	35.1	108.9
.80	1865249.68	2743351.15	.29275115	34	44	1.20	35.1	108.9
.39	1864982.65	2743109.81	.29270924	35	44	.52	35.0	108.9
.63	1864729.07	2742878.33	.29266944	35	44	-.06	35.0	108.9
.35	1864489.11	2742656.51	.29263178	35	44	-.38	35.0	108.9
.54	1864262.45	2742447.07	.29259621	35	44	-.74	35.0	108.9
.42	1864049.10	2742248.96	.29256272	35	44	-1.15	35.0	108.9
.26	1863849.03	2742061.23	.29253132	35	44	-1.63	34.9	108.8
.97	1863662.36	2741885.10	.29250202	35	44	-2.06	34.9	108.8
.66	1863489.18	2741719.35	.29247484	35	44	-2.35	34.9	108.8
.72	1863329.53	2741563.08	.29244979	35	44	-2.47	34.9	108.8
.49	1863183.32	2741416.79	.29242684	35	44	-2.50	34.8	108.8
.98	1863050.54	2741280.49	.29240600	35	44	-2.45	34.8	108.8
.47	1862931.17	2741155.97	.29238726	35	44	-2.34	34.8	108.8
.92	1862825.26	2741039.01	.29237064	35	44	-2.12	34.8	108.8
.30	1862732.73	2740928.18	.29235612	35	44	-1.88	34.8	108.8
.82	1862653.50	2740834.93	.29234368	35	44	-1.69	34.7	108.7
.80	1862587.61	2740754.11	.29233334	35	44	-1.51	34.7	108.7
.83	1862535.09	2740683.76	.29232510	35	44	-1.31	34.7	108.7
.51	1862495.97	2740627.47	.29231896	35	44	-1.06	34.7	108.7
.20	1862470.23	2740570.37	.29231492	35	44	-.76	34.7	108.7
.82	1862457.89	2740527.74	.29231298	35	44	-.46	34.6	108.7
.60	1862458.89	2740498.47	.29231314	35	44	-.16	34.6	108.7
.81	1862473.24	2740473.23	.29231539	35	44	.15	34.6	108.7
.73	1862500.95	2740464.92	.29231974	35	43	.48	34.6	108.7
.75	1862542.01	2740460.16	.29232618	35	43	.79	34.6	108.7
.92	1862596.41	2740468.80	.29233472	35	43	1.10	34.5	108.6
.45	1862664.19	2740492.55	.29234536	35	43	1.43	34.5	108.6
.96	1862745.30	2740521.05	.29235809	35	43	1.74	34.5	108.6
18	1862839.75	2740563.71	.29237292	35	43	2.04	34.5	108.6
87	1862947.49	2740614.53	.29238982	35	43	2.27	34.4	108.6
20	1863068.52	2740673.82	.29240882	35	43	2.44	34.4	108.6
26	1863202.68	2740747.35	.29242991	35	43	2.61	34.4	108.6
58	1863350.52	2740828.80	.29245308	35	43	2.69	34.4	108.6
82	1863511.42	2740920.45	.29247833	35	43	2.67	34.4	108.6
35	1863685.58	2741022.89	.29250567	35	43	2.57	34.3	108.6
55	1863872.97	2741135.18	.29253508	35	43	2.34	34.3	108.5
76	1864073.62	2741257.88	.29256657	35	43	2.03	34.3	108.5
44	1864287.56	2741391.61	.29260015	35	43	1.65	34.3	108.5
69	1864514.79	2741536.52	.29263581	35	43	1.22	34.3	108.5
65	1864755.30	2741691.38	.29267356	35	43	.70	34.2	108.5
24	1865008.97	2741855.99	.29271337	35	43	-.00	34.2	108.5
92	1865275.84	2742029.66	.29275526	35	43	-.86	34.2	108.5
84	1865555.96	2742214.04	.29279922	35	43	-1.82	34.2	108.5
33	1865849.42	2742409.68	.29284526	35	43	-2.78	34.2	108.5
55	1866156.23	2742616.82	.29289344	35	43	-3.75	34.1	108.5
70	1866476.16	2742834.14	.29294365	35	43	-4.96	34.1	108.5

RMS	CENTRAL ANGLE	SCALING RADIUS	MIN. CENTRAL ANGLE
.0050	.29231279	6371451.1661	.29231294

S.7 Orbital Mode Satellite Position. The orbital mode satellite position data listing (two sheets) was obtained during the orbital prediction pass (program GSORB). The following quantities were listed:

Sheet 1:

SAMPL	Cumulative count of samples
H, M, S, MS	Time in hours, minutes, seconds, and milliseconds
LATITUDE	Predicted latitude of the satellite
LONGITUDE	Predicted west longitude of the satellite
HEIGHT	Predicted height of the satellite above the spheroid
RC	Range from the predicted satellite point to the unknown station
AZ	Azimuth of the predicted satellite point with respect to the unknown station
EL	Elevation of the predicted satellite point with respect to the unknown station
RDC	Predicted range rate at the unknown station

Sheet 2:

SAMPL	Cumulative count of samples
H, M, S, MS	Same as sheet 1
RM	Measured range from the unknown station in meters corrected for ionospheric effects, tropospheric refraction, transit time, and calibration
RM - RC	Difference between the measured and predicted ranges
RDM	Measured range rate at the unknown station in meters per second

RDM - RDC	Difference between the measured and the predicted range rates
CORT	Transit time correction
IC	Measured ionospheric correction (correction is subtracted from the range)
CORI	Ionospheric correction from the analytic model in meters (correction is subtracted from the range)
COR	Tropospheric refraction correction (correction is subtracted from the range)

A

UNKNOWN STATION--

GEODETIC SECOR--ORBITAL MODE
GRAND FORKS I

GRAND FOR

SAMPL	H	M	S	MS	LATITUDE	LONGITUDE	HEIGHT
1	0	44	32	9	39.13185	90.74366	917154.34
2	0	44	32	510	39.15804	90.72917	917161.03
3	0	44	33	10	39.18417	90.71470	917167.71
4	0	44	33	510	39.21030	90.70022	917174.40
5	0	44	34	11	39.23648	90.68570	917181.11
6	0	44	34	511	39.26260	90.67119	917187.80
7	0	44	35	11	39.28872	90.65667	917194.50
8	0	44	35	511	39.31484	90.64214	917201.21
9	0	44	36	11	39.34096	90.62760	917207.92
10	0	44	36	511	39.36707	90.61304	917214.63
11	0	44	37	10	39.39313	90.59850	917221.33
12	0	44	37	511	39.41929	90.58389	917228.07
13	0	44	38	11	39.44540	90.56930	917234.79
14	0	44	38	510	39.47145	90.55472	917241.51
15	0	44	39	10	39.49755	90.54011	917248.24
16	0	44	39	510	39.52365	90.52548	917254.98
17	0	44	40	10	39.54974	90.51083	917261.72
18	0	44	40	510	39.57584	90.49618	917268.47
19	0	44	41	10	39.60193	90.48151	917275.22
20	0	44	41	510	39.62802	90.46683	917281.98
21	0	44	42	10	39.65410	90.45214	917288.74
22	0	44	42	511	39.68024	90.43740	917295.51
23	0	44	43	11	39.70632	90.42268	917302.28
24	0	44	43	511	39.73240	90.40795	917309.05
25	0	44	44	11	39.75847	90.39321	917315.83
26	0	44	44	511	39.78455	90.37845	917322.60
27	0	44	45	11	39.81062	90.36368	917329.39
28	0	44	45	511	39.83669	90.34890	917336.18
29	0	44	46	11	39.86275	90.33411	917342.97
30	0	44	46	511	39.88881	90.31930	917349.76
31	0	44	47	12	39.91493	90.30445	917356.58
32	0	44	47	512	39.94098	90.28962	917363.38
33	0	44	48	12	39.96704	90.27477	917370.19
34	0	44	48	511	39.99304	90.25994	917376.98
35	0	44	49	11	40.01909	90.24507	917383.80
36	0	44	49	510	40.04508	90.23022	917390.61
37	0	44	50	10	40.07113	90.21532	917397.43
38	0	44	50	510	40.09717	90.20041	917404.26
39	0	44	51	10	40.12321	90.18549	917411.09
40	0	44	51	510	40.14925	90.17055	917417.92
41	0	44	52	10	40.17528	90.15560	917424.76
42	0	44	52	510	40.20131	90.14064	917431.61
43	0	44	53	10	40.22734	90.12566	917438.46
44	0	44	53	510	40.25337	90.11068	917445.31
45	0	44	54	10	40.27939	90.09567	917452.16
46	0	44	54	510	40.30541	90.08066	917459.02
47	0	44	55	10	40.33143	90.06563	917465.88
48	0	44	55	510	40.35745	90.05059	917472.75
49	0	44	56	10	40.38346	90.03554	917479.62
50	0	44	56	510	40.40947	90.02047	917486.50
51	0	44	57	10	40.43548	90.00539	917493.38
52	0	44	57	510	40.46148	89.99030	917500.26
53	0	44	58	10	40.48748	89.97519	917507.15
54	0	44	58	510	40.51348	89.96007	917514.04
55	0	44	59	10	40.53948	89.94494	917520.93

GRAND FORKS ORBIT B 504

PAGE 1

HEIGHT	RC	AZ	EL	RDC
917154.34	1497917.63	150.67	32.59	-3532.92
917161.03	1496150.41	150.54	32.65	-3521.84
917167.71	1494392.27	150.42	32.71	-3510.74
917174.40	1492639.68	150.30	32.78	-3499.60
917181.11	1490889.19	150.17	32.84	-3488.39
917187.80	1489147.80	150.05	32.90	-3477.17
917194.50	1487412.03	149.92	32.96	-3465.91
917201.21	1485681.89	149.80	33.03	-3454.61
917207.92	1483957.42	149.67	33.09	-3443.27
917214.63	1482238.63	149.55	33.15	-3431.88
917221.33	1480528.97	149.42	33.21	-3420.48
917228.07	1478818.18	149.29	33.28	-3409.00
917234.79	1477116.55	149.16	33.34	-3397.49
917241.51	1475424.08	149.03	33.40	-3385.97
917248.24	1473733.99	148.91	33.46	-3374.39
917254.98	1472049.70	148.78	33.52	-3362.76
917261.72	1470371.23	148.65	33.58	-3351.10
917268.47	1468698.61	148.52	33.65	-3339.39
917275.22	1467031.85	148.39	33.71	-3327.64
917281.98	1465370.98	148.25	33.77	-3315.85
917288.74	1463716.01	148.12	33.83	-3304.02
917295.51	1462063.67	147.99	33.89	-3292.12
917302.28	1460420.58	147.86	33.95	-3280.21
917309.05	1458783.46	147.72	34.02	-3268.26
917315.83	1457152.33	147.59	34.08	-3256.26
917322.60	1455527.21	147.46	34.14	-3244.23
917329.39	1453908.11	147.32	34.20	-3232.15
917336.18	1452295.06	147.19	34.26	-3220.03
917342.97	1450688.08	147.05	34.32	-3207.87
917349.76	1449087.19	146.91	34.38	-3195.67
917356.58	1447489.23	146.78	34.44	-3183.41
917363.38	1445900.60	146.64	34.51	-3171.12
917370.19	1444318.11	146.50	34.57	-3158.80
917376.98	1442744.85	146.36	34.63	-3146.46
917383.80	1441174.82	146.22	34.69	-3134.05
917390.61	1439614.03	146.09	34.75	-3121.63
917397.43	1438056.33	145.95	34.81	-3109.14
917404.26	1436504.89	145.80	34.87	-3096.61
917411.09	1434959.73	145.66	34.93	-3084.04
917417.92	1433420.86	145.52	34.99	-3071.42
917424.76	1431888.32	145.38	35.05	-3058.77
917431.61	1430362.10	145.24	35.11	-3046.07
917438.46	1428842.25	145.10	35.17	-3033.33
917445.31	1427328.78	144.95	35.23	-3020.55
917452.16	1425821.71	144.81	35.29	-3007.73
917459.02	1424321.05	144.66	35.35	-2994.87
917465.88	1422826.84	144.52	35.41	-2981.97
917472.75	1421339.09	144.37	35.47	-2969.02
917479.62	1419857.83	144.23	35.53	-2956.03
917486.50	1418383.07	144.08	35.59	-2943.01
917493.38	1416914.83	143.93	35.65	-2929.94
917500.26	1415453.14	143.78	35.71	-2916.83
917507.15	1413998.01	143.64	35.77	-2903.67
917514.04	1412549.47	143.49	35.82	-2890.48
917520.93	1411107.54	143.34	35.88	-2877.25

A

UNKNOWN STATION--

GEODETIC SECOR--ORBITAL MODE
GRAND FORKS I

GRAND FORKS

SAMPL	H	M	S	MS	RM	RM-RC	RDM	RDM-RDC
1	0	44	32	9	1497920.13	2.50	-3536.04	-3.12
2	0	44	32	510	1496154.56	4.15	-3524.40	-2.57
3	0	44	33	10	1494395.42	3.16	-3512.03	-1.30
4	0	44	33	510	1492642.91	3.23	-3500.01	-.41
5	0	44	34	11	1490896.20	7.01	-3488.73	-.34
6	0	44	34	511	1489154.53	6.73	-3478.18	-1.01
7	0	44	35	11	1487418.01	5.98	-3466.71	-.80
8	0	44	35	511	1485687.51	5.62	-3454.80	-.19
9	0	44	36	11	1483963.45	6.03	-3442.89	.37
10	0	44	36	511	1482245.26	6.63	-3431.55	.34
11	0	44	37	10	1480532.17	3.21	-3420.43	.05
12	0	44	37	511	1478824.57	6.39	-3408.56	.44
13	0	44	38	11	1477123.43	6.88	-3396.21	1.29
14	0	44	38	510	1475428.87	4.79	-3384.47	1.50
15	0	44	39	10	1473739.87	5.89	-3373.45	.94
16	0	44	39	510	1472055.78	6.09	-3363.04	-.28
17	0	44	40	10	1470376.66	5.43	-3352.37	-1.28
18	0	44	40	510	1468703.40	4.79	-3340.94	-1.56
19	0	44	41	10	1467035.76	3.91	-3329.43	-1.79
20	0	44	41	510	1465374.03	3.06	-3317.40	-1.55
21	0	44	42	10	1463718.53	2.53	-3305.52	-1.50
22	0	44	42	511	1462068.79	5.12	-3293.58	-1.45
23	0	44	43	11	1460425.21	4.62	-3281.53	-1.31
24	0	44	43	511	1458787.28	3.80	-3269.48	-1.22
25	0	44	44	11	1457155.79	3.46	-3256.67	-.40
26	0	44	44	511	1455530.75	3.55	-3244.23	-.01
27	0	44	45	11	1453912.02	3.91	-3231.98	.17
28	0	44	45	511	1452299.20	4.14	-3220.38	-.35
29	0	44	46	11	1450691.96	3.87	-3209.20	-1.32
30	0	44	46	511	1449090.17	2.97	-3197.53	-1.86
31	0	44	47	12	1447494.21	4.98	-3185.22	-1.81
32	0	44	47	512	1445904.62	4.02	-3171.54	-.41
33	0	44	48	12	1444322.48	4.37	-3157.55	1.25
34	0	44	48	511	1442747.52	2.56	-3144.06	2.40
35	0	44	49	11	1441179.22	4.40	-3130.98	3.07
36	0	44	49	510	1439616.88	2.86	-3118.97	2.66
37	0	44	50	10	1438060.55	4.21	-3106.94	2.20
38	0	44	50	510	1436510.33	5.44	-3095.31	1.30
39	0	44	51	10	1434965.42	5.69	-3083.68	.35
40	0	44	51	510	1433426.65	5.78	-3071.81	-.39
41	0	44	52	10	1431893.70	5.39	-3060.88	-2.11
42	0	44	52	510	1430366.00	3.90	-3047.79	-1.72
43	0	44	53	10	1428845.03	2.78	-3034.17	-.83
44	0	44	53	510	1427331.69	2.91	-3019.95	.60
45	0	44	54	10	1425826.22	4.51	-3006.08	1.66
46	0	44	54	510	1424326.10	5.05	-2994.39	.48
47	0	44	55	10	1422831.88	5.04	-2982.13	-.16
48	0	44	55	510	1421343.91	4.82	-2969.40	-.38
49	0	44	56	10	1419862.44	4.61	-2956.13	-.10
50	0	44	56	510	1418387.87	4.80	-2942.58	.43
51	0	44	57	10	1416920.27	5.44	-2929.35	.29
52	0	44	57	510	1415458.80	5.47	-2917.20	-.38
53	0	44	58	10	1414003.34	5.33	-2904.84	-1.17
54	0	44	58	510	1412553.77	4.30	-2892.10	-1.62
55	0	44	59	10	1411111.01	3.47	-2878.31	-1.06

RDM	RDM-RDC	CORT	IC	COR1	COR
-3536.04	-3.12	-17.67	11.67	8.36	4.64
-3524.40	-2.57	-17.59	12.33	8.35	4.63
-3512.03	-1.30	-17.51	12.33	8.33	4.62
-3500.01	-.41	-17.43	11.00	8.32	4.62
-3488.73	-.34	-17.35	11.67	8.31	4.61
-3475.18	-1.01	-17.28	11.00	8.30	4.60
-3466.71	-.80	-17.20	11.67	8.29	4.59
-3454.80	-.19	-17.12	11.67	8.27	4.59
-3442.89	.37	-17.04	10.33	8.26	4.58
-3431.55	.34	-16.97	10.33	8.25	4.57
-3420.43	.05	-16.89	10.33	8.24	4.57
-3408.56	.44	-16.81	9.00	8.23	4.56
-3396.21	1.29	-16.73	10.33	8.22	4.55
-3384.47	1.50	-16.66	10.33	8.20	4.54
-3373.45	.94	-16.58	11.00	8.19	4.54
-3363.04	-.28	-16.51	10.33	8.18	4.53
-3352.37	-1.28	-16.44	11.00	8.17	4.52
-3340.94	-1.56	-16.37	10.33	8.16	4.52
-3329.43	-1.79	-16.29	11.67	8.15	4.51
-3317.40	-1.55	-16.22	10.33	8.14	4.50
-3305.52	-1.50	-16.14	11.67	8.12	4.50
-3293.58	-1.45	-16.06	10.33	8.11	4.49
-3281.53	-1.31	-15.99	11.00	8.10	4.48
-3269.48	-1.22	-15.91	10.33	8.09	4.47
-3256.67	-.40	-15.83	10.33	8.08	4.47
-3244.23	-.01	-15.75	10.33	8.07	4.46
-3231.98	.17	-15.68	9.67	8.06	4.45
-3220.38	-.35	-15.60	11.00	8.05	4.45
-3209.20	-1.32	-15.53	8.33	8.04	4.44
-3197.53	-1.86	-15.46	9.00	8.02	4.43
-3185.22	-1.81	-15.38	11.00	8.01	4.43
-3171.54	-.41	-15.30	9.67	8.00	4.42
-3157.55	1.25	-15.21	9.67	7.99	4.42
-3144.06	2.40	-15.13	10.33	7.98	4.41
-3130.98	3.07	-15.05	10.33	7.97	4.40
-3118.97	2.66	-14.98	11.67	7.96	4.40
-3106.94	2.20	-14.90	10.33	7.95	4.39
-3095.31	1.30	-14.83	11.00	7.94	4.38
-3083.68	.35	-14.76	9.67	7.93	4.38
-3071.81	-.39	-14.69	11.67	7.92	4.37
-3060.88	-2.11	-14.62	11.00	7.91	4.36
-3047.79	-1.72	-14.54	10.33	7.90	4.36
-3034.17	-.83	-14.46	11.67	7.89	4.35
-3019.95	.60	-14.38	11.00	7.88	4.35
-3006.08	1.66	-14.30	9.67	7.87	4.34
-2994.39	.48	-14.23	10.33	7.86	4.33
-2982.13	-.16	-14.15	10.33	7.85	4.33
-2969.40	-.38	-14.08	12.33	7.84	4.32
-2956.13	-.10	-14.00	10.33	7.83	4.32
-2942.58	.43	-13.92	9.67	7.82	4.31
-2929.65	.29	-13.85	9.67	7.81	4.30
-2917.20	-.38	-13.77	9.67	7.80	4.30
-2904.84	-1.17	-13.70	10.33	7.79	4.29
-2892.10	-1.62	-13.63	9.67	7.78	4.29
-2878.31	-1.06	-13.55	11.00	7.77	4.28

APPENDIX T

GEODETIC SECOR

DATA PROCESSING COMPUTER PROGRAMS

NO. H5(PROJECT: Geodetic SECOR
TITLE: Geodetic SECOR Data List
CATEGORY: Special IDENTIFICATION: Program EXAMI
CODE: Fortran II CDC - 1604
PROGRAMMER: G.W. Rutherford DATE: Feb., 1963
PURPOSE: To list recorded Geodetic SECOR data including resolved ranges and rates.

USAGE:

1. Calling Sequence: Program LIST
2. Arguments: None
3. Inputs: (a). Control card with 2 three digit integers; the first specifies the number of samples to skip between printouts, and the second is the number of data tapes being used.
   (b). 1 through 99 data tapes in SECOR format (See figure T-1A.)
4. Outputs: Lists-

1 mark	1st difference
Quality mark	Very fine channel
Station number	Fine channel
Run number	Coarse channel
Month	Very coarse channel
Day	Extended Range channel
Hour (24 hr. clock)	D1-IC
Minutes	R - D2
Seconds	R - D3
Milliseconds	R - D4
Range (meters)	

5. Routines Called: RESOLVE, FORMAT
6. Linkage: None

METHOD:

The range data are corrected and compiled into unambiguous range words by the FORMAT and RESOLVE subroutines. Only every i^{th} sample is considered where i is the first integer on the control card (i.e., i-1 samples are skipped). When an end of file is encountered, the program will begin a new tape or terminate, depending on whether or not the second control integer has been satisfied.

REMARKS:

For continuous lists of more than one tape, the tapes must be mounted on successive units always beginning with unit 1.
An alternate output format replaces R-D2, R-D3 and R-D4 by the difference between the overlap bits.

A

TAPE INPUT RECORD

C	P	MS_0	IC_{-2}	X_0	R_{-2}	$D1_{-2}$
C	P	MS_1	IC_{-1}	X_1	R_{-1}	$D1_{-1}$
C	P	MS_2	IC_0	$D2_0$	R_0	$D1_0$
C	P	MS_3	IC_1	$D2_1$	R_1	$D1_1$
C	P	MS_4	IC_2	$D2_2$	R_2	$D1_2$
C	P	MS_5	IC_3	$D2_3$	R_3	$D1_3$
C	P	MS_6	IC_4	$D2_4$	R_4	$D1_4$
C	P	MS_7	IC_5	$D2_5$	R_5	$D1_5$
C	P	MS_8	IC_6	$D2_6$	R_6	$D1_6$
C	P	MS_9	IC_7	$D2_7$	R_7	$D1_7$
C	P	S_0	$D4_0$	$D3_0$	ER_0	X_2
C	P	S_1	$D4_1$	$D3_1$	ER_1	X_3

B

COMMON IRNT(27)

OUTPUT FORMAT

		(47)		7	6	5	4	3	2	1	0
1	Q										0
2	STATION								2	1	0
3	RUN								2	1	0
4	MONTH							3	2	1	0
5	DAY					5	4	3	2	1	0
6	HOUR						4	3	2	1	0
7	MINUTES					5	4	3	2	1	0
8	SECONDS					5	4	3	2	1	0
9	MSEC	9	8	7	6	5	4	3	2	1	0
10	RANGE	7	6	5	4	3	2	1	0	-1	-2
11	BLANK										
12	VF			7	6	5	4	3	2	1	0

B

C	P	S_1	$D4_1$	$D3_1$	ER_1	X_3	12	VF			[illegible]	6	5	4	3	2	1	0
C	P	S_2	$D4_2$	$D3_2$	ER_2	X_4	13	FN	BITS 8 THRU 11									
C	P	S_3	$D4_3$	$D3_3$	ER_3	X_5	14	CS	BITS 12 THRU 15									
C	P	S_4	$D4_4$	$D3_4$	ER_4	X_6	15	VC	BITS 16 THRU 18									
C	P	S_5	$D4_5$	$D3_5$	ER_5	X_7	16	ER	BITS 19 THRU 22									
C	P	M_0	$D4_6$	$D3_6$	ER_6	X_8	17	IC - D1			7	6	5	4	3	2	1	0
C	P	M_1	$D4_7$	$D3_7$	ER_7	S	18	R - D2			7	6	5	4	3	2	1	0
C	P	M_2	MO_3	MO_2	MO_1	MO_0	19	R - D3			7	6	5	4	3	2	1	0
C	P	M_3	ST_2	ST_1	ST_0	T	20	R - D4			7	6	5	4	3	2	1	0
C	P	M_4	RU_2	RU_1	RU_0	Q	21	REF.	7	6	5	4	3	2	1	0	-1	-2
C	P	M_5	DA_4	DA_3	DA_2	DA_1	22	D1	7	6	5	4	3	2	1	0	-1	-2
C	P	HR_0	DA_8	DA_7	DA_6	DA_5	23	D2			7	6	5	4	3	2	1	0
C	P	HR_1					24	D3			7	6	5	4	3	2	1	0
C	P	HR_2					25	D4			7	6	5	4	3	2	1	0
C	P	HR_3					26	IC	7	6	5	4	3	2	1	0	-1	-2
C	P	HR_4					27											

Figure 1-1. [illegible]COR Data Input Record and Output Format

NO. G130 PROJECT: Geodetic SECOR

TITLE: Geodetic SECOR Range Resolution

CATEGORY: Special IDENTIFICATION: Subroutine RESOLVE

CODE: CODAP CDC - 1604

PROGRAMMER: G.W. Rutherford DATE: Feb., 1963

PURPOSE: To compile the unambiguous range word from recorded SECOR data.

USAGE:

1. Calling Sequence: CALL RESOLVE
2. Arguments: None
3. Inputs: Data stored in IRNT in the format output by the Geodetic SECOR Format program, Figure T-1B with sectors 10 through 16 are blank.
4. Outputs: See figure T-1B
5. Routines Called: None
6. Linkage: COMMON A, B, C, TN(7), IRNT (27), TMP(9)

METHOD:

Compilation of the final range word is effected by combining the corrected channel data recorded on the SECOR magnetic tape. The method used is as follows:

1. D_1 is subtracted from the reference $(R-D_1=VF)$ to obtain the very fine data.
2. D_1 is subtracted from D_2 $(D_2-D_1)=FN)$ to obtain the fine uncorrected data.
3. The 4 least significant bits (LSB) of the fine uncorrected data are subtracted from the 4 most significant bits (MSB) of the very fine data, the MSB of the difference is repeated for 4 more bits to make an 8-bit word which is called the fine difference (FND).
4. The fine difference is then added to the fine uncorrected to form the fine corrected data.
5. D_1 is subtracted from D_3 to obtain the coarse uncorrected data.
6. The coarse uncorrected data is corrected using the fine corrected data in the same manner as described in steps 3 and 4.
7. D_4 is subtracted from D_2 to obtain the very coarse uncorrected data.
8. The very coarse uncorrected data is corrected using the coarse corrected data in the same manner as in steps 3 and 4 except that there are 5 bits overlap.
9. The extended range data is corrected by using the very coarse corrected data in the same manner as in steps 3 and 4. (Note that the extended range word needs no translation.)

The correction process can correct for a difference in the overlapping bits of plus $\left(\frac{2^n}{2}\right)$ bits, and minus $\left(\frac{2^n}{2} - 1\right)$ bits where n is the number of overlapping bits. The most significant bit of the difference between words to be corrected is repeated to form an 8-bit word for the following reason:

When a 1 appears as the most significant bit of the difference, it indicates that the subtrahend is larger than the minuend, and therefore, should be subtracted from the subtrahend (which is the word being corrected). By repeating this 1 to complete an 8-bit word, we have the complement of the number which should be subtracted from the subtrahend to give the corrected word. However,

by adding the complement to the subtrahend, we have performed an effective subtraction.

After all the corrections have been made on the data words, the composite 25-bit range word is made up of the following: the

4 most significant bits of ER
3 most significant bits of VC
4 most significant bits of CS
4 most significant bits of FN
10 bits of the VF.

(NOTE: Refer to appendix B of this report for additional information on range resolution.)

NO. G120 PROJECT: Geodetic SECOR
TITLE: Geodetic SECOR Format Conversion
CATEGORY: Special IDENTIFICATION: Subroutine FORMAT
CODE: CODAP CDC - 1604
PROGRAMMER: G. W. Rutherford DATE: Feb., 1963
PURPOSE: To rearrange input SECOR data into a meaningful format for processing.

USAGE:

1. Calling Sequence: CALL FORMAT
2. Arguments: None
3. Inputs: One Geodetic SECOR data record stored in COMMON IN(7) See figure T-1A for input tape format.
4. Outputs: Formatted data record stored in COMMON IRNT(27) as shown in figure T-1 except that locations 10 through 16 are left blank to be completed by the range resolution program.
5. Routines Called: None
6. Linkage: COMMON A, B, C, N(7), IRNT(27), TMP(9)

METHOD:

This program is simply a series of mask and shift operations to group the data bits recorded on each information channel into a binary word occupying a unique memory cell. These words are stored in successive locations except for seven blank locations which allow for the insertion of a resolved range word and its components.

REMARKS

This program is usually used in conjunction with the RESOLVE program which resolves and inserts the range word into the list.

NO: G200 PROJECT: Geodetic SECOR
TITLE: Geodetic SECOR Editing and Smoothing
CATEGORY: Data Processing IDENTIFICATION: Program PASS 2
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To input a raw SECOR tape from one station and produce an output tape with the raw data plus edited and smoothed data.
USAGE:

1. Card Input
   (1) Title Card - 80 columns 10A8

   (2) Indicator Card: 16I5
   1. Logical unit for input tape
   2. Logical unit for output tape
   3. Block length
   4. IC Switch: 1 = apply IC, 2 = don't apply IC
   5. Edit and Smooth switch: 1 = yes, 2 = no
   6. Edit span length, starting span length
   7. Noise gate
   8. Maximum average second difference
   9. Maximum number of successive bad samples
   10. Smoothing output point
   11. Overlap, smoothing span
   12. Degree polynomial for filter
   13. List 1 option: 1 = yes, 2 = no
   14. Not used
   15. Not used
   16. Not used

   (3) Time Card 12I5
   First time (H, M, S, MS), Last time (H, M, S, MS), Delta time (H, M, S, MS).

   (4) Calibration Card 2(E17.10,3X)
   Range calibration
   IC calibration

   Multiple time spans may be processed by repeating cards (1) through (4).

2. Magnetic Tape Assignment

   The logical units are assigned according to indicator 1 and indicator 2. Generally 1 is the input unit and 2 the output unit.

3. Printout
   a. Input Data
   The input data is listed once for reference.
   b. List 1
   This is a listing of the unprocessed data which may be deleted (and generally is) by use of indicator 13. The format is identical to that of EXAM1.
   c. List 2
   This is a listing of the processed data. The following is a brief description of the various columns:

(1) Time - the time recorded at the station is listed in hours, minutes, seconds, and milliseconds.

(2) Raw Range - the raw range from the input tape.

(3) Edited Range - the range output by the editing process plus the input range calibration.

(4) Smoothed Range - the range output from the smoothing filter.

(5) Residual - the difference between the edited range (input to smoothing filter) and the smoothed range.

(6) Edit Correction - the correction applied to raw range to obtain edited range (minus calibration). With one exception, the edit correction will be an integral multiple of the least significant ambiguity (i.e., 256 meters). If a data sample is "bad" (i.e., cannot be reduced within the noise tolerance by using an integral number of ambiguities) then the edit correction is indicated with a "9.0." In this case the edited range will be either a predicted value or the measured value depending upon the number of successive bad samples which have occurred.

(7) Edited First Difference - the difference between the edited ranges $(\Delta R_E)_i = (R_E)_{i+1} - (R_E)_i$

(8) Smoothed First Difference - the difference between the smoothed ranges $(\Delta R_S)_i = (R_S)_{i+1} - (R_S)_i$

(9) Range Rate - the time derivation of the smooth range data. The units are meters/sec.

(10) Range Acceleration - the second time derivative of the smooth range data. The units are meters/sec^2.

(11) Measured Ionospheric Correction - the ionospheric correction derived from VFIC and VF including the IC calibration.

(12) Quality Indicator -
- = No correction
A = Ambiguous sample
B = Bad sample

d. Tabulation at End of Block

At the end of each block three values are printed; (a) the number of bad samples, (b) the algebraic sum of the integral number of ambiguities used to correct the data of the block, (c) the RMS error for the block.

e. Notes Regarding Listing

(1) Because of the overlapping of data in the editing and smoothing process, the overlap portion will be processed twice. Because of this, the first samples of each block will only indicate the results of the second processing.

(2) The values indicated in d. are not for the previous block but start beyond the overlap and continue into the overlap of the next block.

4. Magnetic Tape Output Format

1 Quality mark
2 Station number
3 Run number
4 Month
5 Day
6 Hour
7 Minute
8 Second
9 Millisecond
10 Raw Range
11 Raw first difference
12 VF
13 FN
14 CS
15 VC
16 ER
17 D1-IC
18 R-D2
19 R-D3
20 R-D4
21 Reference
22 D1
23 D2
24 D3
25 D4
26 IC
27 Edited range
28 Edited range plus calibration and IC
29 Raw first difference from edit routine
30 Edited first difference
31 Edit correction
32 Smoothed range
33 Smooth first difference
34 Residual
35 Range rate
36 Range acceleration
37 IC correction (including calibration)
38
|
|
|
50

The output tape is in 1604 FORTRAN 63 format with all words expressed in floating point format.

REMARKS:

1. In processing multiple time spares, a time span of (overlap) x Δt should be left between spans.

DCW/kkb

NO: G205 PROJECT: Geodetic SECOR
TITLE: Block Input for PASS 2
CATEGORY: Special IDENTIFICATION: Subroutine BKINP
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To input a block of data for processing by PASS 2.
USAGE:

1. Calling Sequence

CALL BKINP (TF, TL, DTM, NIN, NUM, KSTART, NUMIN, DATA, TML)

2. Parameters

TF	first time (seconds)
TL	last time (seconds)
DTM	time increment
NIN	logical input unit
NUM	number of samples in block
KSTART	starting sample number
NUMIN	number of samples input
DATA	storage array (50)
TML	actual last time input

3. Common Linkage

COMMON A, B, C, IN(7), IRNT(27), TMP(9)

REMARKS:

DCW:kkb

No. G210 PROJECT: Geodetic SECOR
TITLE: Compute IC Correction for PASS 2
CATEGORY: Special Purpose IDENTIFICATION: Subroutine CORIC
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To use the R-Dl channel, the IC calibration to compute the measured IC correction.

USAGE: 1. Calling Sequence

Call CORIC (DATA, CAL)

DATA A 50x1 array containing the PASS 2 data.
CAL The IC calibration

2. Common Linkage

COMMON A, B, C, IN(7), IRNT(27), TMP(9)

REMARKS:

DCW/wj

No. G215 PROJECT: Geodetic SECOR
TITLE: First Listing for PASS 2
CATEGORY: Special Purpose IDENTIFICATION: Subroutine LIST1
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To provide the option 1 listing for PASS 2 (i.e. edit & smooth data)

USAGE: 1. Calling Sequence

Call LIST1(NPR, KSTART, DATA, TITLE)

NPR - No. samples to print

KSTART - Starting sample in DATA block

DATA - A 50x array containing data to be listed.

TITLE - A 10x1 array containing the page title.

DCW/wj

No. G220
TITLE: Second Listing for PASS 2
CATEGORY: Special Purpose
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To provide the option 2 listing for PASS 2 (i.e. raw data list).

PROJECT: Geodetic SECOR
IDENTIFICATION: Subroutine LIST2

USAGE: 1. Calling Sequence

Call LIST2(NPR, KSTART, DATA, TITLE)

NPR - No. samples to print.

KSTART - Starting sample in DATA block

DATA - A 50x array containing data to be listed

TITLE - A 10x1 array containing the page title.

REMARKS:

DCW/wj

NO. G225 PROJECT: Geodetic SECOR, SHIRAN
TITLE: Data Editing Subroutine
CATEGORY: Data Reduction IDENTIFICATION: Subroutine EDITSR
CODE: Fortran II CDC - 1604
PROGRAMMER: Dennis Wilson DATE: July 17, 1963
PURPOSE: To edit a block of data stored in memory.
USAGE:

1. Calling Sequence: CALL EDITSR (ARRAY, LEDIT, LOUT, NSAMP, NSPAN, AMB, SNOISE, DEL2MX, BADMAX, NBAD, NAMB, LAKE)
2. Arguments:
   - ARRAY - A two dimensional array, the first subscript specifying the word within a sample and the second subscript being the sample number. Maximum array size (48,100)
   - LEDIT - An integer specifying the location within a sample of the data to be edited.
   - LOUT - An integer specifying the first location within a sample where the results of the editing process will be stored.
   - NSAMP - An integer giving the number of samples in ARRAY to be edited.
   - NSPAN - An integer giving the number of samples to be used: (1) to determine a "good" starting point; (2) to predict a new first difference
   - AMP - A floating point number giving the value of the least significant ambiguity (eg. 256.0 meters in the case of Geodetic SECOR).
   - SNOISE - The noise gate; that is, the maximum noise value to be tolerated by the program.
   - DEL2MX - The maximum average second difference within SPAN for which editing will begin.
   - BADMAX - The maximum number of successive bad samples which will be tolerated.
   - NBAD - The number of "bad" samples encountered
   - NAMB - An algebraic sum of the number of ambiguities edited
   - LAKE - The number of the first sample
3. Inputs: ARRAY, LEDIT, LOUT, NSAMP, NSPAN, AMB, SNOISE, DEL2MX, BADMAX, LAKE
4. Outputs: ARRAY, NBAD, NAMB
5. Routines Called: EDIT, CCNSMP

METHOD:

1. The first NSAMP data samples are used to determine an average second difference. This average second difference is compared with DEL2MX. If it exceeds it, the span is then shifted one sample. The process is continued until NSAMP "good" samples are found.
2. The NSAMP good data samples are used to predict the first difference for the succeeding sample (an end point prediction is made using a least squares first degree polynomial). The predicted and measured first differences and the noise gate are used to detect ambiguities. One of three cases will result: (a) the sample is good as it stands; (b) there are an integral number of

ambiguities to remove; (c) the sample is extraneous.
3. An extraneous sample will result in the predicted first difference being used subject to the condition that no more than BADMAX successive extraneous samples will be tolerated.
If BADMAX is exceeded, the program will begin again as in Step 1.
4. The corrected data sample will be returned where the raw sample was found. Beginning at LOUT the following information is stored back into ARRAY:

LOUT - Corrected data sample
LOUT+1 - Raw first difference
LOUT+2 - Corrected first difference
LOUT+3 - Correction

In the case where a sample is "bad," the number 9.0 will be recorded as the correction.
5. Two counters are output. The first is the number of "bad" samples encountered and the second is the number of least significant ambiguities corrected. This second counter is algebraic, i.e., an ambiguity added is counted positively and an ambiguity subtracted is counted negatively.

REMARKS:

If more than one block of continuous data is to be edited, prior information may be inserted by overlapping the blocks of data such that the first NSPAN samples of a block have previously been edited.

NO: GL30 PROJECT: SHIRAN

TITLE: Edit One Sample Subroutine

CATEGORY: Data Reduction IDENTIFICATION: Subroutine EDIT

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Dennis Wilson DATE: July 17, 1963

PURPOSE: To use parameters of the CALL statement to:

(1). Remove integral multiple of the least significant ambiguity from the data,

(2). To reject spurious data.

USAGE:

1. Calling Sequence: CALL EDIT (XM, XPR, DELXP, COUNTB, SNOISE, AMB1, DELXM, DELXC, XC, CORR)

2. Arguments.

XM	=	Measured data sample
XPR	-	Previous edited sample
DELXP	-	Predicted first difference
COUNTB	-	Number of successive "bad" samples
SNOISE	-	Noise gate
AMB1	-	Least significant ambiguity
DELXM	-	Measured first difference
DELXC	-	Corrected first difference
XC	-	Corrected data sample
CORR	-	Correction

3. Inputs: XM, XPR, DELXP, COUNTB, SNOISE, AMB1
4. Outputs: COUNTB, DELXM, DELXC, XC, CORR
5. Routines Called: None
6. Linkage: None

METHOD:

The input values are used to edit the data. One of three things will happen:

1. The data will be good and require no correction;
2. The data contains an integral number of ambiguities;
3. The data is extraneous and cannot be edited, in which case COUNTB is incremented, CORR = 9.0, DELXC = DELXP.

REMARKS:

Primarily used with the EDIT5R.

NO. 55 PROJECT: Fishbowl

TITLE: Smoothing

CATEGORY: Utility IDENTIFICATION: Subroutine CCNSMR

CODE: Fortran - 62 CDC - 1604

PROGRAMMER: Terry Yuen DATE: 6-27-63

PURPOSE: To compute smoothing coefficients and smooth input data.

USAGE:

1. Calling Sequence: CALL CCNSMR (N, L, K, DELT, XM, COEF, SMX)
2. Arguments:
   - N - Total number of equally spaced data samples within an input span (N_{mx} = 101)
   - L - The order of the derivative (L_{max} = 2)
   - K - The degree of the polynomials (K_{max} = 3)
   - M - The desired output position of the interval (M = 1, if the left most point of the interval is desired)
   - DELT - The time between each successive data point
   - COEF - Smoothing coefficients
   - AMX - Output point
3. Inputs:
4. Outputs:
5. Routines Called: None
6. Linkage: DIMENSION XM(101), CCEF(101)

METHOD:

REMARKS:

This subroutine is primarily used for smoothing data; if the use requires only smoothing coefficients, subroutine SCR should be used.

TY/kkb

NO. UT002 PROJECT: Utility

TITLE: Least Squares Moving Filter Coefficients Computation

CATEGORY: Filtering IDENTIFICATION: Subroutine SCR

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Terry Yuen DATE: June 1964

PURPOSE: To determine coefficients to be used in a least squares smoothing routine.

USAGE:

1. Calling Sequence: CALL SCR (N, L, K, M, COEF)
2. Arguments:

N	-	Total number of equally spaced data samples within an input span
L	-	The order of the derivative (Lmax=2)
K	-	The degree of the polynomial (Kmax=3)
M	-	The desired output point of the interval (M=1 if it is in the left-most point of the interval).
COEF	-	Coefficients computed by this subroutine used to reduce the varians and to obtain the desired output quantity.

3. Input: N, L, K
4. Output: M, COEF
5. Routines Called:

METHOD: This subroutine computes a set of coefficients which are used for fitting a given function. The number of coefficients depends on the total number of an input span of samples. The number of coefficients is nominally limited to 501 points.

REMARKS: Revision of original routine to accommodate additional coeffici nts. (LBP)

No. 3290 PROJECT: Geodetic SECOR

TITLE: List ES Tape

CATEGORY: Special Purpose IDENTIFICATION: Program EXAM2

CODE: Fortran 63 CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE: To provide a listing of a Geodetic SECOR ES Tape

USAGE: 1. Card Input

(1) Title - 80 columns 10A8

(2) Number of tapes, number to skip 2I5

2. Magnetic Tape Assignment

One ES tape on logical unit 1

3. Printout

(See FA332 - no raw list)

REMARKS:

1. No provision for multiple tapes.

DCJ/wj

NO. G300 PROJECT: Geodetic SECOR
TITLE: Simultaneous Mode Satellite Positioning
CATEGORY: Data Processing IDENTIFICATION: Program PASS 3
CODE: FORTRAN 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To input three or four ES tapes, time synchronize them and generate a satellite position (SP) tape.
USAGE:

1. Card Input
   - (1) Title card 10A8
   - (2) Indicator card 16I5
     1. Input code
     2. Number of tapes to synch
     3. IC correction code
     4. Number of samples to skip
     15. Slope for IC model
     16. Electron density for IC model
   - (3) Time interval
     First time (H, M, S, MS) 12I5
     Last time "
     Delta time "
   - (4-7) Base station locations
     (latitude, longitude, height, name) 3(E17.10, 3X)
   - (8) Range Calibration 4(E17.10, 3X)
   - (9) IC calibration 4(E17.10, 3X)

2. Magnetic Tape Assignment
   Three or four input ES tapes are mounted on logical units 1 through 3 or 4. The output SP tape is mounted on logical unit 5.

3. Printout
   - a. Listing of input cards
   - b. Page 1 - station data
     - (1) Sample number (1-54)
     - (2) Time
       The time from tape 1 is listed unless a time drift on 1 has occurred in which case it is the time from tape 2.
     - (3) Trackers
       The four numbers indicate which of the four tapes were time synched at this point (e.g. 1234, 1230, or 1240, etc.)
     - (4) Range 1
       The range to the satellite from station 1 (name appears in heading) as determined from the input survey and the solution using stations 123.

(5) AZ1

The azimuth of the satellite from station 1.

(6) EL1

The elevation of the satellite from station 1.

(7-15) The information of (4-6) is repeated for the other three stations.

c. Page 2 - satellite position

(1) Sample number (corresponds to that of page 1)

(2) Time (same as page 1)

(3-5) LATITUDE, LONGITUDE, HEIGHT

The latitude, west longitude, and height above the spheroid as determined from the solution using stations 123.

(6-8) XE, YE, ZE

The equatorial coordinates as determined from solution 123.

(9-11) EQ. VELOCITY

The satellite velocity in equatorial coordinates as determined from range and range rate data from stations 123.

d. Page 3 - corrections

(1) Sample number (corresponds to that of page 1)

(2-5) TROPO. REFR. CORR.

The tropospheric refraction range correction determined from the model for stations. (Subtracted from ranges.)

(6-9) MEASURED IC

The measured IC for the four stations (subtracted from ranges.)

(10-13) COMPUTED IC

The IC correction computed from the IC model using input slope and electron density for the four stations (subtracted from ranges).

(14-17) TRANSIT TIME CORR.

The transit time correction for the four stations (added to ranges).

e. Page 4 - permuted satellite position

(Only applies and is printed if four tapes are synced)

(1) Sample number (corresponds to that of page 1)

(2-4) LSSQ OF PERMUTED SOLUTIONS

The average latitude, west longitude, and height using the four combinations of three tracking sites.

(5-16) VARIATION OF PERMUTED SOLUTIONS FROM LSSQ COMBINATION

The difference between the average solution above and each of the four individual solutions is taken and the difference in latitude, longitude, and height is printed.

4 Output Tape Format

1	Time (decimal seconds)	
2	Run	
3	Month	
4	Day	
5	Number of trackers	
6		
7		
8		
9		
10		
11	Station Number	STATION 1
12	Range	
13	Range Rate	
14	Range Acceleration	
15	Smoothing Residual	
16	Measured IC	
17	Range + Tropo + IC + TT	
18		
19		
20		
21		
22		
23		
24		
25	(same format as 11)	STATION 2
26		
27		
28		
29		
30		
31		
32		
33		
34		
35		
36	(same format as 11)	STATION 3
37		
38		
39		
40		

41		
42		
43		
44		
45	(same format as 1)	STATION 4
46		
47		
48		
49		
50		
51	X_E	
52	Y_E	
53	Z_E	
54	$\dot{X}_E$	EQUATORIAL COORDINATES USING THREE TRACKERS
55	$\dot{Y}_E$	
56	$\dot{Z}_E$	
57	$\ddot{X}_E$	
58	$\ddot{Y}_E$	
59	$\ddot{Z}_E$	
.		
.		
.		
70	Latitude	
71	Longitude	STATIONS 123
72	Height	
73	Latitude	
74	Longitude	STATIONS 124
75	Height	
76	Latitude	
77	Longitude	STATIONS 134
78	Height	
79	Latitude	
80	Longitude	STATIONS 234
81	Height	
82	Latitude	
83	Longitude	AVERAGE SOLUTION
84	Height	
.		
.		
.		
100		

REMARKS:

1. The input codes are

   1 - 1,2,3

   2 - 1,2,4

   3 - 1,3,4

   4 - 2,3,4

   5 - 1,2,3,4

2. The units are degrees and meters

3. The unknown station (if any) is number four.

4. Failure of the times on the four (or three) tapes to agree within 20 ms results in a diagnostic (time drift on unit) to be printed

No. G305 PROJECT: Geodetic SECOR
TITLE: Tape Time Sync and Search
CATEGORY: Special Purpose IDENTIFICATION: Subroutine SYNC or Subroutine SEARCH
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: SYNC: To synch three or four ES tapes to the first time.
SEARCH: To search the three or four ES tapes for the next time.

USAGE: 1. Calling Sequence

Call SYNC (NTP, NSKP, TIME, DATAIN, TFOUND, IK, NSYN, TBS)

OR Call SEARCH (NTP, NSKP, TIME, DATAIN, TFOUND, IK, NSYN, TBS)

NTP A code indicating which tapes are to be called
1 = 123   4 = 234
2 = 124   5 = 1234
3 = 134

NSKP No samples to skip

TIME A 3x1 array giving the first, last and delta time. For SEARCH TIME (1) is the desired time.

DATAIN A 50x4 array of input data in SP tape format.

TFOUND The time found on tape 1.

IK A 4x1 array giving the tape units being called.

NSYN Number of tape units signed.

TBS A 4x1 array giving the relative time bias of the four (or three) tapes.

REMARKS: 1. Buffering is overlapped.

DCW/wj

No. G310 | PROJECT: Geodetic SECOR
TITLE: Permuted Solutions
CATEGORY: General Purpose | IDENTIFICATION: Subroutine PERMUT
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To compute from the ranges of four trackers the four possible three-range solutions.

USAGE: 1. Calling Sequence

Call PERMUT (STA, R, RV, RVAV, RES)

STA	A 3x4 array consisting of the latitude, longitude and height of the four trackers.
R	A 4x1 array consisting of the four ranges.
RV	A 3x4 array consisting of the four solutions in latitude, longitude, height.
RVAV	A 3x1 array giving the average solution in latitude, longitude and height.
RES	A 3x4 array giving the difference in latitude, longitude and height of each solution from the average solution.

REMARKS:

1. Units - meters, degrees
2. Longitude is west longitude.
3. The order of the solutions is:

   1, 2, 3
   1, 2, 4
   1, 3, 4
   2, 3, 4

DCW/wj

No. 9315 PROJECT: Geodetic SECOR

TITLE: Computation of Velocity and Acceleration from R_i, $\dot{R}_i$, $\ddot{R}_i$

CATEGORY: General Purpose IDENTIFICATION: Subroutine TSTVA

CODE: Fortran 63 CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE: To use range rate and acceleration from each of three tracking sites to determine velocity and acceleration.

USAGE: 1. Calling Sequence

Call TSTVA (STA, RV, R, RD, RDD, V, A)

STA A 3x3 array consisting of tracker coordinates relative to some cartesian coordinate system.

i.e. $\begin{bmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{bmatrix}$

RV A 3x1 array consisting of the x,y,z coordinates of the vehicle in the same cartesian coordinate system.

R, RD, RDD The range, range rate and range acceleration observed at the tracker.

V, A Two 3x1 arrays giving the velocity and acceleration in the cartesian coordinate system.

REMARKS:

1. The coordinate system and the units are arbitrary.

DCW/wj

NO. C326 PROJECT:

TITLE: Rotation to Tracker Plane System

CATEGORY: Utility IDENTIFICATION: Subroutine ROTATE

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Dennis Wilson DATE: May 18, 1963

PURPOSE: To rotate a Cartesian coordinate system centered at base station one into a new system such that the X,Y plane passes through two other base stations and the rotation is about the X and Y axes only.

USAGE:

1. Calling Sequence: CALL ROTATE (ARRAY1,ARRAY2)
2. Arguments: ARRAY1 - A 3x3 array of the station locations (r1 station one will be (0,0,0)). The first subscript refers to XYZ and the second to the station number.

   ARRAY2 - A 3x3 array of the rotation matrix to rotate from the old to the new system.
3. Input: ARRAY1
4. Output: ARRAY2
5. Routines Called: None

METHOD:

The rotation matrix, T1, is evaluated as follows:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

where: $\tan\beta = \left(\dfrac{Z_2Y_3 - Z_3Y_2}{X_2Y_3 - X_3Y_2} \right)$

and: $\tan\alpha = \left(\dfrac{X_2Z_2 - X_3Z_2}{X_2Y_3 - X_3Y_2} \right) \cos\beta$

REMARKS:

NO. C0110 PROJECT: General

TITLE: Compute Ionospheric Refraction

CATEGORY: Utility IDENTIFICATION: Subroutine IONCR

CODE: Fortran 62 CDC-1604

PROGRAMMER: Fred C. Forbes, Jr. DATE: March, 1964

PURPOSE: To compute the ionospheric refraction correction to ranging using an empirical model of the ionosphere.

USAGE:

1. Calling Sequence: CALL IONCR(R,SIN,DRINO)
2. Arguments:

   R - Slant range in feet

   SIN - Sine of elevation angle

   DRINO - Range correction in feet.
3. Inputs: R,SIN
4. Outputs: DRINO
5. Routines Called: None
6. Linkage: None

METHOD:

REMARKS:

NO. 84 PROJECT: SHIRAN

TITLE:

CATEGORY: IDENTIFICATION: Subroutine RAYGEO

CODE: Fort[illegible] - II CDC - 1604

PROGRAMMER: [illegible]art Ebert DATE: 3-16-63

PURPOSE:

USAGE:

1. Calling Sequence: CALL RAYGEO (S, H, G, R, GD, X, I)
2. Arguments:
   - S - Actual distance from the station to the device
   - H - Height of the device above the sphere surface
   - G - Height of the station above the sphere surface
   - R - Radius of the sphere
   - GD - Arc length on the sphere between radii extending to the station and the device from the sphere center
   - X - Angle in radians between the same two radii
   - I - Error return with a value of one if S is less than the difference between H and G, or if S is greater than R
3. Inputs: S, H, G, R
4. Outputs: GD, X, I
5. Routines Called: None
6. Linkage: None

METHOD:

The Cos x term is replaced by its series expansion, the 1 term removed and X set equal to Cos x. [illegible] is solved for and the error is used to correct X. This procedure is repeated until the error is less than 1×10^{-9} radians. Four terms in the series are used so that the accuracy decreases for X greater than .25 radians.

REMARKS:

Functional Relationships:

$$S^2 = A^2 + B^2 - 2AB\cos X$$

$$A = R + H$$

$$B = R + G$$

$$GD = R \cdot X$$

NO. G700 PROJECT: Geodetic SECOR

TITLE: Orbital Mode Satellite Position

CATEGORY Data Processing IDENTIFICATION: Program GSORB

CODE: FORTRAN 63 CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE To use input injection vectors to predict the satellite position at times found on the remote site ES tape. An output tape in the format of the simultaneous mode SP tape is produced.

USAGE:

1. Card Input

   (I) Orbital Perturbation data

CO	4(E17.10.3X)
FMD	4(E17.10.3X)
NO. FMASS, EA	I20.2(E17.10.3X)

   (This set of cards is input only once and is generally left attached to the deck)

(1)	Title card	(10A8)
(2)	Times (for ES tape)	
	First time (H, M, S. MS)	
	Last " "	(12I5)
	Delta " "	
(3)	Indicator card	(16I5)

   1. Number of unknown stations (usually 4)
   2. Number of samples to skip
   3. H } tenths of seconds - integration and rectification intervals
   4. DT }
   15. Slope } for IC model
   16. Electron density }

(4)	Station Location Card	3(E17.10, 3X), 4A5
(5)	Injection time (H, M.S, MS)	4I5
(6)	Injection position vector (EQ coord.)	3(E17.10, 3X)
(7)	Injection velocity vector (EQ coord.)	3(E17.10.3X)
(8)	Calibration constants	3(E17.10.3X)
	Range, IC, time (seconds)	

   (For more than one time interval repeat cards 4-8)

2. Magnetic Tape Assignment

   One ES tape for the unknown station on logical unit 1.

   One output tape (PES) on logical unit 2.

3 Printout

   a Listing of the Input Cards

b. Page 1

(1) SAMPL

A cumulative count of samples

(2) Time

(3) Latitude

Predicted latitude of the satellite

(4) Longitude

Predicted west longitude of the satellite

(5) HEIGT

Predicted height of satellite above spheroid.

(6) RC

The range from the predicted satellite point to the unknown station.

(7) AZ

The azimuth of the predicted satellite point with respect to the unknown station.

(8) EL

The elevation o' the predicted satellite point with respect to the unknown station.

(9) RDC

The predicted range rate at the unknown station.

c. Page 2

(1) SAMPL

(2) Time

(3) RM

Measured range from the unknown station in meters corrected for ionospheric effects, tropospheric refraction, transit time, and calibration.

(4) RM-RC

The difference between the measured and predicted ranges.

(5) RDM

The measured range rate at the unknown station in meters/sec

(6) RDM-RDC

The difference between the measured and the predicted range rates.

(7) CORT

The transit time correction

(8) IC

The measured ionospheric correction (correction is subtracted from the range.)

(9) <u>CORI</u>

The ionospheric correction from the analytic model in meters (correction is subtracted from the range).

(10) <u>COR</u>

The tropospheric refraction correction. (Correction is subtracted from the range.)

4. <u>Output Tape Format</u>

See PASS 3 description. Sufficient data is packed in this format to allow use by PASS 4.

REMARKS:

1. Units are degrees, meters, and meters sec.
2. The first time for the ES tape must exceed the injection time.

NO: G510 PROJECT: Geodetic SECOR
TITLE: Inverse Geodetic Problem
CATEGORY: Utility IDENTIFICATION: Subroutine GDSIC
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To use the latitude and longitude for two points on the earth's surface given with respect to some reference spheroid to compute the geodesic, A_{12}, A_{21} where A_{ij} is the azimuth from i to j.

USAGE:

1. Calling Sequence

   CALL GDSIC (XLAT1, XLONG1, XLAT2, XLONG2, A12, A21, SGD)

2. Parameter List

XLAT1, XLONG1	latitude and longitude of point 1 (east longitude, degrees)
XLAT2, XLONG2	latitude and longitude of point 2 (east longitude, degrees)
A12, A21	azimuths (degrees CW from north)
SGD	Geodesic (meters)

REMARKS:

1. The method as outlined in the following reference was used:

   Sodano, E. M., General Non-Iterative Solution of the Inverse and Direct Geodetic Problems; Research and Analysis Division, U.S. Army Engineer (GIMRADA); Ft Belvoir, Virginia; April, 1963.

DCW:kb

NO: G511 | PROJECT: Geodetic SECOR

TITLE: Direct Geodetic Problem

CATEGORY: Utility | IDENTIFICATION: Subroutine DIRECT

CODE: Fortran 63 CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE: To compute the latitude and longitude of a point on the earth's surface with respect to some reference spheroid from the latitude and longitude of the other end point, the geodesic, and the azimuth.

USAGE:

1. Calling Sequence

CALL DIRECT (XLAT1, XLONG1, A12, S, XLAT2, XLONG2, A21)

2. Parameter List

XLAT1, XLONG1	latitude and longitude of point 1 (east longitude, degrees)
A12	Azimuth (CW from north) from point 1 to point 2
S	Geodesic (meters)
XLAT2, XLONG2	latitude and longitude of point 2 (east longitude, degrees)
A21	Azimuth (CW from north) from point 2 to point 1.

REMARKS:

1. The method as outlined in the following reference was used:

Sodano, E. M., General Non-Iterative Solution of the Inverse and Direct Geodetic Problems; Research and Analysis Division, U.S. Army Engineer (GIMRADA); Ft. Belvoir, Virginia; April, 1963.

DCW:kkb

No. G500 PROJECT: Geodetic SECOR
TITLE: Geodetic SECOR Line Crossing
CATEGORY: Data Processing IDENTIFICATION: Program GSLINE
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To determine the geodetic distance between two points using a satellite line crossing.

USAGE: 1. Card Input

(1) Title - 80 columns 10A8
(2) Indicator Card 16I5

IND1 - Span length
IND2 - Station #1
IND3 - Station #2
IND4 - Print intermediate results 1=yes, 2-no
IND5 - no. samples to skip
IND6 - Satellite solution
1 = 1,2,3
2 = 1,2,4
3 = 1,3,4
4 = 2,3,4

(3,4) Station Locations 3(E17.10,3X)
(5) First time (H,M,S,MS), Last time (H,M,S,MS), Delta time (H,M,S,MS) 12I5

2. Magnetic Tape Assignment

One SP tape on logical unit HD1
(Density = 200)

3. Printout

The input cards are listed. The span of data about the minimum geodetic sum is listed. The computed geodesic and measured minimum sum distance are listed.

REMARKS:

1. The satellite solution should be chosen to avoid using both stations in the satellite solution.

2. If no crossing occurs, the output will be meaningless.

DCW/wj

No. G515 PROJECT: Geodetic SECOR

TITLE: Determine Line Distances

CATEGORY: General IDENTIFICATION: Program LINE

CODE: Fortran 63 CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE: To provide a listing of the geodesics between points on earth's surface using the GDSIC subroutine.

USAGE: Card Input

Pairs of station location cards.

Latitude, longitude, height, station name 3(E17.10,3X), 4A5

Printout

Input cards, geodesic, azimuths

REMARKS:

1. Either Clark or international spheroids may be used by changing the subroutine GDSIC.

DCW/wj

NO. 410 PROJECT: SHIRAN

TITLE: Latitude, Longitude, Radians to Meters and Azimuth

CATEGORY: General IDENTIFICATION: Subroutine INVERS

CODE: Fortran 63

PROGRAMMER: Robert Ebert DATE: March 19, 1964

PURPOSE: To compute azimuths and base line given the latitudes and longitudes of base stations.

USAGE:

1. Calling Sequence: CALL INVERS(FE1,FE2,F1,F2,A12,A21,S)
2. Arguments:
   - FE1 - Latitude at site 1 (Eastern)
   - FE2 - Latitude at site 2 (Western)
   - F1 - Longitude at site 1 (Eastern)
   - F2 - Longitude at site 2 (Western)
   - A12 - Azimuth from site 1 to site 2 (South to East)
   - A21 - Azimuth from site 2 to site 1 (South to West)
   - S - Distance between sites
3. Inputs: FE1,FE2,F1,F2
4. Outputs: A12,A21,S
5. Routines Called: None

METHOD:

REMARKS: Units are radians and meters.

NO. G600 PROJECT: Geodetic SECOR
TITLE: Punch Equatorial Coordinates on Cards
CATEGORY: Data Processing IDENTIFICATION: Program SPUNCH
CODE: Fortran 63 CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To read a satellite position tape and punch onto cards and list satellite position in Equatorial coordinates.

USAGE:

1. Card Input
   (1) Number of samples to skip (I5)
   (2) First time (H, M, S, MS), last time (H, M, S, MS), delta time (H, M, S, MS) (12I5)

2. Card Output
   (1) Injection vectors -- time, X_E, Y_E, Z_E 4(E17.10,3X)
   (2) Injection vectors -- $\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$ 3(E17.10,3X)
   (3) Satellite position -- time, X_E, Y_E, Z_E 4(E17.10,3X)

3. Printout Format
   (Same as 2)
   The number of input samples is output at end.

REMARKS:

DCW/kkb

NO: G605 PROJECT: Geodetic SECOR

TITLE: Punch Equatorial Coordinates and Velocity Onto Cards

CATEGORY: Data Processing IDENTIFICATION: Program VUNCH

CODE: Fortran 63, CDC 1604

PROGRAMMER: Dennis Wilson

PURPOSE: To read a satellite position tape and punch onto cards and list satellite position and velocity in equatorial coordinates.

USAGE:

1. Card Input

(1)	Number of samples to skip	(I5)
(2)	First time (H, M, S, MS), last time (H, M, S, MS), delta time (H, M, S, MS)	(12I5)

2. Card Output

(1)	Injection vectors--time, X_E, Y_E, Z_E	4(E17.10,3X)
(2)	Injection vectors-- $\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$	3(E17.10,3X)
(3)	Satellite position -- time, X_E, Y_E, Z_E	4(E17.10,3X)
(4)	Satellite Velocity -- $\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$, time	4(E17.10,3X)

[(3) and (4) repeated to last time]

3. Printout Format

(same as 2)

The number of input samples is output at end.

REMARKS:

DCW:kkb

No. 0619 PROJECT: Geodetic SECOR
TITLE: Compute Gravitational Acceleration
CATEGORY: General Purpose IDENTIFICATION: Subroutine GRAVITY
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To compute the gravitational acceleration at a point above the earth's surface using zonal harmonics. Constants of Y. Kozai are used for the computation.

USAGE: 1. Calling Sequence

CALL GRAVITY (RP, R, AC)

RP - A 3x1 array consisting of the unit vector from the earth's center to the point;

$$\text{i.e.,} \begin{cases} RP(1) = \cos\varphi \cos\lambda \\ RP(2) = \cos\varphi \sin\lambda \\ RP(3) = \sin\varphi \end{cases}$$

R - The range from the center of the earth to the point.

AC - A 3x1 array consisting of the equatorial components of the acceleration.

REMARKS: 1. Units are feet.

2. Two versions of the subroutine:
(1) the total acceleration is computed
(2) the first harmonic (i.e., $1/r^2$) term is deleted so that only the perturbations to a two-body field are retained.

3. The inputs are referenced to the international spheroid.

DCW:kkb

No. 3406 PROJECT: Geodetic SECOR
TITLE: Unknown Station Solution - 3, 3
CATEGORY: Data Processing IDENTIFICATION: Program PASS 432
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To compute the position of an unknown fourth station from **two** spans of satellite position and range data.

USAGE: 1. Card Input

(1)	Title Card - 80 columns	10A8
(2,3)	Time Cards: first time (H,M,S,MS) last time (H,M,S,MS), delta time (H,M,S,MS), Logical tape number (1, 2, 3)	12I5, 15X, I5
(4)	Unknown station location (latitude, longitude, height)	3(E17.10, 3X)

2. Magnetic Tape Assignment

One or two SP tapes assigned to HD1, **and/or** HD2 corresponding to time cards.

3. Printout

The unknown station position is computed with each succeeding set of **two** satellite position. The results of each computation are listed with the deviation from the input position and the average solution. This is preceded by a summary including input data, average deviation from the survey position and RMS error.

REMARKS:

1. The shortest of the three spans controls the number of solutions attempted.

2. Two spans may be taken off one SP tape as long as the earliest span timewise is in the deck first.

DCW/wj

NO. G321 PROJECT: LRSS

TITLE: Two Range and Height Solution

CATEGORY: Utility IDENTIFICATION: Subroutine TWBASEC

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Dennis Wilson DATE: July 22, 1963

PURPOSE: To compute the location of a target, given the ranges from two known trackers and the height of the target.

USAGE:

1. Calling Sequence: CALL TWBASEC(ECB, RAN1,RAN2,H,SIGN,POST)
2. Arguments:
   - ECB - A 3x2 array consisting of the two trackers' geodetic latitude, longitude(west), and height
   - RAN1,
   - RAN2 - The ranges from the two trackers to the target
   - SIGN - The sign associated with the solution
   - POST - A 3x1 array consisting of the target's geodetic latitude longitude (west), and height
3. Inputs: POSB, RAN1,RAN2, H, SIGN
4. Outputs: POST
5. Routines Called: MTXP, MTXT, QUTS, ECGD
6. Linkage: None

METHOD: See LRSS "Program Description Document FADAC," by Autonetics.

REMARKS: All units are meters or degrees.

NO. 415 PROJECT: Geodetic SECOR
TITLE: Compute Two-body Prediction in Inertial Coordinates
CATEGORY: Trajectory IDENTIFICATION: Subroutine TWBVIN
CODE: FORTRAN 63 CDC 1604, 3600
PROGRAMMER: F. C. Forbes, Jr. DATE: 3-31-64
PURPOSE: With the injection vectors R_E V_E and a time interval given, predict the position and velocity vectors based on Keplerian two-body motion at a time increment DT forward of R_E V_E.

USAGE: Calling Sequence: Call TWBVIN (RE, VE, DT, RI, VI)

Inputs: RE(3), VE(3) = position and velocity vectors of a vehicle in freefall expressed in earth centered, equatorial coordinates and in units of feet and feet/second.

DT = time interval in seconds over which the two-body prediction is to be performed.

Outputs: RI(3), VI(3) = predicted position and velocity vectors of a vehicle expressed in a non-rotating, space fixed (i.e., inertial) earth centered coordinate system and in units of feet and feet/second.

Routines Called: None

Linkage: Explicit transfer

METHOD: Kepler's equation of mean motion is iterated to give the change in eccentric anomaly, which in turn is used to compute integration constants and then the desired forward predicted vectors.

REMARKS: The iteration of Kepler's equation is based on a convergence test given by

$$\left|\Delta E_{i+1} - \Delta E_i\right| - K \Rightarrow 0$$

where $K = 0.0000001 * \Delta E_1$

ΔE = change in eccentric anomaly.

A variable K allows control of truncation and maintenance of prediction accuracy. This routine is very accurate and is used primarily in conjunction with ENCKE's method of trajectory prediction.

NO: TPC06 PROJECT: Bluerock

TITLE: Net Perturbations

CATEGORY: Impact Prediction IDENTIFICATION: Subroutine NETPT

CODE: Fortran 62 CDC - 1604

PROGRAMMER: L.Bruce Palmer DATE: June 1964

PURPOSE: To compute acceleration components of vehicle in free-fall, given position and velocity.

USAGE:

1. Calling Sequence: CALL NETPT (RO,VO,AC,KP)
2. Arguments:

RO(3) - Position vector in equatorial coordinates
VO(3) - Velocity vector in equatorial coordinates
AC(3) - Acceleration vector in equatorial coordinates
KP - Code: KP = 1 Add drag and lift effects
KP = 2 Do not add drag and lift effects
FMD - Table of drag coefficients vs. mach. speed
NO - Number of values in FMD array
CO - Fit coefficients used in determining acoustical velocity
FMASS - Mass of vehicle (lbs)
EA - Eff ctive cross-sectional area of vehicle (FT^2)

3. Inputs: RO, VO, KP
4. Outputs: AC
5. RoutinesCalled: GRAVITY, EQGD, ADEN, ACVEL, DRAG
6. Linkage: COMMON/PERT/FMD(20,2),NO,FMASS,EA,CO(4,6)

METHOD:

REMARKS: Units are in feet and seconds.

NO. TP 005 · PROJECT: Bluerock
TITLE: Predict Free-fall Position and Velocity with Perturbations
CATEGORY: Trajectory Prediction IDENTIFICATION: Subroutine TBWPT
CODE: Fortran 62 CDC-1604
PROGRAMMER: Fred C. Forbes, Jr. DATE: January, 1964
PURPOSE: Given injection vectors predict the position and velocity vectors of a vehicle in free-fall at a future time.

USAGE:

1. Calling Sequence: CALL TBWPT(RQ,VQ,CT,DT,RN,VN,KP)
2. Arguments:

RQ(3), VQ(3)	- Vehicle injection vectors in equatorial coordinates
CT	- Time interval between rectifications of acceleration
DT	- Time to be predicted ahead
RN(3), VN(3)	- Vehicle position and velocity vectors at time DT ahead of injection in equatorial coordinates
KP	- Code: KP=1 Lift and drag effects added to acceleration KP=2 Lift and drag effects not added

3. Inputs: RQ,VQ,CT,DT
4. Outputs: RN,VN
5. Routines Called: TWBVIN,NETPT
6. Linkage:

METHOD:

REMARKS: Units are feet and seconds.

No. 2 PROJECT: P-69

TITLE: EARTH'S RADIUS

CATEGORY: [illegible] IDENTIFICATION: Subroutine RADIUS

CODE: Fortran CP [illegible]-1604

PROGRAMMER: Darrel L[illegible] DATE: 8 Nov 1963

PURPOSE: Finds the earth's radius and normal from a geodetic or geocentric latitude.

USAGE:
1. Calling Sequence: CALL RADIUS(XLAT,GDRAD,GCRAD,GDNORM,GCNORM)
2. Arguments:

XLAT	-Latitude in radians
GDRAD	-Earth's radius in meters if latitude is geodetic
GCRAD	-Earth's radius in meters if latitude is geocentric
GDNORM	-Earth's normal in meters if latitude is geodetic
GCNORM	-Earth's normal in meters if latitude is geocentric

3. Inputs: XLAT
4. Outputs: GDRAD, GCRAD, GDNORM, GCNORM
5. Routines used: None

METHOD: $\text{normal} = a/(1-e^2 \text{SIN}^2 \Phi_D)^{1/2}$ $\quad$ $\text{radius} = (b^2 + L^2 e^2 \text{COS}^2 \Phi_D)^{1/2}$

$$\Phi_D = \text{ARCTAN}((a/b)^2 \text{TAN}(\Phi_C))$$

Remarks: Assumes the constants of Clarke's Spheroid of 1866.

No. G400 PROJECT: Geodetic SECOR
TITLE: Unknown Station Solution - 3, 3
CATEGORY: Data Processing IDENTIFICATION: Program PASS 4
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To compute the position of an unknown fourth station from three spans of satellite position and range data.

USAGE: 1. Card Input

(1)	Title Card - 80 columns	10A8
(2,3,4)	Time Cards: first time (H,M,S,MS) last time (H,M,S,MS), delta time (H,M,S,MS), Logical tape number (1, 2, 3)	12I5, 15X, I5
(5)	Unknown station location (latitude, longitude, height)	3(E17.10, 3X)

2. Magnetic Tape Assignment

Two or three SP tapes assigned to HD1, HD2, and/or HD3 corresponding to time cards.

3. Printout

The unknown station position is computed with each succeeding set of three satellite position. The results of each computation are listed with the deviation from the input position and the average solution. This is preceded by a summary including input data, average deviation from the survey position and RMS error.

REMARKS:

1. The shortest of the three spans controls the number of solutions attempted.

2. Two spans may be taken off one SP tape as long as the earliest span timewise is in the deck first.

DCW/wj

NO. 3805 PROJECT: Geodetic SECOR
TITLE: List Packed Geodetic SECOR Tape
CATEGORY: Data Processing IDENTIFICATION: Program EXAMPCK
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To produce a listing of one or more sets of station data from a packed tape.

USAGE: 1. Card Input

(1) Title - 80 columns 10A8

(2) Indicators 16I5

IND1-IND8 Desired data 1=yes, 2=no

IND9- No. samples to skip

2. Magnetic Tape Assignment

One input tape on logical unit one.

3. Printout

The listing is by station with the station name (from the tape) at the top of the page and 55 samples. One such page is output for each station indicated.

REMARKS: 1. Program begins listing with the first sample on the tape and terminates when the end of file is encountered.

DCW:kb

NO. G800 PROJECT: Geodetic SECOR
TITLE: Time Synch and Pack ES Tapes
CATEGORY: Data Processing IDENTIFICATION: Program PACKES
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To input two to eight Geodetic SECOR ES tapes and produce one or more packed output tapes. Data from the stations is time synchronized as it is packed.

USAGE: 1. Card Input

(1) Title - 80 columns 10A8

(2) Indicators 16I5

IND1-IND8 = input tapes used;
2=no, 1=yes
IND9-No. samples to skip
IND10-No. output tapes

(3) Time Card 12I5

First time (H,M,S,MS)
Last time (H,M,S,MS)
Delta time (H,M,S,MS)

(4) - (11) Station Calibration Cards 3(E17.10, 3X), 12X, A8

Range Calibration
IC Calibration
Time Calibration (sec)
Station Name

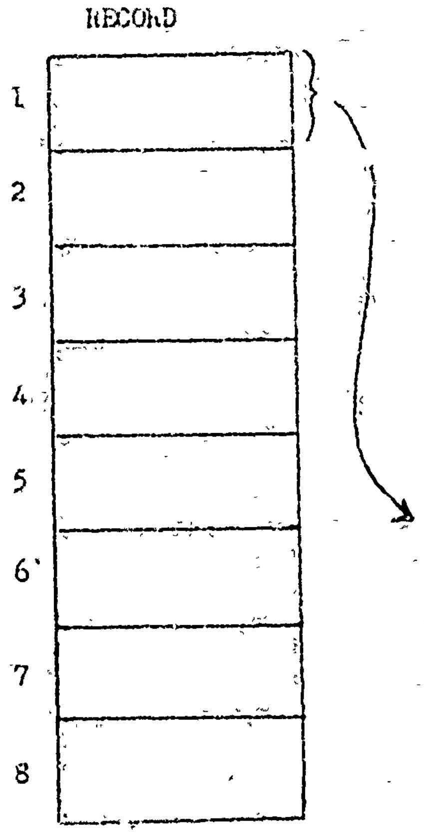
2. Magnetic Tape Assignment

Any logical unit 1-8 may be used for input as indicated on indicator card. The output tapes are mounted starting on logical unit 9.

3. Printout

Input cards are listed along with self-explanatory indications of tape synch.

TAPE FORMAT: The generated output tape record consists of eight blocks of data. Each block is formated identically and consists of data from different input tapes.



1. DATA CODE 1 = data, 2 = no data
2. STATION NAME - 1604 BCD - 8 characters
3. RUN NUMBER
4. MONTH
5. DAY
6. TIME (Decimal Seconds)
7. RANGE - Edited and smoothed
8. RANGE RATE
9. SMOOTHING RESIDUAL
10. MEASURED IC
11. RANGE - Raw
12. CALIBRATION - from edit
13. EDIT CORRECTION
14. RANGE ACCELERATION
15. NOT USED

REMARKS:

1. Data blocks not containing data are zeroed except for the first word which contains a floating point 2.
2. The output tapes are terminated with an end of file.
3. If the first sample is bad, the range calibration will be an error by -9.0.
4. If the IC is not locked, an erroneous IC calibration may result.
5. The maximum number of samples which may be recorded low density ~ 5000 or ~8 mm of data @ 10 samples/sec.

DCW:kkb

NO. C' 113 FCF PROJECT: Fi·.bowl
TITLE: Iteratively Fit Ionospheric Refraction
CATEGORY: General IDENTIFICATION: Program IONITR
CODE: Fortran 62, F63 CDC-1604
PROGRAMMER: Fred C. Forbes, Jr. DATE:
PURPOSE: To iteratively solve for FMAX, BIAS, and CONTROL CONSTANT

USAGE:

1. ...ng Sequence: Program IONITR
2. ...nts:
3. ...

CARDS	DESCRIPTION	FORMAT
(1)	Title	10A8
(2)	Indicator Card:	(3I1,17X,3(E17.10,3X))
	NSOL(1) = Code: NSOL(1)=1 if calibrated	
	NSOL(1)=0 if not calibrated	
	NSOL(2)=1 if FMAX is calibrated	
	NSOL(2)=0 if FMAX is not calibrated	
	NSOL(3)=1 if slope constant	
	NSOL(3)=0 if no slope calibration	
(3)	R,H,DRM,ICONT	(3(E17.10,3X),15X,I5)
	R = Range, in meters	
	H = Height in meters	
	DRM = Ionospheric correction	
	ICONT = Code: ICONT=1 end of data	
	ICONT=0 continue data input	

4. Routines Called: IONO,MTXP,MTXA,MTXI

METHOD: Least squares adjustment of IC_{meas} with calibration, FMAX, and/or slope.

REMARKS:

NO. 0412 PROJECT: General

TITLE: Compute Precise Trajectories and Orbits from fit to Equatorial Coordinates

IDENTIFICATION: Program PCMPTJ

CATEGORY: Trajectory

CODE: Fortran 62 CDC-1604 DATE: 2-1-64

PROGRAMMER: Fred C. Forbes, Jr.

PURPOSE: Compute precise trajectories and orbits with compensation for 2nd, 3rd and 4th zonal harmonics of the earth, atmospheric drag, and lift. Optional initial conditions of time, position and velocity vectors or two positions and apogee height are provided as well as provision for iterative least squares fit (with or without weighting) to the vehicle position coordinates. Equatorial coordinates are listed and topocentric coordinates are computed and listed with respect to any number of sites on the earth's surface. The final orbital parameters are adjusted to input equatorial coordinates.

USAGE:	CARDS	DESCRIPTION	FORMAT
Inputs:	(1)	Title	A 80 Hollerith
	(2)	IOPTN,NO,ITER,KP,KWT,IDTI,IUNITS*	5I3,I5,I3
		IOPTN = initial conditions option 001 - time with two positions and apogee height 002 - time with equatorial position and velocity vectors	
		NO = number of input position vectors to be fit by iterative least squares NO 200	
		ITER = number of iterations of the fit	
		KP = perturbation option indicator 001 - drag and lift 002 - no drag and lift	
		KWT = least squares weighting option 001 - input weighting 002 - no weighting	
		IDTI = maximum interval between rectification in perturbation computation times 10	
Option 1	(3)	TI = time of epoch	E17.10
Option 1	(4)	CO = latitude, longitude, height of epoch	3(E17.10,3X)
Option 1	(5)	CI = latitude, longitude, height of second point on trajectory	3(E17.10,3X)
Option 1	(6)	AL = apogee height (AL is located between CO and CL)	E17.10
Option 2	(3)	TI,RE=time and position vector at epoch where RE is the equatorial coordinates	4E20.10

*IUNITS = units code of input injection vectors(see(3)) for code ID).

NO. 0412 Cont.

Option 2 (4) VE = equatorial velocity components at epoch — 3 E20.10

KP=001 (7)-(16) FMD = table of mach speed versus drag coefficients — 4(E17.10,3x)

KP=001 (17)-(23) table of curve fitting coefficients — 4(17.10,3x)

KP=001 (24) NOT, FMASS, EA — I20,2(E17.10,3x
- NOT = number of coefficients in table
- FMASS = vehicle mass in pounds
- EA = vehicle cross sectional area in ft^2

(25) TM, TRJP = trajectory points for fitting — 4 E20.10
- TM = time in seconds
- TRJP = X_E, Y_E, Z_E of vehicle

(26) Same as 25 until NO (see item 2) are input

KWT=001 (27) COSITE = latitude, longitude, height of local origin for error propagation — 3(E17.10,3x)

KWT=001 (28) IEQU, SITE — I3,X17, 3(E17.10,3x)
- IEQU = equipment selection code
  - 001 = range
  - 002 = height
  - 003 = L direction cosine
  - 004 = M direction cosine
  - 005 = azimuth
  - 006 = elevation
- SITE = latitude, longitude, and height of equipment site

KWT=001 (29) EM, ICONT = error model — 5 E17.8,I5
- EM(1) = equipment error
- EM(2) = tropospheric refraction
- EM(3) = scale factor
- EM(4) = site survey
- EM(5) = ionospheric refraction for range and height equipment
- EM(5) = baseline length for L and M
- ICONT = continuation code
  - ICONT(0)= continue input of EM
  - ICONT(1)= indicates last site input

KWT=001 (30), (31) Same as (28) and (29) for all tracking equipment

(32) TO, DT, TF = times for forward predictions — 3(E17.10,3x)
- TO = first time
- DT = time interval
- TF = final time of prediction ((TF-TO) /DT <1000) per run)

(33) IPUNCH, IUNITS — 2I3
- IPUNCH = punch output code
  - 000 = no punch output
  - 001 = punch output
- IUNITS = output units code
  - 001 = feet
  - 002 = meters
  - 003 = statute miles (5280')
  - 004 = nautical miles (6076.10333')
  - 005 = yards

NO. 0412 Cont.

(36)	TITLE = site name card	A80 Hollerith
(37)	CS = latitude, longitude, height of points on earth to which trajectory points will be referred.	3(E17.10,3x)
(38), (39)	Same as (36) and (37) for all desired sites.	

Output: From Fitting:

All input data are dumped for reference. On successive iterations, the adjusted injection vectors, as well as the difference between each predicted point and the corresponding input data are listed. The difference between the two-body prediction of position and velocity and the total field prediction values are listed. The error propagation (inverse weighting) in the XYZ coordinates are also listed when weighting is called out.

In Equatorial Coordinates:

Point number, hours, minutes, seconds, total seconds, XYZ, $\dot{X}\dot{Y}\dot{Z}$, latitude, longitude, height, and associated titles.

In Topocentric Coordinates:

Point number, hours, minutes, seconds, XYZ, $\dot{X}\dot{Y}\dot{Z}$, in local east-north-up coordinates, slant range, surface range, range rate, azimuth, elevation angle, and height AMSL with titles.

Method: Trajectory predictions are computed by a method based on computing a reference trajectory with closed form two-body equations and then adding numerically integrated perturbation terms.

In the least squares fitting, unity weighting or weighting based on the propagation of major sources of observational error into position variance and covariance, are optional. See program GENEPQ No. 617 FCF, for method of error propagation.

Remarks: All input is expressed in feet, seconds, degrees, and ratios. In the error propagation refraction is input as the ratio of the residual error to the total (mean atmosphere) correction (i.e., 5% residual would be input as 0.05). Scale factor, baseline length, and site survey are ratios. An error in site survey of 1 PNM would be input as 0.000001.

If punch output is desired, the source deck should be modified to give the desired output.

Reference should be made to the program listings for all necessary subroutines and memory usage.

NO. 402 PROJECT: FISHBOWL

TITLE: Predict Two-Body Position, Velocity, Partial Derivatives

CATEGORY: Trajectory IDENTIFICATION: Subroutine TBPVU

CODE: Fortran 62, 63 CDC 1604, 3600

PROGRAMMER: F. C. Forbes, Jr. DATE: 1-10-62

PURPOSE: Predict two-body position and velocity vectors and optionally the (3x6) matrix of two-body partial derivatives with respect to position expressed in units of feet, feet/second, and in an **inertial** coordinate system.

USAGE:

1. Calling Sequence: CALL TBPVS (TO, TM, VEC, PTL, NOTE)
2. Arguments:

   INPUTS: TO = time of epoch or injection in seconds
   TM = time of forward predicted data in seconds

   OUTPUTS: VEC(6) = array of position and velocity components in **inertial** coordinates and in feet and feet/second.

$$PTL(18) = \begin{bmatrix} \frac{\partial X}{\partial X_o} & \frac{\partial X}{\partial Y_o} & \cdots & \frac{\partial X}{\partial \dot{Z}_o} \\ \frac{\partial Y}{\partial X_o} & \frac{\partial Y}{\partial Y_o} & \cdots & \cdot \\ \frac{\partial Z}{\partial X_o} & \cdot & \cdots & \frac{\partial Z}{\partial \dot{Z}_o} \end{bmatrix}$$ = partial derivatives of predicted position vector with respect to injection vectors $R_o V_o$

   NOTE = option indicator
   1 = compute VEC only
   2 = compute VEC and PTL

3. Routines Called: MTXP, MTXT
4. Linkage: Subroutine TRJK or TRJKX
   COMMON/TRJCNTS/CU(3), PO(6), RO, AL, AIS, AAIS, GAMO, EATO, ECC, EO, DBO(6), DAI(6), WE, DGO(6), PTLI(3,6), VECI(6), LMINV(3,3)

METHOD: Iterate Kepler's equation based on the time of prediction to compute the change in the eccentric anomaly. Integration constants are then computed and used to form the predicted quantities.

REMARKS: This routine is used in conjunction with subroutine TRJK or TRJKX, which computes all necessary constants used in TBPVS. TBPVS is used in trajectory fitting, error propagation, and simulation programs.

NO. CO 413 FCF PROJECT: Geodetic SECOR
TITLE: Adjust Range and Velocity to Input Trajectory Points using Precise Trajectory Predictions IDENTIFICATION: Subroutine PTRJFT
CATEGORY: Trajectory Prediction
CODE: Fortran 62 DATE:
PROGRAMMER: Fred C. Forbes, Jr.
PURPOSE: To predict a trajectory path using initial injection vectors.

USAGE:

1. Calling Sequence: CALL PTRJFT(TO,RO,VO,NO,TM,TRJP,ITER,KP,KWT,DTI)
2. Arguments:
   - NO - No. input position vectors to be fit by iterative least squares NO< 200
   - ITER - No. iterations of the fit
   - KP - Perturbation option number: 001 = drag and lift; 002 = no drag and lift
   - KWT - Least squares weighting option: 001 = input weighting; 002 = no weighting
   - TRJP - Latitude, longitude and height above mean sea level of vehicle
   - TO - First time for prediction
   - DTI - Time interval
   - TM - Time of forward predicted data in seconds
   - RO - Adjusted range injection vector
   - VO - Adjusted velocity injection vector
3. Inputs: NO,ITER,KP,KWT,TRJP,TO,DTI,TM
4. Outputs: RO,VO
5. Routines Called: EQCOOR,TBWPT,TBPVS,MTXT,MTXP,GEP,MTXA,INSS

METHOD: Trajectory predictions are computed with closed form two-body equations adding numbrically integrated perturbation terms.

REMARKS: Input is expressed in feet, seconds and degrees.

NO. C0412A

PROJECT: Geodetic SECOR

TITLE: Call in Drag Coefficients

CATEGORY: Trajectory

IDENTIFICATION: Subroutine DRGCOF

CODE: F62, 63

PROGRAMMER: F. C. Forbes, Jr.

DATE:

PURPOSE: To read in drag coefficients

USAGE:

1. Calling Sequence: CALL DRGCOF(KP)
2. Arguments: KP = option indicator
   1 = read in coefficients
   2 = used as a dummy routine
3. Inputs:
4. Outputs:
5. Routines Called: none
6. Linkage: COMMON/PERT/FMD(20, 2), NO, FMASS, EA, CO(4, 6)

METHOD:

REMARKS:

FCF/kkb

NO. G320 PROJECT:

TITLE: Three Range Solution

CATEGORY: [illegible] IDENTIFICATION: Subroutine SOLUT

CODE: Fortran 62 [illegible] - 1604

PROGRAMMER: [illegible] DATE: May 17, 1962

PURPOSE: To compute a target's position relative to some Cartesian frame, given the location of three trackers in this reference frame, the three ranges from the trackers to the target, and the sign to be associated with the z coordinate of the target in the s coordinate system. The z-axis of the s system is normal to a plane passing through the three trackers and directed "up."

USAGE:

1. Calling Sequence: CALL SOLUT (STA, R, SIGN, X, Y, Z)
2. Arguments: STA - A 3x3 array of the base station positions in the arbitrary X,Y,Z system. The first subscript 1,2,3 refers to X,Y,Z and the second subscript indicates the base station number (1,2,3).

   R - A 3x1 array of the ranges from base stations to the target. The subscript refers to the base station number.

   SIGN - Either +1.0 or -1.0 to assign the sign of z in the s system.

   X,Y,Z - The coordinates of the target in the system in which the base station locations were entered.
3. Input: STA, R, SIGN
4. Output: X,Y,Z
5. Routines Called: TARGET, ROTATE

METHOD:

The subroutine uses the TARGET and ROTATE subroutines to determine the target position. The coordinates are translated to base station one and rotated into the s system using the ROTATE subroutine. The solution is then made by the TARGET subroutine and the results transformed back to the original system.

REMARKS:

NO. 3225 PROJECT Geodetic SECOR

TITLE Three Range Solution--Plane System

CATEGORY Utility ID Subroutine TARGET

CODE Fortran 62 CDC 1604

PROGRAMMER Dennis Wilson DATE May 17, 1963

PURPOSE

To compute the position of some target using ranges from three base stations. The results are relative to a plane coordinate system whose x-y plane passes through the three base stations and whose center is at base station one.

USAGE

1. Calling Sequence CALL TARGET(STA,R1,R2,R3,SIGNZ,X,Y,Z)

2. Arguments

STA	A 3x3 matrix of the xyz positions of the three base stations in the plane coordinate system. NOTE: STA(1,1)=STA(2,1)=STA(3,1)= STA(3,2)=STA(3,3)=0
R1,R2,R3	The three ranges
SIGNZ	The sign of the z coordinate
X,Y,Z	The target's position in the plane system

3. Input STA,R1,R2,R3,SIGNZ

4. Output X,Y,Z

METHOD

$$x = K_1 R_1^2 + K_2 R_2^2 + K_3 R_3^2 + K_4$$

$$y = K_5 R_1^2 + K_6 R_2^2 + K_7 R_3^2 + K_8$$

$$z = \pm\left[R_1^2 - x^2 - y^2\right]^{\frac{1}{2}}$$

REMARKS

Program SOLUT uses this subroutine and converts between the local system and plane system

No. 0390 PROJECT: Geodetic SECOR
TITLE: List Satellite Position Tape
CATEGORY: Special Purpose IDENTIFICATION: Program EXAMSP
CODE: Fortran 63, CDC 1604
PROGRAMMER: Dennis Wilson
PURPOSE: To list all or part of the data on a Geodetic SECOR SP tape.

USAGE: 1. <u>Card Input</u>

(1) Title - 80 columns 10A8
(2) Indicator Card

IND1 = no. samples to skip 3I5
IND2 = print option 1 1 = yes
IND3 - print option 2 2 = no

(3) Times - first time (H, M, S, MS), 12I5
last time (H, M, S, MS),
delta time (H, M, S, MS)

2. <u>Magnetic Tape Assignment</u>

One SP tape on logical unit HD1

3. <u>Printout</u>

(1) Input data
(2) Time, satellite position, ...
(3) Option 1: range data from each station
(4) Option 2: permuted solutions

REMARKS:

1. The two print options may be included or not independently.

DCW/wj

NO. 62
TITLE: Quad Test for the Azimuth from North to East
CATEGORY: Utility
IDENTIFICATION: Subroutine QUTS
CODE: Fortran II CDC - 1604
PROGRAMMER: Fred Forbes
DATE: 6-28-63
PROJECT:
PURPOSE:
USAGE:

1. Calling Sequence: CALL QUTS (X, Y, A)
2. Arguments: X - X-component
   Y - Y-component
   A - Radians
3. Inputs: X, Y
4. Output: A
5. Routines Called: None

METHOD:

REMARKS:

NO. U 2 PROJECT:
TITLE: Form Unit Vector
CATEGORY: Utility IDENTIFICATION: Subroutine UTVT
CODE: Fortran II CDC - 1604
PROGRAMMER: Fred Forbes DATE: 2-16-63
PURPOSE:

USAGE:

1. Calling Sequence: CALL UTVT (A, B, I)
2. Arguments: A - Vector

   B - $\frac{A_I}{|A|_I}$

   I - Total elements
3. Input: A, I
4. Output: B
5. Routines Called: None
6. Linkage: DIMENSION A(1), B(1)

METHOD:

NO.: U 3
TITLE: Form the Sum of the Squares
CATEGORY: Utility
CODE: Fortran II CPC - 1604
PROGRAMMER: Fred Forbes
PURPOSE:

PROJECT:

IDENTIFICATION: Subroutine SMSQ

DATE: 2-12-63

USAGE:

1. Calling Sequence: CALL SMSQ (A, B, I)
2. Arguments: A - Vector
   B - $\Sigma (A_I)^2$
   I - Total elements
3. Input: A, I
4. Output: B
5. Routines Called: None
6. Linkage: DIMENSION A(1)

METHOD:

NO. 414

TITLE: Form Scaler Matrix Product

CATEGORY: Utility

CODE: Fortran II CDC - 1604

PROGRAMMER: Fred Forbes

PURPOSE:

PROJECT:

IDENTIFICATION: Subroutine SCMT

DATE: 2-12-63

USAGE:

1. Calling Sequence: CALL SCMT (A, B, C, I)
2. Arguments:
   - A - Scaler
   - B - Matrix
   - C - A*B
   - I = Total elements
3. Input: A, B, I
4. Output: C
5. Routines Called: None

Linkage: DIMENSION B(1), C(1)

METHOD:

NO. U.5 PROJECT:
TITLE: Compact an Array into a Larger Array
CATEGORY: Utility IDENTIFICATION: Subroutine MTXC
CODE: Fortran II CDC - 1604
PROGRAMMER: Fred Forbes DATE: 6-28-63
PURPOSE:

USAGE:

1. Calling Sequence: CALL MTXC (A, B, IA, IB, IC, ID, IE)
2. Arguments: A - Composite array
   B - Segment of array
   IA - Rows in A
   IB - Columns to skip in A
   IC - A column elements to skip to first B element
   ID - Rows in B
   IE - Columns in B
3. Input:
4. Output:
5. Routines Called: None
6. Linkage: DIMENSION: A(1), B(1)

METHOD:

REMARKS:

NO. U 6

TITLE: Magnitude of a Vector

CATEGORY: Utility

CODE: Fortran I, CDC - 1604

PROGRAMMER: Fred Forbes

PURPOSE:

PROJECT:

IDENTIFICATION: Subroutine MGVT

DATE: 6-28-63

USAGE:

1. Calling Sequence: CALL MGVT (A, B, I)
2. Arguments: A - Vector

   B - $/A/_I$

   I - Total elements
3. Input: A, I
4. Output: B
5. Routines Called: None
6. Linkage: DIMENSION A(1)

METHOD:

NO. U 7

TITLE: Matrix Addition

CATEGORY: Utility

CODE: Fortran II CDC - 1604

PROGRAMMER: Fred Forbes

PURPOSE:

PROJECT:

IDENTIFICATION: Subroutine MTXA

DATE: 6-28-63

USAGE:

1. Calling Sequence: CALL MTXA (A, B, C, I)
2. Arguments A - Matrix
   B - Matrix
   C - $A_i + B_i$
   I - Total elements
3. Input: A, B, I
4. Output: C
5. Routines Called: None
6. Linkage: DIMENSION A(1), B(1), C(1)

METHOD:

NO. U 8
TITLE: Matrix Subtraction
CATEGORY: Utility
CODE: Fortran II   CDC - 1604
PROGRAMMER: Fred Forbes
REMARKS:

PROJECT:

IDENTIFICATION: Subroutine MTXS

DATE: 6-28-63

USAGE:

1. Calling Sequence: CALL MTXS (A, B, C, I)
2. Arguments: A - Matrix
   B - Matrix
   C = $A_i - B_i$
   I - Total elements
3. Input: A, B, I
4. Output: C
5. Routines Called: None
6. Linkage: DIMENSION A(1), B(1), C(1)

METHOD:

NO. U 9

TITLE: Matrix Transpose

CATEGORY: Utility

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Fred Forbes

PROJECT:

IDENTIFICATION: Subroutine MTXT

DATE: 28 June 1963

PURPOSE: To transpose a NxM matrix to a MxN matrix.

USAGE:

1. Calling Sequence: CALL MTXT (A, B, N, M)
2. Arguments: A - Matrix (NxM)
   B - Matrix (MxN)
   N - Rows of matrix A, columns of matrix B
   M - Columns of matrix A, rows of matrix B
3. Input: A(NxM)
4. Output: B(MxN)
5. Routines Called: None
6. Linkage: DIMENSION A(1), B(1)

METHOD:

REMARKS:

NO. U 10

PROJECT:

TITLE: Matrix Product

IDENTIFICATION: Subroutine MTXP

CATEGORY: Utility

CODE: Fortran 62 CDC - 1604

PROGRAMMER: G. Rutherford

PURPOSE: To multiply Matrix A by B and store results in C.

USAGE:

1. Calling Sequence: CALL MTXP (A, B, C, N, M, K)
2. Arguments:
   - A - Matrix (NxM)
   - B - Matrix (MxK)
   - C - Matrix (NxK): C = A * B
   - N = Rows in Matrix A, rows in Matrix C
   - M - Columns in Matrix A, rows in Matrix B
   - K - Columns in Matrix B, columns in Matrix C
3. Input: A(NxM), B(MxK)
4. Output: C(NxK)
5. Routines Called:
6. Linkage: DIMENSION A(1), B(1), C(1)

METHOD:

REMARKS:

NO. U 11
TITLE: Vector Multiplication
CATEGORY: Utility
CODE: Fortran 62 CDC - 1604
PROGRAMMER: Dennis Wilson

PROJECT: [illegible]
IDENTIFICATION: Subroutine VECMPY
DATE: November 20, 1963

PURPOSE: To form the vector product of two vectors (cross product).

USAGE:
1. Calling Sequence: CALL VECMPY (R1,R2,R3)
2. Arguments: R1,R2,R3 - 3x1 arrays for the three vectors
3. Input: R1,R2
4. Output: R3
5. Routines Called: None

METHOD: $\vec{R}_3 = \vec{R}_1 \times \vec{R}_2$

REMARKS:

NO. U 12 PROJECT: SHIRAN

TITLE: Matrix Inversion and Linear Solution

CATEGORY: Matrix Operations IDENTIFICATION: Subroutine MATINV

CODE: Fortran 62 CDC - 1604

PROGRAMMER: (From the UCSD Library) DATE: 22 June 1963

PURPOSE: To invert a matrix up to 20x20 and solve the matrix equation if desired.

USAGE:

1. Calling Sequence: CALL MATINV (A,N,B,M,W)
2. Arguments: A_{out} - Equals A_{in} inverted

   N - The size of the matrix

   B_{out} - Equals B_{in} times A_{out}

   M - Control; 0 if B is not to be computed, 1 if it is to be computed

   W - The characteristic determinant
3. Input:
4. Output:
5. Routines Called:

METHOD: A more complete description is available in the University of California at San Diego write-up. A copy of this is available in the Master File of this series.

REMARKS: The A space should be reserved as 920,N), B(20), and W(1).

NO. 113
TITLE: Rotation Matrix
CATEGORY: Utility
CODE: Fortran II CDC - 1604
PROGRAMMER: Dennis Wilson

PROJECT: ODVAR

IDENFICATION: Subroutine ROTMX

DATE: August 13, 1963

PURPOSE: To form a rotation matrix given the angle and the axis about which to rotate.

USAGE:

1. Calling Sequence: CALL ROTMX(I1,I2,I3,ANGRAD, ROT)
2. Arguments: I1,I2,I3 - Indicators: = 0 axis about which rotation is made
   = 1 the other two axes
   ANGRAD - The angle in radians
   ROT - A 3x3 rotation matrix
3. Inputs:
4. Outputs:
5. Routines Called: None
6. Linkage:

METHOD:

<u>Example</u>: To rotate by an angle Alpha about the Y-axis to form matrix A;

CALL ROTMX (1,0,1,ALPHA,A)

$$A = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

REMARKS:

NO. U 14 PROJECT: ODVAR
TITLE: Quadrant Test - Polar Angle
CATEGORY; Utility IDENTIFICATION: Subroutine QUAD
CODE: Fortran II CDC - 1604
PROGRAMMER: Dennis Wilson DATE: August 8, 1963
PURPOSE: To compute the polar angle (CCW from X-axis) from the X and Y components.

USAGE:

1. Calling Sequence: CALL QUAD (X,Y,ANGDEG,ANGRAD)
2. Arguments: X,Y - X and Y components in any system of units
   ANGDEG - Angle in degrees CCW from X-axis
   ANGRAD - Angle in radians CCW from X-axis
3. Inputs:
4. Outputs:
5. Routines Called: None
6. Linkage: None

METHOD: Arctangent function and quadrant test.

REMARKS: X and/or Y may be zero.

NO. U 15 PROJECT: General
TITLE: Compute Equatorial Coordinates and Rotation Matrix
CATEGORY: Geometric IDENTIFICATION: Subroutine EQCOOR
CODE: Fortran 62,63 CDC - 1604
PROGRAMMER: F.C. Forbes, Jr. DATE: 2-6-64

PURPOSE: Compute equatorial coordinates and topocentric to equatorial rotation matrix from the geodetic latitude, longitude, and height above mean sea level.

USAGE:

1. Calling Sequence: CALL EQCOOR (COOR, XYZ, GD, MTFT)
2. Arguments:
   - COOR -3 element array -- geodetic latitude, east longitude in degrees, and height AMSL in feet.
   - XYZ - 3 element array -- equatorial coordinates in meters or feet
   - GD -9 element array -- local east-north-up to equatorial rotation matrix
   - MTFT - units option on XYZ output
     - 1 = meters
     - 2 = feet
3. Inputs: COOR, MTFT
4. Outputs: XYZ, GD

METHOD: See coding.

REMARKS: The Clark spheroid of 1866 is assumed for all computations. Degrees and feet are input and feet or meters are optionally output.

NO. U 16 PROJECT:

TITLE: Earth-centered Coordinates to Geodetic Coordinates

CATEGORY: Utility IDENTIFICATION: Subroutine ECGD

CODE: Fortran 62 CDC - 1604

PROGRAMMER: Dennis Wilson DATE: May 16, 1963

PURPOSE: To use geocentric coordinates of a point to determine the geodetic latitude, longitude and height.

USAGE:

1. Calling Sequence: CALL ECGD (X,Y,Z,SLAT,SLONG,HT)
2. Arguments: X,Y,Z - Location of a point in geocentric coordinates. X-axis is in the equitorial plane and is through the prime meridian; the Z-axis is along the minor axis of the geode and passes through the north pole; Y is chosen to form a right-handed system.

   SLAT, SLONG, HT - The geodetic coordinates of the point. SLAT is the latitude (positive north of equator) in degrees. SLONG is the longitude (west longitude) in degrees. HT is the height above the geode along the local normal in meters.
3. Input: X,Y,Z
4. Output: SLAT, SLONG, HT
5. Routines Called: None

METHOD: The calculation uses the method described on pp.15-16 of ASTIA document #90538. This is an iterative solution for determining latitude and height. Constants for the Clark Spheroid of 1866 were used.

REMARKS: Versions using both Clark and International Spheroids are available.

NO. C0205 PROJECT: General
TITLE: Invert 3x3 Matrix
CATEGORY: Utility; IDENTIFICATION: Subroutine MTXI
CODE: Fortran 62, Fortran 63, CDC-1604, 3600
PROGRAMMER: F.C. Forbes Jr. DATE: January, 1964
PURPOSE: To invert a 3x3 matrix.

USAGE:

1. Calling Sequence: CALL MTXI(A,B,DETERM)
2. Arguments:
   - A(9) - 3x3 matrix
   - B(9) - Inverse matrix of A
   - DETERM - Determinant of A
3. Inputs: A
4. Outputs: B,DETERM
5. Routines Called: None
6. Linkage: None

METHOD:

REMARKS:

NO. CO 114 FCF PROJECT: Fishbowl
TITLE: Compute Empirical Ionospheric Refraction
CATEGORY: General IDENTIFICATION: Subroutine IONO
CODE: Fortran 62, F63 CDC-1604
PROGRAMMER: Fred C. Forbes, Jr. DATE:
PURPOSE:

USAGE:

1. Calling Sequence: CALL IONO(RM,SIN,DRINO,FMAX,CK,FREQ)
2. Arguments:

RM	- Slant Range in meters
SIN	- Sine of Elevation Angle
DRINO	- Range correction in meters
FMAX	- Maximum electron density in the F2 layer
CK	- Control constant
FREQ	- Carrier frequency in mc/sec

3. Inputs: RM,SIN,FMAX,CK,FREQ
4. Outputs: DRINO
5. Routines Called: None

METHOD:

REMARKS: I/O in units of meters with frequency in mc/sec, FMAXx10^{-12}

NO. CO 203 FCF

PROJECT: General

TITLE: Compute Vector Dot Product

CATEGORY: Utility

IDENTIFICATION: Subroutine CCXPD

CODE: Fortran 62, F63 CDC-1604

PROGRAMMER: Fred C. Forbes, Jr.

DATE:

PURPOSE:

USAGE:

1. Calling Sequence: CALL CCXPD (RO,R1,COSV,VR,ROM,RIM)
2. Arguments:

   RO - 3x1 input array
   R1 - 3x1 input array
   ROM - Magnitude of vector RO
   R1M - Magnitude of vector R1
   COSV- Cosine of the angle between RO and R1
   VR - Angle between RO and R1 ($\cos^{-1}$ (COSV))
3. Inputs: RO, R1
4. Outputs: COSV, VR, ROM, R1M
5. Routines Called: None

METHOD:

REMARKS:

NO. CO 307 FCF

PROJECT: Fishbowl

TITLE: Compute Rotation Matrix

CATEGORY: General

IDENTIFICATION: Subroutine VECROT

CODE. Fortran 62, F63 CDC-1604

PROGRAMMER: Fred C. Forbes, Jr.

DATE:

PURPOSE: Given vectors, compute rotation matrix by vector cross product.

USAGE:

1. Calling Sequence: CALL VECROT(RO,R1,VO,V1,GVR)
2. Arguments:

   RO - 3x1 input array
   R1 - 3x1 input array
   VO - 3x1 input array
   V1 - 3x1 input array
   GVR - 3x3 rotation matrix

3. Inputs: RO, R1, VO, V1
4. Outputs: GVR
5. Routines Called: MTXI, MTXP

METHOD:

REMARKS:

NO. C0501 PROJECT: Geodetic SECOR
TITLE: Convert Seconds to Hours, Minutes, and Seconds to Midnight
CATEGORY: General IDENTIFICATION: Subroutine TIMEC
CODE: F-62, 63
PROGRAMMER: F. C. Forbes, Jr. DATE:
PURPOSE: To convert seconds to hours, minutes and seconds
USAGE:

1. Calling Sequence: CALL TIMEC(T_{SEC}, HR, T_{MIN}, SEC)
2. Arguments:

   T_{SEC} - input time in seconds
   HR - hours
   T_{MIN} - minutes
   SEC - seconds

3. Inputs: T_{SEC}
4. Outputs: HR, T_{MIN}, SEC
5. Routines Called: none
6. Linkage: none

METHOD:

REMARKS:

FCF/kkb

NO. 414 PROJECT: FISHBOWL

TITLE: Predict Two-Body Position, Velocity, Partial Derivatives

CATEGORY: Trajectory IDENTIFICATION: Subroutine TBPVS

CODE: Fortran 62, 63 CDC 1604, 3600

PROGRAMMER: F. C. Forbes, Jr. DATE: 1-10-62

PURPOSE: Predict two-body position and velocity vectors and optionally the (3x6) matrix of two-body partial derivatives with respect to position expressed in units of feet, feet/second, and in an equatorial coordinate system.

USAGE:

1. Calling Sequence: CALL TBPVS (TO, TM, VEC, PTL, NOTE)
2. Arguments:

INPUTS: TO = time of epoch or injection in seconds
TM = time of forward predicted data in seconds

OUTPUTS: VEC(6) = array of position and velocity components in equatorial coordinates and in feet and feet/second.

$$PTL(18) = \begin{bmatrix} \frac{\partial X}{\partial X_o} & \frac{\partial X}{\partial Y_o} & \cdots & \frac{\partial X}{\partial \dot{Z}_o} \\ \frac{\partial Y}{\partial X_o} & \frac{\partial Y}{\partial Y_o} & \cdots & \cdot \\ \frac{\partial Z}{\partial X_o} & \cdot & \cdots & \frac{\partial Z}{\partial \dot{Z}_o} \end{bmatrix}$$ = partial derivatives of predicted position vector with respect to injection vectors $R_o V_o$

NOTE = option indicator
1 = compute VEC only
2 = compute VEC and PTL

3. Routines Called: MTXM, MTXT
4. Linkage: Subroutine TRJK or TRJKX
Common/TRJKS/CU(3), PI, WE, RO, PO(6), AI, AIS, AAIS, GAMC, BATO, ECC, EO, DBC(6), DAI(6), DCC(6)

METHOD: Iterate Kepler's equation based on the time of prediction to compute the change in the eccentric anomaly. Integration constants are then computed and used to form the predicted quantities.

REMARKS: This routine is used in conjunction with subroutine TRJK or TRJKX, which computes all necessary constants used in TBPVS. TBPVS is used in trajectory fitting, error propagation, and simulation programs.

NO. CO 406 FCF

PROJECT: Fishbowl

TITLE: Compute Two-Body Partials and Vectors

CATEGORY: Trajectory

IDENTIFICATION: Subroutine TBPV

CODE: Fortran 62, F63 CDC-1604

PROGRAMMER: Fred C. Forbes, Jr.

DATE:

PURPOSE:

USAGE:

1. Calling Sequence. CALL TBPV(TO,TM,VEC,P,NOTE)
2. Arguments:

   TO - Time of epoch or injection in seconds
   TM - Time of forward predicted data in seconds
   VEC - Array of position and velocity components
   NOTE - Option indicator: 1 = Compute VEC only
   2 = Compute VEC and P

   P(18)- a 3x3 array:

$$\begin{bmatrix} \frac{\partial X}{\partial X_o} & \frac{\partial X}{\partial Y_o} & \cdots\cdots & \frac{\partial X}{\partial Z_o} \\ \frac{\partial Y}{\partial X_o} & \frac{\partial Y}{\partial Y_o} & \cdots\cdots & \\ \frac{\partial Z}{\partial X_o} & \cdots\cdots & \cdots\cdots & \frac{\partial Z}{\partial Z_o} \end{bmatrix}$$

3. Inputs: TO,TM,NOTE
4. Outputs: VEC, P
5. Routines Called: MTXT, MTXP
6. Linkage: COMMON/TRJKS/CU(3),PI,WE,RO,PO(6),AI,AIS,AAIS,GAMO,BATO, ECC,EO,DBO(6),DAI(6),DGO(6)

METHOD:

REMARKS:

NO. C0405 PROJECT: Geodetic SECOR

TITLE: Computation of Trajectory Constants

CATEGORY: Utility IDENTIFICATION: Subroutine TRJK

CODE: Fortran 62 CDC-1604 DATE: June, 1964

PROGRAMMER: Fred C. Forbes

PURPOSE: To compute trajectory constants and/or two body partials given position and velocity vectors in equatorial coordinates.

USAGE:

1. Calling Sequence: CALL TRJK (RE, VE)
2. Arguments:

   RE(3),
   VE(3) - Position and velocity of a vehicle in free-fall expressed in equatorial coordinates
   CU(3) - Canonical units for length, velocity, and time
   PIE - Pi
   WE - Earth's rotational velocity (radians/sec)
   RO(3),
   VO(3) - RE, VE represented in inertial coordinates and in canonical units
   EC - Eccentricity of ellipse = e
   EO - Eccentric anomaly at initial time = E_o
   AAIS - Mean motion = $1/a^{3/2}$
   AI - $1/a$
   AIS - $1/a^{1/2}$
   GAMO - e sin E_o
   BATO - e cos E_o
   DBO(6),
   BAI(6),
   DGO(6) - Two body partials
3. Inputs: RE, VE
4. Outputs: CU(3), PIE, WE, RO(3), VO(3), AI, AIS, AAIS, GAMO, BATO, EC, EO, DBO(6), DAI(6), DGO(6)
5. Routines Called: QUTS
6. Linkage: COMMON/TRJKS/CU(3), PIE, WE, ROM, RO(3), VO(3), AI, AIS, AAIS, GAMO, BATO, EC, EO, DBO(6), DAI(6), DGO(6)

METHOD:

REMARKS: Units are in feet and seconds.

NO. CO206 PROJECT: General
TITLE: Invert Ortho 6x6 Matrix by Partitioning
CATEGORY: Utility IDENTIFICATION: Subroutine INSS
CODE: Fortran CDC-1604
PROGRAMMER: F.C. Forbes, Jr. DATE: January, 1964
PURPOSE: To invert an orthogonal 6x6 matrix.

USAGE:

1. Calling Sequence: CALL INSS (A,B)
2. Arguments:
   A(6) - 6x6 input matrix
   B(6) - Inverse matrix of A
3. Inputs: A
4. Outputs: B
5. Routines Called: MTXT, MTXI, MTXP, MTXS
6. Linkage:

METHOD:

REMARKS:

NO. CO 417 FCF PROJECT: Geodetic SECOR

TITLE: Adjust RO and VO to Position and Velocity

CATEGORY: Trajectory IDENTIFICATION: Subroutine PVTRJF

CODE: Fortran 62, F63

PROGRAMMER: Fred C. Forbes, Jr. DATE:

PURPOSE:

USAGE:

1. Calling Sequence: CALL PVTRJF(TO,RO,VO,NO,TM,TRJP,ITER,KP,DTI)
2. Arguments:

TO	- Time of epoch or injection in seconds
RO	- Adjusted range injection vector
VO	- Adjusted velocity injection vector
NO	- Number of input position vectors to be fit by iterative least squares NO < 200
ITER	- Number of iterations
DTI	- Time interval (seconds)
TRJP	- Latitude, longitude and height above mean sea level of vehicle
TM	- Time of forward predicted data in seconds

3. Inputs: TO, NO, ITER, DTI, TRJP, TM
4. Outputs: RO, VO
5. Routines Called: TRJK, TBWPT, TBPV, MTXT, MTXP, MTXA, INSS

METHOD:

REMARKS:

NO. TP001 PROJECT: Bluerock

TITLE: Compute Air Density at Altitude (FT)

CATEGORY: Impact Prediction IDENTIFICATION: Subroutine ADEN

CODE: Fortran 63 CDC-1604

PROGRAMMER: L. BrucePalmer DATE: June 1964

PURPOSE: To compute the air density at a given altitude.

USAGE:

1. Calling Sequence: CALL ADEN(H,X)
2. Arguments:

   H = Height above sea level (FT)

   X = Air density (lb/ft^3)

   CO = Table of four coefficients for six third degree polynomials
3. Inputs: H,CO
4. Outputs: X
5. Routines Called: None
6. Linkage: COMMON/PERT/FMD(20,2),NO,FMASS,EA,CO(4,6)

METHOD: Evaluation of one of six predetermined third degree polynomial curve fit todata, depending upon height.

REMARKS:

NO. TP003 PROJECT: Bluerock
TITLE: Sound Velocity at Altitude (FT)
CATEGORY: Impact Prediction IDENTIFICATION: Subroutine ACVEL
CODE: Fortran 62 CDC - 1604
PROGRAMMER: L. Bruce Palmer DATE: June, 1964
PURPOSE: To compute the velocity of sound at a given altitude.

USAGE:

1. Calling Sequence: CALL ACVEL (H,X)
2. Arguments:
   H = Height above sea level (FT)
   X = Acoustical velocity (Ft/Sec)
3. Inputs: H
4. Outputs: X
5. Routines Called: None
6. Linkage: None

METHOD: Evaluation of predetermined curve fit to values.

REMARKS:

NO. TP002 PROJECT: Bluerock

TITLE: Interpolate for Drag Coefficient

CATEGORY: Impact Prediction IDENTIFICATION: Subroutine DRAG

CODE: Fortran 62 CDC-1604

PROGRAMMER: L.Bruce Palmer DATE: June 1964

PURPOSE: To determine the drag coefficient for a given mach speed.

USAGE:

1. Calling Sequence: CALL DRAG(DCOEF ,FMACH)
2. Arguments:
   - DCOEF - Drag coefficient
   - FMACH - Mach speed
   - FMD - Table of drag coefficients vs. Mach speed
   - NO - Number of coefficients in FMD
3. Inputs: FMACH, FMD, NO
4. Outputs: DCOEF
5. Routines Called: None
6. Linkage: COMMON/PERT/FMD(20,2),NO,FMASS,EA,CO(4,6)

METHOD: Table look-up.

REMARKS:

NO. 416 PROJECT: General

TITLE: Compute Precise Trajectories and Orbits from fit to Equatorial Coordinates and Velocity

IDENTIFICATION: Program PVCMPT

CATEGORY: Trajectory

CODE: Fortran 62 CDC-1604

DATE: 2-1-64

PROGRAMMER: Fred C. Forbes, Jr.

PURPOSE: Compute precise trajectories and orbits with compensation for 2nd, 3rd and 4th zonal harmonics of the earth, atmospheric drag, and lift. Optional initial conditions of time, position and velocity vectors or two positions and apogee height are provided as well as provision for iterative least squares fit (with or without weighting) to the vehicle position coordinates. Equatorial coordinates are listed and topocentric coordinates are computed and listed with respect to any number of sites on the earth's surface. The final orbital parameters are adjusted to input equatorial coordinates and velocity.

USAGE:

	CARDS	DESCRIPTION	FORMAT
Inputs:	(1)	Title	A 80 Hollerith
	(2)	IOPTN,NO,ITER,KP,KWT,IDTI,IUNITS*	5I3,I5,I3
		IOPTN = initial conditions option 001 - time with two positions and apogee height 002 - time with equatorial position and velocity vectors	
		NO = number of input position vectors to be fit by iterative least squares NO 200	
		ITER = number of iterations of the fit	
		KP = perturbation option indicator 001 - drag and lift 002 - no drag and lift	
		KWT = least squares weighting option 001 - input weighting 002 - no weighting	
		IDTI = maximum interval between rectification in perturbation computation times 10	
Option 1	(3)	TI = time of epoch	E17.10
Option 1	(4)	CO = latitude, longitude, height of epoch	3(E17.10,3X)
Option 1	(5)	CI = latitude, longitude, height of second point on trajectory	3(E17.10,3X)
Option 1	(6)	AL = apogee height (AL is located between CO and CL).	E17.10
Option 2	(3)	TI,RE=time and position vector at epoch where RE is the equatorial coordinates	4E20.10

*IUNITS = units code of input injection vectors(see(33) for code ID).

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Option 2 (4) VE = equatorial velocity components at epoch — 3 E20.10

KP=001 (7)-(16) FMD = table of mach speed versus drag coefficients — 4(E17.10,3x)

KP=001 (17)-(23) table of curve fitting coefficients — 4(17.10,3x)

KP=001 (24) NOT, FMASS, EA — I20,2(E17.10,3x)

NOT = number of coefficients in table
FMASS = vehicle mass in pounds
EA = vehicle cross sectional area in ft^2

(25) TM, TRJP = trajectory points for fitting — 4 E20.10

TM = fime in seconds
TRJP = X_E, Y_E, Z_E of vehicle, $\dot{X}_E$, $\dot{Y}_E$, $\dot{Z}_E$ of vehicle

(26) Same as 25 until NO (see item 2) are input

KWT=001 (27) COSITE = latitude, longitude, height of local origin for error propagation — 3(E17.10,3x)

KWT=001 (28) IEQU, SITE — I3,X17, 3(E17.10,3x)

IEQU = equipment selection code
001 = range
002 = height
003 = L direction cosine
004 = M direction cosine
005 = azimuth
006 = elevation

SITE = latitude, longitude, and height of equipment site

KWT=001 (29) EM, ICONT = error model — 5 E17.8,I5

EM(1) = equipment error
EM(2) = tropospheric refraction
EM(3) = scale factor
EM(4) = site survey
EM(5) = ionospheric refraction for range and height equipment
EM(5) = baseline length for L and M

ICONT = continuation code
ICONT(0)= continue input of EM
ICONT(1)= indicates last site input

KWT=001 (30), (31) Same as (28) and (29) for all tracking equipment

(32) TO, DT, TF = times for forward predictions — 3(E17.10,3x)

TO = first time
DT = time interval
TF = final time of prediction ((TF-TO) /DT <1000) per run)

(33) IPUNCH, IUNITS — 2I3

IPUNCH = punch output code
000 = no punch output
001 = punch output

IUNITS = output units code
001 = feet
002 = meters
003 = statute miles (5280')
004 = nautical miles (6076.10333')
005 = yards

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(36) TITLE = site name card — A80 Hollerith

(37) CS = latitude, longitude, height of points on earth to which trajectory points will be referred. — 3(E17.10,3x)

(38), (39) Same as (36) and (37) for all desired sites.

Output: From Fitting:

All input data are dumped for reference. On successive iterations, the adjusted injection vectors, as well as the difference between each predicted point and the corresponding input data are listed. The difference between the two-body prediction of position and velocity and the total field prediction values are listed. The error propagation (inverse weighting) in the XYZ coordinates are also listed when weighting is called out.

In Equatorial Coordinates:

Point number, hours, minutes, seconds, total seconds, XYZ, $\dot{X}\dot{Y}\dot{Z}$, latitude, longitude, height, and associated titles.

In Topocentric Coordinates:

Point number, hours, minutes, seconds, XYZ, $\dot{X}\dot{Y}\dot{Z}$, in local east-north-up coordinates, slant range, surface range, range rate, azimuth, elevation angle, and height AMSL with titles.

Method: Trajectory predictions are computed by a method based on computing a reference trajectory with closed form two-body equations and then adding numerically integrated perturbation terms.

In the least squares fitting, unity weighting or weighting based on the propagation of major sources of observational error into position* variance and covariance, are optional. See program GENEPQ No. 617 FCE, for method of error propagation.

* and velocity

Remarks: All input is expressed in feet, seconds, degrees, and ratios. In the error propagation refraction is input as the ratio of the residual error to the total (mean atmosphere) correction (i.e., 5% residual would be input as 0.05). Scale factor, baseline length, and site survey are ratios. An error in site survey of 1 PNM would be input as 0.000001.

If punch output is desired, the source deck should be modified to give the desired output.

Reference should be made to the program listings for all necessary subroutines and memory usage.